Electromagnetics:
Electromagnetic Field Theory

Electromagnetic Wave Polarization

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Lecture Outline

- Definition of Electromagnetic Wave Polarization
- Linear, Circular and Elliptical Polarization
- Combinations of Linear and Circular Polarization
- Poincaré Sphere
- Polarization Explicit Form
- Be Careful About Conventions

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Definition of Electromagnetic Wave Polarization

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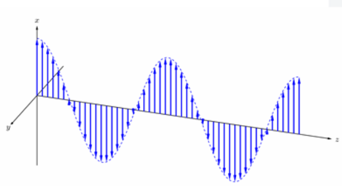
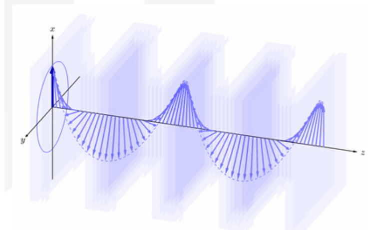
What is Polarization?

Polarization is that property of an electromagnetic wave which describes the time-varying direction and relative magnitude of the electric field.

$$\vec{E}(\vec{r}) = \vec{P} e^{-j\vec{k}\bullet\vec{r}}$$

Polarization Vector





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polarization: (of an electromagnetic wave). The locus of the tip of the electric field vector observed over time at a fixed point in space in a plane orthogonal to the wave normal. *See also*: **circularly polarized wave; elliptically polarized wave; linearly polarized wave; parallel polarization; perpendicular polarization**.

NOTE 1—Elliptical polarization is the most general case.

NOTE 2—The polarization of an electromagnetic wave is defined by the tilt angle, the axial ratio, and the sign of the axial ratio, which expresses the sense of rotation of the polarization ellipse.

linearly polarized wave: An electromagnetic wave for which the locus of the tip of the electric field vector is a straight line in a plane orthogonal to the wave normal.

circularly polarized wave: An electromagnetic wave for which the locus of the tip of the instantaneous electric field vector is a circle in a plane orthogonal to the wave normal. This circle is traced at a rate equal to the angular frequency of the wave with a left-hand or right-hand sense of rotation. *See also:* **left-hand circularly polarized (LHCP) wave**; **right-hand circularly polarized (RHCP) wave**.

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Why is Polarization Important?

- Different polarizations can behave differently in a device
- Orthogonal polarizations will not interfere with each other
- Polarization becomes critical when analyzing devices on the scale of a wavelength
- Focusing properties of lenses can depend on polarization
- Reflection/transmission can depend on polarization
- Frequency of resonators can depend on polarization
- Cutoff conditions for filters, waveguides, etc., can depend on polarization

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Linear Polarization (LP)

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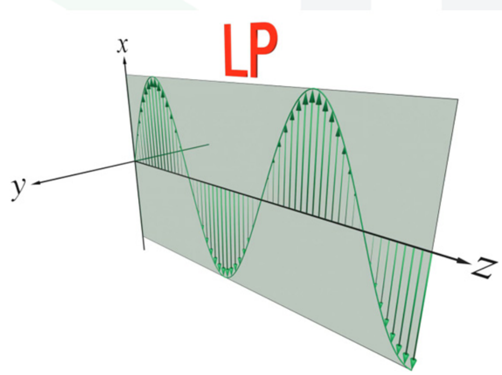
Linearly Polarized Wave: An electromagnetic wave for which the locus of the tip of the electric field vector is a straight line in a plane orthogonal to the wave normal.



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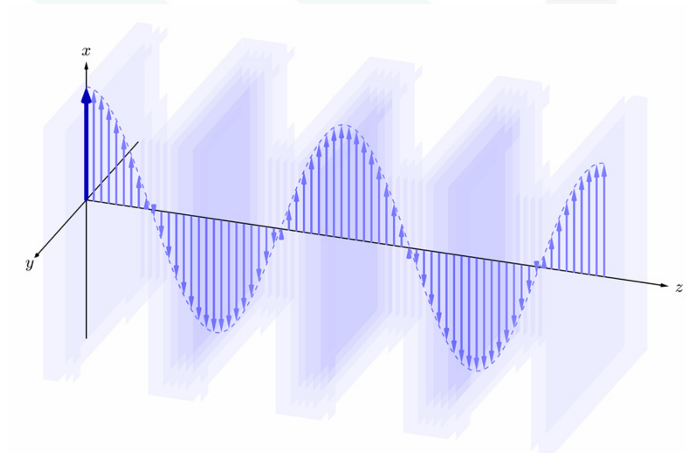
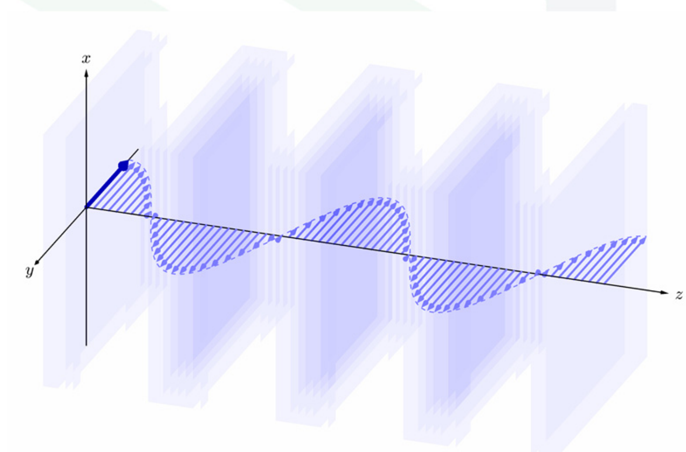
Wave Linearly Polarized Along x

$$\vec{E} = \left(E_0 \hat{a}_x\right) e^{-jkz}$$



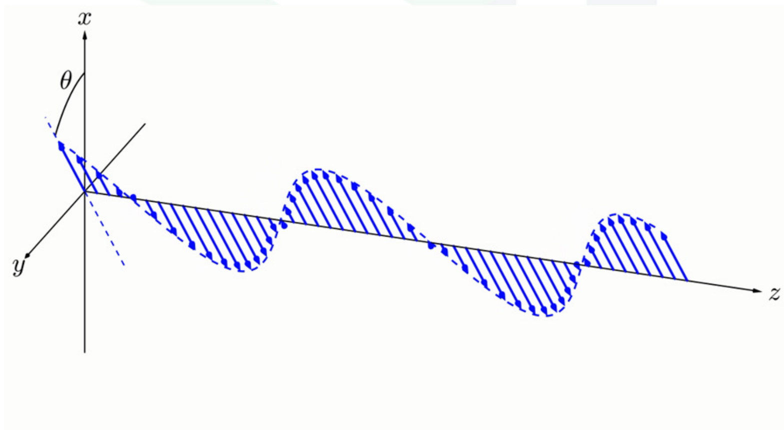
Wave Linearly Polarized Along y

$$\vec{E} = \left(E_0 \hat{a}_y\right) e^{-jkz}$$



Wave Linearly Polarized with Tilt θ

$$\vec{E} = E_0\left(\cos\theta\hat{a}_x + \sin\theta\hat{a}_y\right)e^{-jkz}$$



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Circular Polarization

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left-hand circularly polarized (LHCP) wave: A circularly or an elliptically polarized electromagnetic wave for which the electric field vector, when viewed with the wave approaching the observer, rotates clockwise in space.

NOTE 1—This definition is consistent with observing a counterclockwise rotation when the electric field vector is viewed in the direction of propagation.

right-hand circularly polarized (RHCP) wave: A circularly or an elliptically polarized electromagnetic wave for which the electric field vector, when viewed with the wave approaching the observer, rotates counter-clockwise in space.

NOTE 1—This definition is consistent with observing a clockwise rotation when the electric field vector is viewed in the direction of propagation.

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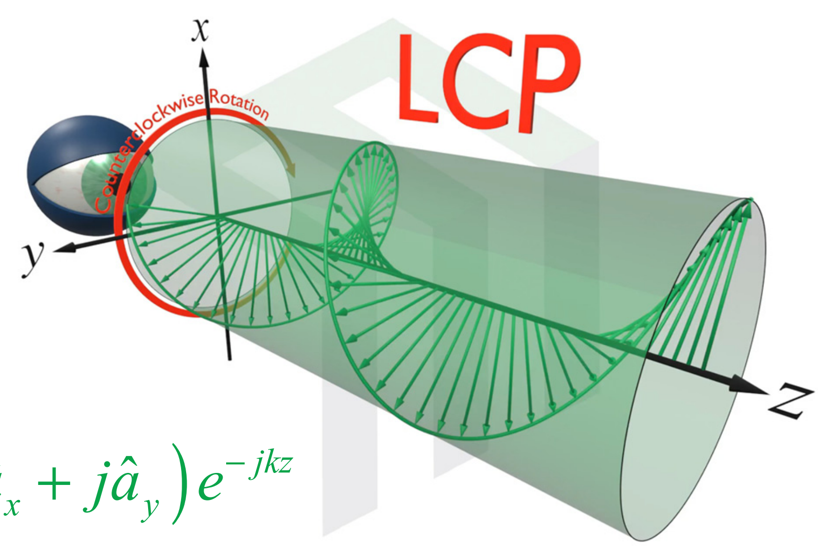
Lefthand Circular Polarization (LCP)



$$\vec{E} = E_0\left(\hat{a}_x + j\hat{a}_y\right)e^{-jkz}$$

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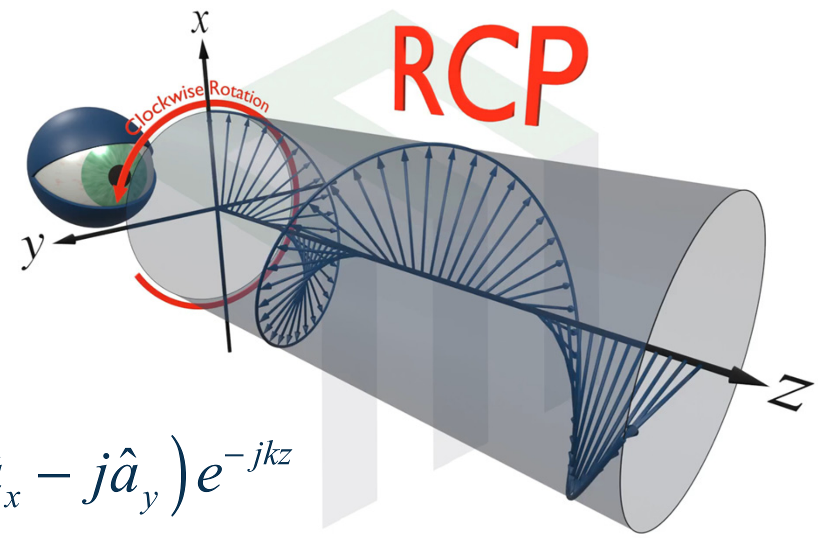
Righthand Circular Polarization (RCP)



$$\vec{E} = E_0\left(\hat{a}_x - j\hat{a}_y\right)e^{-jkz}$$

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Elliptical Polarization

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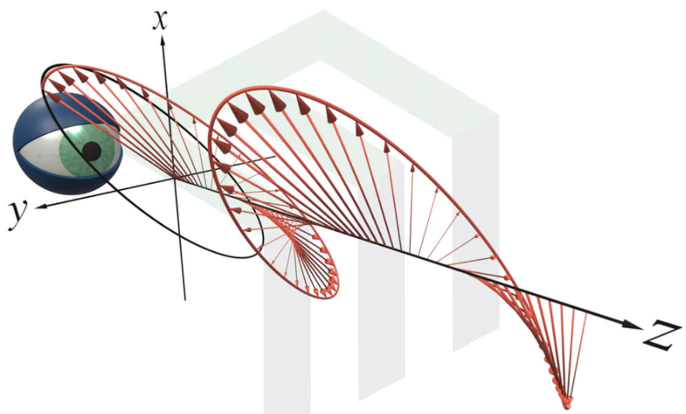
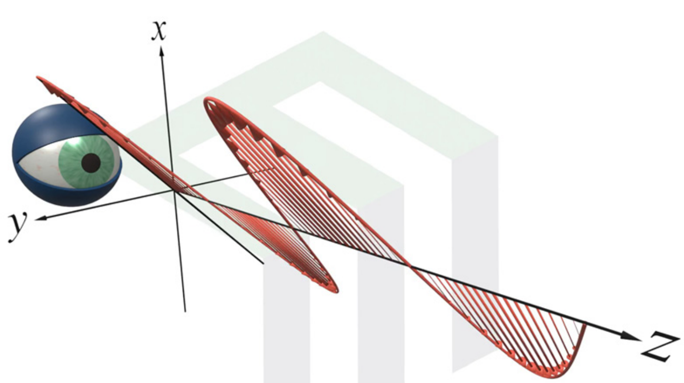
Elliptical Polarization (EP)

An electromagnetic wave has elliptical polarization if the electric field rotates with time to form an ellipse.



Continuum of Elliptical Polarizations

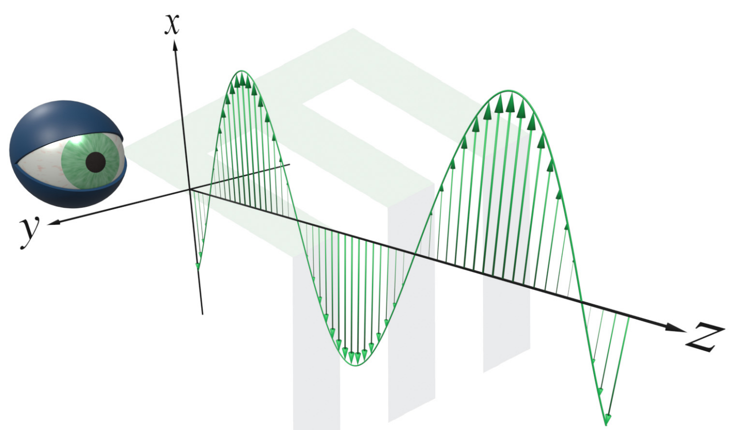
Observe that *linear polarization* and *circular polarization* are just special cases of *elliptical polarization*. In some sense, everything is elliptically polarized.



Combinations of Linear and Circular Polarization

Two Linear Polarizations Can Give Circular Polarization

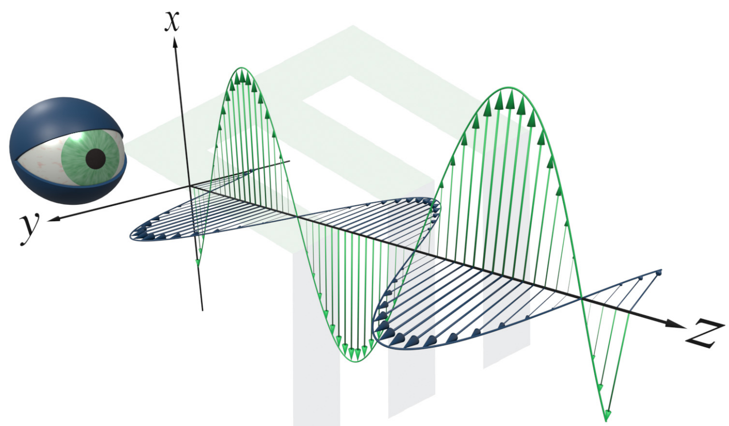
Let there be a first LP wave in the x direction.



$$\vec{E}_1 = \left(E_0 \hat{a}_x\right) e^{-jkz}$$

Two Linear Polarizations Can Give Circular Polarization

Let there be a second LP wave in the y direction.



$$\vec{E}_1 = \left(E_0 \hat{a}_x\right) e^{-jkz}$$

$$\vec{E}_2 = \left(E_0 \hat{a}_y\right) e^{-jkz}$$

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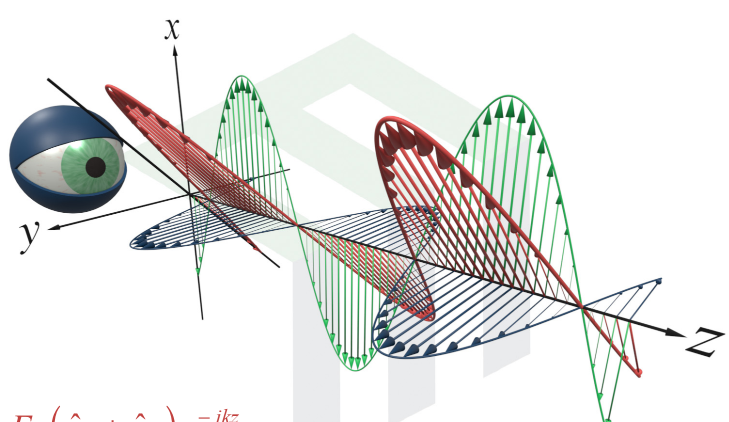
Two Linear Polarizations Can Give Circular Polarization

If the two LP waves are added, they form a composite LP wave at 45°.



$$\vec{E}_1 = \left(E_0 \hat{a}_x\right) e^{-jkz}$$

$$\vec{E}_2 = \left(E_0 \hat{a}_y\right) e^{-jkz}$$

$$\vec{E}_{1+2} = E_0 \left(\hat{a}_x + \hat{a}_y\right) e^{-jkz}$$

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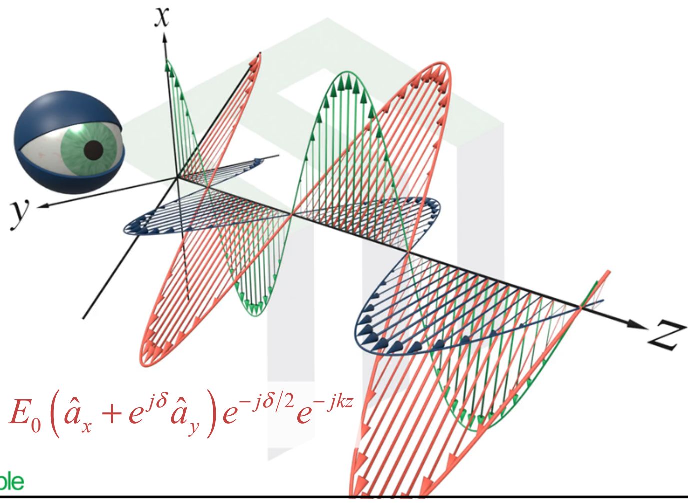
Two Linear Polarizations Can Give Circular Polarization

If the phase of the LP waves is adjusted, a continuum of polarizations results.



$$\vec{E}_1 = \left(E_0 e^{-j\delta/2}\hat{a}_x\right)e^{-jkz}$$

$$\vec{E}_2 = \left(E_0 e^{+j\delta/2}\hat{a}_y\right)e^{-jkz}$$

$$\vec{E}_{1+2} = E_0\left(\hat{a}_x + e^{j\delta}\hat{a}_y\right)e^{-j\delta/2}e^{-jkz}$$

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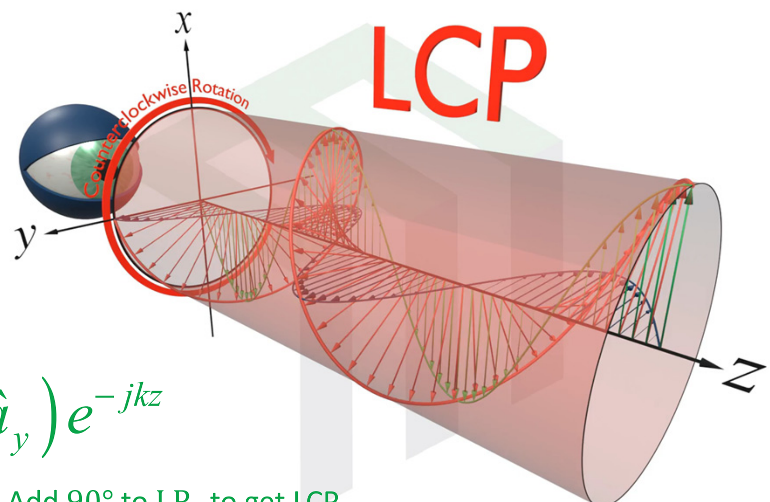
$\mathrm{LP}_x + \mathrm{LP}_y(+90°) = \mathrm{LCP}$

Two orthogonal LP waves will form a CP wave if they have equal amplitude and are $\pm 90°$ out of phase.



$$\vec{E}_{\mathrm{LCP}} = E_0\left(\hat{a}_x + j\hat{a}_y\right)e^{-jkz}$$

Add 90° to LP_y to get LCP.

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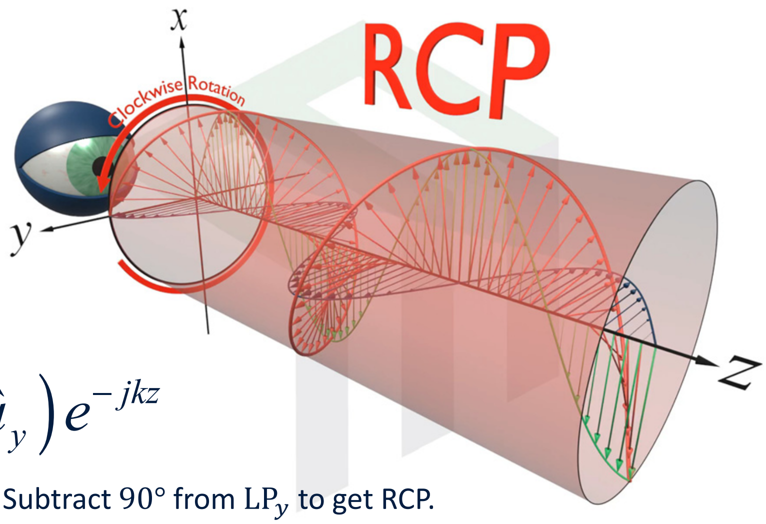
$\mathrm{LP}_x + \mathrm{LP}_y(-90°) = \mathrm{RCP}$

Two orthogonal LP waves will form a CP wave if they have equal amplitude and are $\pm 90°$ out of phase.



$$\vec{E}_{\mathrm{RCP}} = E_0 \left(\hat{a}_x - j\hat{a}_y \right) e^{-jkz}$$

Subtract 90° from LP_y to get RCP.

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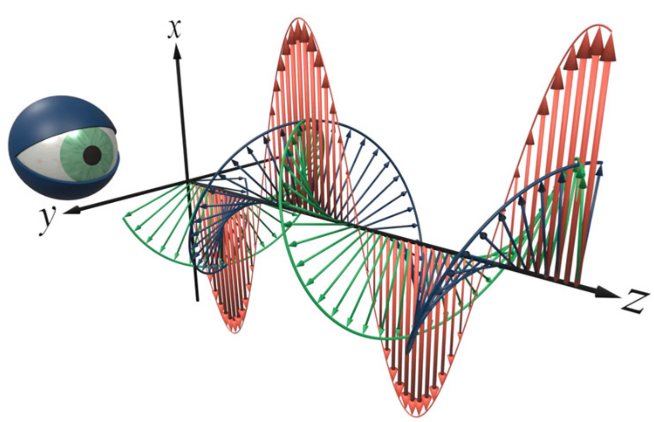
$\mathrm{LCP} + \mathrm{RCP}(\theta) = \mathrm{LP}$ with tilt $\theta/2$

As the phase difference θ between the LCP and RCP waves changes, the resulting polarization is LP with tilt angle θ.



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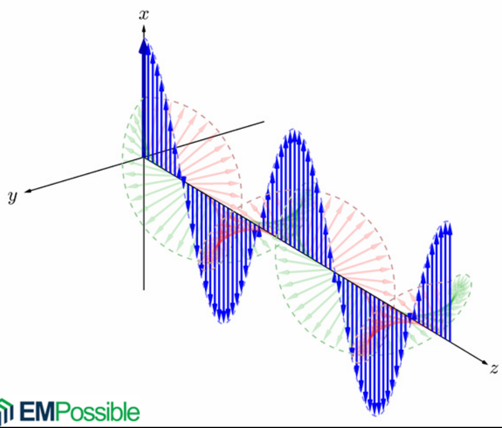
Derivation of LCP + RCP = LP

$$\cos\alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2} \qquad \sin\alpha = \frac{e^{j\alpha} - e^{-j\alpha}}{2j}$$



Start by adding an LCP and RCP wave.

$$E_0\left(\hat{a}_x + j\hat{a}_y\right)e^{-jkz} + E_0\left(\hat{a}_x - j\hat{a}_y\right)e^{j\theta}e^{-jkz}$$

Collect common vector components.

$$E_0\left[\left(1 + e^{j\theta}\right)\hat{a}_x + j\left(1 - e^{j\theta}\right)\hat{a}_y\right]e^{-jkz}$$

Factor out an $e^{j\theta/2}$ term.

$$E_0\left[\left(e^{-j\frac{\theta}{2}} + e^{j\frac{\theta}{2}}\right)\hat{a}_x + j\left(e^{-j\frac{\theta}{2}} - e^{j\frac{\theta}{2}}\right)\hat{a}_y\right]e^{j\frac{\theta}{2}}e^{-jkz}$$

Apply Euler's identities.

$$2E_0\left[\cos\left(\frac{\theta}{2}\right)\hat{a}_x + \sin\left(\frac{\theta}{2}\right)\hat{a}_y\right]e^{j\frac{\theta}{2}}e^{-jkz}$$

This is linear polarization (LP) because all components are in phase.

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Poincaré Sphere

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Poincaré Sphere

Any polarization of a wave can be mapped to a unique point on the Poincaré sphere.

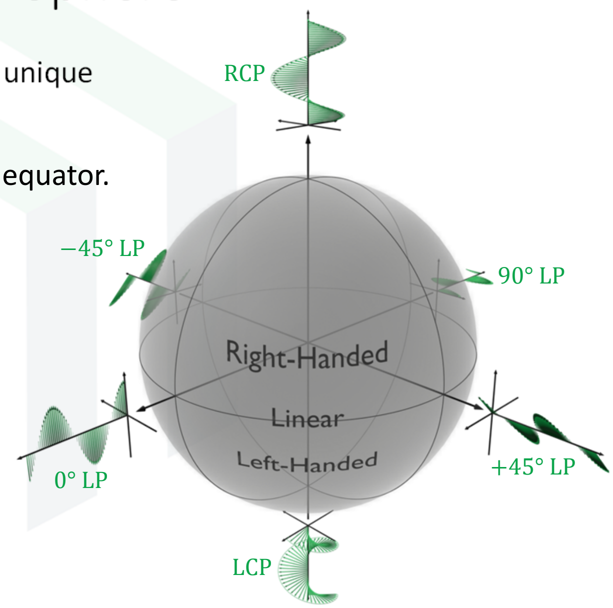
All *linear polarizations* are mapped around the equator.

Circular polarizations are at the poles.

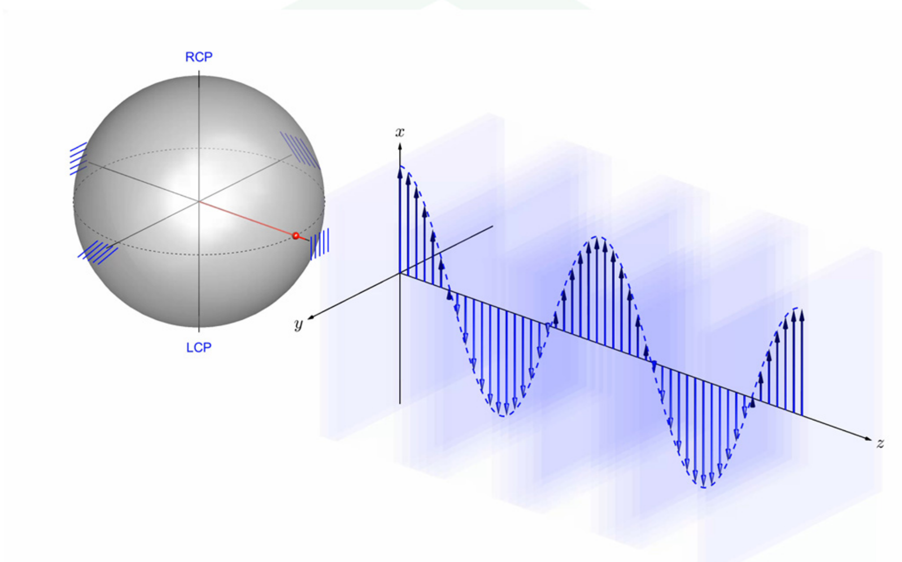
Northern hemisphere is all *righthanded polarizations*. Southern hemisphere is all *lefthanded polarizations*.

Everywhere else are *elliptical polarizations*.

Points on opposite sides of the sphere are orthogonal.



Continuum of All Polarizations



Polarization Explicit Form

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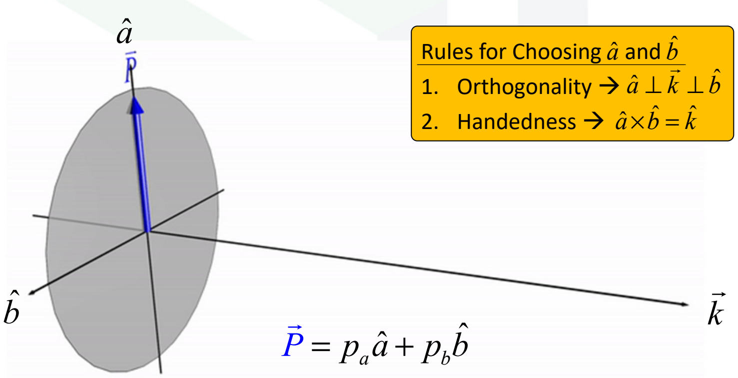
Possibilities for Wave Polarization

Recall that $\vec{E} \perp \vec{k}$ so the polarization vector $\vec{P}$ must fall within the plane perpendicular to $\vec{k}$.

The polarization can be decomposed into two orthogonal directions, $\hat{a}$ and $\hat{b}$.



Rules for Choosing $\hat{a}$ and $\hat{b}$

1. Orthogonality → $\hat{a} \perp \vec{k} \perp \hat{b}$
2. Handedness → $\hat{a} \times \hat{b} = \hat{k}$

$$\vec{P} = p_a \hat{a} + p_b \hat{b}$$

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Explicit Form to Convey Polarization

The expression for the electromagnetic wave can now be written as

$$\vec{E}(\vec{r}) = \vec{P}e^{-j\vec{k}\bullet\vec{r}} = \left(p_a\hat{a} + p_b\hat{b}\right)e^{-j\vec{k}\bullet\vec{r}}$$

p_a and p_b are in general complex numbers to convey the amplitude and phase of each of these components.

$$p_a = E_a e^{j\phi_a} \qquad p_b = E_b e^{j\phi_b}$$

Substituting these expression for p_a and p_b into the wave expression gives

$$\vec{E}(\vec{r}) = \left[E_a e^{j\phi_a}\hat{a} + E_b e^{j\phi_b}\hat{b}\right]e^{-j\vec{k}\bullet\vec{r}} = \left[E_a\hat{a} + E_b e^{j(\phi_b-\phi_a)}\hat{b}\right]e^{j\phi_a}e^{-j\vec{k}\bullet\vec{r}}$$

Interpret $\phi_b - \phi_a$ as the phase difference between p_a and p_b.

$$\delta = \phi_b - \phi_a$$

Interpret ϕ_a as the phase common to both p_a and p_b.

$$\theta = \phi_a$$

The final expression is: $\vec{E}(\vec{r}) = \left(E_a\hat{a} + E_b e^{j\delta}\hat{b}\right)e^{j\theta}e^{-j\vec{k}\bullet\vec{r}}$

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Determining Polarization of a Wave

To determine polarization, it is most convenient to write the expression for the wave that makes polarization explicit.

$$\vec{E}(\vec{r}) = \left(E_a\hat{a} + E_b e^{j\delta}\hat{b}\right)e^{j\theta}e^{-j\vec{k}\bullet\vec{r}}$$

$E_a \equiv$ amplitude along $\hat{a}$
$E_b \equiv$ amplitude along $\hat{b}$
$\delta \equiv$ phase difference
$\theta \equiv$ common phase

Given E_a, E_b, and δ the following table can be used to identify the polarization of the wave…

Polarization Designation	Mathematical Definition
Linear Polarization (LP)	$\delta = 0°$
Circular Polarization (CP)	$\delta = \pm 90°$ and $E_a = E_b$
Right-Hand CP (RCP)	$\delta = -90°$ and $E_a = E_b$
Left-Hand CP (LCP)	$\delta = +90°$ and $E_a = E_b$
Elliptical Polarization	Everything else

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Be Careful About Conventions

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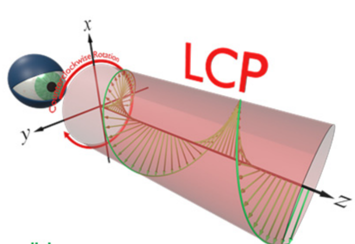
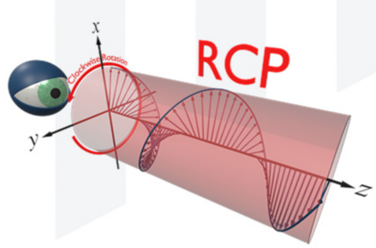
Conventions Used in This Course

The *negative sign convention* defines a wave travelling in the $+z$ direction to be written as

$$\cos(\omega t - kz) \qquad\qquad e^{-jkz}$$

IEEE defines LCP as counterclockwise rotation of the electric field at a fixed point as observed from the source. RCP is defined as clockwise rotation of the electric field at a fixed point as observed from the source.





The convention for LCP and RCP is independent of sign convention.

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