

Problem Description

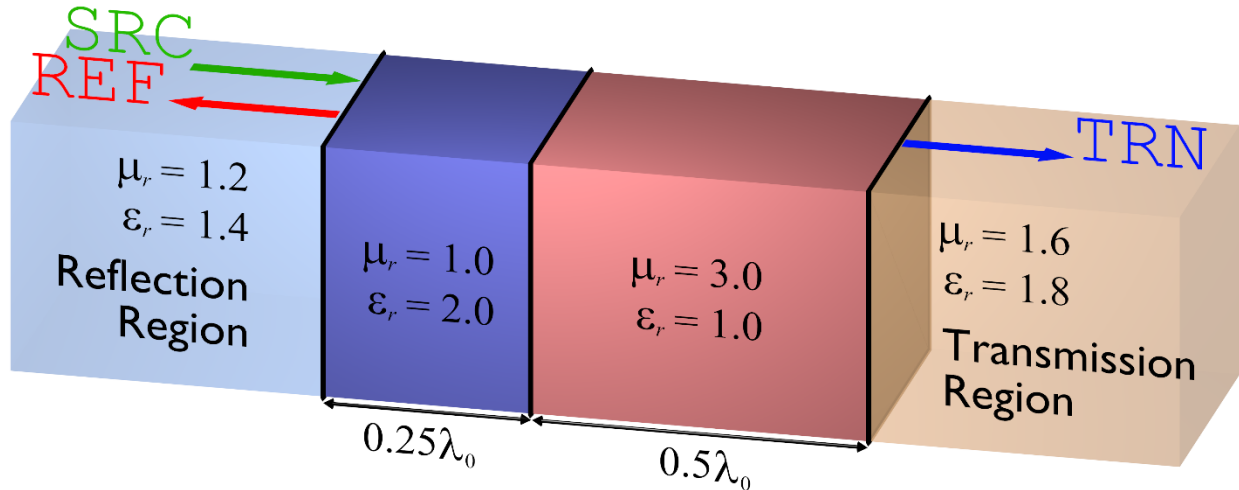
We start with the following parameters:

$$\begin{aligned}
 \lambda_0 &= 2.7 \\
 \theta &= 57^\circ \\
 \phi &= 23^\circ \\
 P_{\text{TE}} &= 1/\sqrt{2} \\
 P_{\text{TM}} &= j/\sqrt{2} \\
 \mu_{r,\text{ref}} &= 1.2 \\
 \varepsilon_{r,\text{ref}} &= 1.4 \\
 \mu_{r,\text{tm}} &= 1.6 \\
 \varepsilon_{r,\text{tm}} &= 1.8
 \end{aligned} \tag{1}$$

The device is composed of two layers with the following parameters each:

$$\text{Layer 1: } \mu_{r,1} = 1.0, \varepsilon_{r,1} = 2.0, L_1 = 0.25\lambda_0 \tag{2}$$

$$\text{Layer 2: } \mu_{r,2} = 3.0, \varepsilon_{r,2} = 1.0, L_2 = 0.5\lambda_0 \tag{3}$$



Step-by-Step Benchmarking Aid

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*****
*                               SETUP                               *
*****
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$$k_0 = 2.3271$$

$$k_x = 1.0006$$

$$k_y = 0.4247$$

$$Q_h = \begin{matrix} 0.4250 & 1.1804 \\ -2.0013 & -0.4250 \end{matrix}$$

$$V_h = \begin{matrix} 0.0000 - 0.4250i & 0.0000 - 1.1804i \\ 0.0000 + 2.0013i & 0.0000 + 0.4250i \end{matrix}$$

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*****
*                               ITERATE THROUGH LAYERS           *
*****
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-----> Layer 1 of 2

$$Q = \begin{matrix} 0.4250 & 0.9987 \\ -1.8196 & -0.4250 \end{matrix}$$

$$k_z = 0.9046$$

$$\text{OMEGA} = \begin{matrix} 0.0000 + 0.9046i & 0.0000 + 0.0000i \\ 0.0000 + 0.0000i & 0.0000 + 0.9046i \end{matrix}$$

$$V = \begin{matrix} 0.0000 - 0.4698i & 0.0000 - 1.1040i \\ 0.0000 + 2.0114i & 0.0000 + 0.4698i \end{matrix}$$

$$X = \begin{matrix} 0.1493 + 0.9888i & 0.0000 + 0.0000i \\ 0.0000 + 0.0000i & 0.1493 + 0.9888i \end{matrix}$$

$$A = \begin{bmatrix} 2.0049 & -0.0427 \\ -0.0427 & 2.0873 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.0049 & 0.0427 \\ 0.0427 & -0.0873 \end{bmatrix}$$

$$D = \begin{bmatrix} 2.0057 - 0.0003i & -0.0445 + 0.0006i \\ -0.0445 + 0.0006i & 2.0916 - 0.0013i \end{bmatrix}$$

$$S11 = \begin{bmatrix} 0.0039 - 0.0006i & -0.0398 + 0.0060i \\ -0.0398 + 0.0060i & 0.0808 - 0.0121i \end{bmatrix}$$

$$S12 = \begin{bmatrix} 0.1490 + 0.9880i & 0.0005 + 0.0017i \\ 0.0005 + 0.0017i & 0.1480 + 0.9848i \end{bmatrix}$$

$$S21 = \begin{bmatrix} 0.1490 + 0.9880i & 0.0005 + 0.0017i \\ 0.0005 + 0.0017i & 0.1480 + 0.9848i \end{bmatrix}$$

$$S22 = \begin{bmatrix} 0.0039 - 0.0006i & -0.0398 + 0.0060i \\ -0.0398 + 0.0060i & 0.0808 - 0.0121i \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} 0.1490 + 0.9880i & 0.0005 + 0.0017i \\ 0.0005 + 0.0017i & 0.1480 + 0.9848i \end{bmatrix}$$

$$SG11 = \begin{bmatrix} 0.0039 - 0.0006i & -0.0398 + 0.0060i \\ -0.0398 + 0.0060i & 0.0808 - 0.0121i \end{bmatrix}$$

$$SG12 = \begin{bmatrix} 0.1490 + 0.9880i & 0.0005 + 0.0017i \\ 0.0005 + 0.0017i & 0.1480 + 0.9848i \end{bmatrix}$$

$$\text{SG21} = \begin{matrix} 0.1490 + 0.9880i & 0.0005 + 0.0017i \\ 0.0005 + 0.0017i & 0.1480 + 0.9848i \end{matrix}$$

$$\text{SG22} = \begin{matrix} 0.0039 - 0.0006i & -0.0398 + 0.0060i \\ -0.0398 + 0.0060i & 0.0808 - 0.0121i \end{matrix}$$

-----> Layer 2 of 2

$$\text{Q} = \begin{matrix} 0.1417 & 0.6662 \\ -0.9399 & -0.1417 \end{matrix}$$

$$\text{kz} = 1.3485$$

$$\text{OMEGA} = \begin{matrix} 0.0000 + 1.3485i & 0.0000 + 0.0000i \\ 0.0000 + 0.0000i & 0.0000 + 1.3485i \end{matrix}$$

$$\text{V} = \begin{matrix} 0.0000 - 0.1051i & 0.0000 - 0.4941i \\ 0.0000 + 0.6970i & 0.0000 + 0.1051i \end{matrix}$$

$$\text{X} = \begin{matrix} -0.4583 - 0.8888i & 0.0000 + 0.0000i \\ 0.0000 + 0.0000i & -0.4583 - 0.8888i \end{matrix}$$

$$\text{A} = \begin{matrix} 3.8324 & 0.2579 \\ 0.2579 & 3.3342 \end{matrix}$$

$$\text{B} = \begin{matrix} -1.8324 & -0.2579 \\ -0.2579 & -1.3342 \end{matrix}$$

$$\text{D} = \begin{matrix} 4.3436 - 0.7182i & 0.3604 - 0.1440i \\ 0.3604 - 0.1440i & 3.6475 - 0.4401i \end{matrix}$$

$$\text{S11} = \begin{matrix} 0.6997 - 0.2262i & 0.0517 - 0.0014i \\ 0.0517 - 0.0014i & 0.5998 - 0.2235i \end{matrix}$$

$$S_{12} = \begin{pmatrix} -0.2093 - 0.6406i & 0.0311 + 0.0390i \\ 0.0311 + 0.0390i & -0.2693 - 0.7160i \end{pmatrix}$$

$$S_{21} = \begin{pmatrix} -0.2093 - 0.6406i & 0.0311 + 0.0390i \\ 0.0311 + 0.0390i & -0.2693 - 0.7160i \end{pmatrix}$$

$$S_{22} = \begin{pmatrix} 0.6997 - 0.2262i & 0.0517 - 0.0014i \\ 0.0517 - 0.0014i & 0.5998 - 0.2235i \end{pmatrix}$$

$$D = \begin{pmatrix} 0.1506 + 0.9886i & -0.0163 - 0.0190i \\ -0.0163 - 0.0190i & 0.1822 + 1.0253i \end{pmatrix}$$

$$F = \begin{pmatrix} -0.2117 - 0.6413i & 0.0471 + 0.0518i \\ 0.0471 + 0.0518i & -0.3027 - 0.7414i \end{pmatrix}$$

$$SG_{11} = \begin{pmatrix} -0.5961 + 0.4214i & -0.0840 + 0.0085i \\ -0.0840 + 0.0085i & -0.4339 + 0.4051i \end{pmatrix}$$

$$SG_{12} = \begin{pmatrix} 0.6020 - 0.3046i & -0.0431 + 0.0534i \\ -0.0431 + 0.0534i & 0.6852 - 0.4078i \end{pmatrix}$$

$$SG_{21} = \begin{pmatrix} 0.6020 - 0.3046i & -0.0431 + 0.0534i \\ -0.0431 + 0.0534i & 0.6852 - 0.4078i \end{pmatrix}$$

$$SG_{22} = \begin{pmatrix} 0.6971 - 0.2216i & 0.0672 - 0.0211i \\ 0.0672 - 0.0211i & 0.5673 - 0.1808i \end{pmatrix}$$

 * EXTERNAL REGIONS *

$$V_{ref} = \begin{pmatrix} 0.0000 - 0.5017i & 0.0000 - 0.8012i \\ 0.0000 + 1.7702i & 0.0000 + 0.5017i \end{pmatrix}$$

$$SR_{11} = \begin{pmatrix} -0.0800 & 0.0761 \\ 0.0761 & -0.2269 \end{pmatrix}$$

$$\text{SR12} = \begin{bmatrix} 1.0800 & -0.0761 \\ -0.0761 & 1.2269 \end{bmatrix}$$

$$\text{SR21} = \begin{bmatrix} 0.9200 & 0.0761 \\ 0.0761 & 0.7731 \end{bmatrix}$$

$$\text{SR22} = \begin{bmatrix} 0.0800 & -0.0761 \\ -0.0761 & 0.2269 \end{bmatrix}$$

$$\text{ST11} = \begin{bmatrix} 0.2060 & 0.0440 \\ 0.0440 & 0.1209 \end{bmatrix}$$

$$\text{ST12} = \begin{bmatrix} 0.7940 & -0.0440 \\ -0.0440 & 0.8791 \end{bmatrix}$$

$$\text{ST21} = \begin{bmatrix} 1.2060 & 0.0440 \\ 0.0440 & 1.1209 \end{bmatrix}$$

$$\text{ST22} = \begin{bmatrix} -0.2060 & -0.0440 \\ -0.0440 & -0.1209 \end{bmatrix}$$

$$\text{SG11} = \begin{bmatrix} -0.6018 + 0.3062i & -0.0043 + 0.0199i \\ -0.0043 + 0.0199i & -0.5935 + 0.2678i \end{bmatrix}$$

$$\text{SG12} = \begin{bmatrix} 0.5766 - 0.3110i & -0.0919 + 0.0469i \\ -0.0919 + 0.0469i & 0.7542 - 0.4016i \end{bmatrix}$$

$$\text{SG21} = \begin{bmatrix} 0.7415 - 0.4007i & 0.0716 - 0.0409i \\ 0.0716 - 0.0409i & 0.6033 - 0.3218i \end{bmatrix}$$

$$\text{SG22} = \begin{bmatrix} 0.5861 - 0.3354i & 0.0170 + 0.0042i \\ 0.0170 + 0.0042i & 0.5533 - 0.3434i \end{bmatrix}$$

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*****  
*                               SOLVE SCATTERING PROBLEM                               *  
*****
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P =
-0.2763 + 0.3545i
0.6509 + 0.1505i
0.0000 - 0.5930i

Esrc =
-0.2763 + 0.3545i
0.6509 + 0.1505i

Eref =
0.0519 - 0.2856i
-0.4324 + 0.0780i

Etrn =
-0.0101 + 0.3577i
0.4358 - 0.0820i

Ezref =
0.1866 + 0.3580i

Eztrn =
-0.1343 - 0.2480i

REF =
0.4403

TRN =
0.5597

CON =

1