

```
*****
***** DIRICHLET BOUNDARY CONDITIONS *****
*****
```

```
GRID SIZE: Nx = 1, Ny = 3
RESOLUTION: dx = 1, dy = 2
BC's: DIRICHLET
```

DX =

```
[ 0 0 0 ]
[ 0 0 0 ]
[ 0 0 0 ]
```

D2X =

```
[ 0 0 0 ]
[ 0 0 0 ]
[ 0 0 0 ]
```

DY =

```
[ 0 0.25 0 ]
[ -0.25 0 0.25 ]
[ 0 -0.25 0 ]
```

D2Y =

```
[ -0.5 0.25 0 ]
[ 0.25 -0.5 0.25 ]
[ 0 0.25 -0.5 ]
```

```
-----
GRID SIZE: Nx = 1, Ny = 5
RESOLUTION: dx = 1, dy = 2
BC's: DIRICHLET
```

DX =

```
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
```

D2X =

```
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
```

DY =

```
[ 0 0.25 0 0 0 ]
[ -0.25 0 0.25 0 0 ]
[ 0 -0.25 0 0.25 0 ]
[ 0 0 -0.25 0 0.25 ]
[ 0 0 0 -0.25 0 ]
```

D2Y =

```
[ -0.5 0.25 0 0 0 ]
[ 0.25 -0.5 0.25 0 0 ]
[ 0 0.25 -0.5 0.25 0 ]
[ 0 0 0.25 -0.5 0.25 ]
[ 0 0 0 0.25 -0.5 ]
```

```
-----
GRID SIZE: Nx = 3, Ny = 1
RESOLUTION: dx = 1, dy = 2
BC's: DIRICHLET
```

DX =

```
[ 0 0.5 0 ]
[ -0.5 0 0.5 ]
[ 0 -0.5 0 ]
```

D2X =

```
[ -2 1 0 ]
[ 1 -2 1 ]
[ 0 1 -2 ]
```

DY =

```
[ 0 0 0 ]
[ 0 0 0 ]
[ 0 0 0 ]
```

D2Y =

```
[ 0 0 0 ]
[ 0 0 0 ]
[ 0 0 0 ]
```

GRID SIZE: Nx = 5, Ny = 1
 RESOLUTION: dx = 1, dy = 2
 BC's: DIRICHLET

DX =

```
[ 0 0.5 0 0 0 ]
[ -0.5 0 0.5 0 0 ]
[ 0 -0.5 0 0.5 0 ]
[ 0 0 -0.5 0 0.5 ]
[ 0 0 0 -0.5 0 ]
```

D2X =

```
[ -2 1 0 0 0 ]
[ 1 -2 1 0 0 ]
[ 0 1 -2 1 0 ]
[ 0 0 1 -2 1 ]
[ 0 0 0 1 -2 ]
```

DY =

```
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
```

D2Y =

```
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
```

GRID SIZE: Nx = 3, Ny = 3
 RESOLUTION: dx = 1, dy = 2
 BC's: DIRICHLET

DX =

```
[ 0 0.5 0 0 0 0 0 0 0 ]
[ -0.5 0 0.5 0 0 0 0 0 0 ]
[ 0 -0.5 0 0 0 0.5 0 0 0 ]
[ 0 0 0 0 0.5 0 0 0 0 ]
[ 0 0 0 -0.5 0 0.5 0 0 0 ]
[ 0 0 0 0 -0.5 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0.5 0 ]
[ 0 0 0 0 0 0 -0.5 0 0.5 ]
[ 0 0 0 0 0 0 0 -0.5 0 ]
```

D2X =

```
[ -2 1 0 0 0 0 0 0 0 ]
[ 1 -2 1 0 0 0 0 0 0 ]
[ 0 1 -2 0 0 0 0 0 0 ]
[ 0 0 0 -2 1 0 0 0 0 ]
[ 0 0 0 1 -2 1 0 0 0 ]
[ 0 0 0 0 1 -2 0 0 0 ]
[ 0 0 0 0 0 0 -2 1 0 ]
[ 0 0 0 0 0 0 0 1 -2 ]
[ 0 0 0 0 0 0 0 0 1 -2 ]
```

Benchmarking Document for FDDER

DY =

```
[ 0 0 0 0.25 0 0 0 0 0 0 ]
[ 0 0 0 0 0.25 0 0 0 0 0 ]
[ 0 0 0 0 0 0.25 0 0 0 0 ]
[ -0.25 0 0 0 0 0 0.25 0 0 0 ]
[ 0 -0.25 0 0 0 0 0 0.25 0 0 ]
[ 0 0 -0.25 0 0 0 0 0 0.25 0 ]
[ 0 0 0 -0.25 0 0 0 0 0 0 ]
[ 0 0 0 0 -0.25 0 0 0 0 0 ]
[ 0 0 0 0 0 -0.25 0 0 0 0 ]
```

D2Y =

```
[ -0.5 0 0 0.25 0 0 0 0 0 ]
[ 0 -0.5 0 0 0.25 0 0 0 0 ]
[ 0 0 -0.5 0 0 0.25 0 0 0 ]
[ 0.25 0 0 -0.5 0 0 0.25 0 0 ]
[ 0 0.25 0 0 -0.5 0 0 0.25 0 ]
[ 0 0 0.25 0 0 -0.5 0 0 0.25 ]
[ 0 0 0 0.25 0 0 -0.5 0 0 ]
[ 0 0 0 0 0.25 0 0 -0.5 0 ]
[ 0 0 0 0 0 0.25 0 0 -0.5 ]
```

 GRID SIZE: Nx = 3, Ny = 5
 RESOLUTION: dx = 1, dy = 2
 BC's: DIRICHLET

DX =

```
[ 0 0.5 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ -0.5 0 0.5 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 -0.5 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0.5 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 -0.5 0 0.5 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 -0.5 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0.5 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 -0.5 0.5 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 -0.5 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0.5 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 -0.5 0.5 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 -0.5 0 0.5 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 -0.5 0 0.5 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 -0.5 0 ]
```

D2X =

```
[ -2 1 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 1 -2 1 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 1 -2 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 -2 1 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 1 -2 1 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 1 -2 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 -2 1 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 1 -2 1 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 -2 1 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 1 -2 1 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 -2 1 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 1 -2 ]
```

DY =

```
[ 0 0 0 0.25 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0.25 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0.25 0 0 0 0 0 0 0 0 ]
[ -0.25 0 0 0 0 0 0.25 0 0 0 0 0 0 0 ]
[ 0 -0.25 0 0 0 0 0 0.25 0 0 0 0 0 0 ]
[ 0 0 -0.25 0 0 0 0 0 0.25 0 0 0 0 0 ]
[ 0 0 0 -0.25 0 0 0 0 0 0.25 0 0 0 0 ]
[ 0 0 0 0 -0.25 0 0 0 0 0 0.25 0 0 0 ]
[ 0 0 0 0 0 -0.25 0 0 0 0 0 0.25 0 0 ]
[ 0 0 0 0 0 0 -0.25 0 0 0 0 0 0.25 0 ]
[ 0 0 0 0 0 0 0 -0.25 0 0 0 0 0 0.25 ]
[ 0 0 0 0 0 0 0 0 -0.25 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 -0.25 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 -0.25 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 -0.25 0 0 ]
```

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D2Y =

```
[ -0.5  0  0  0.25  0  0  0  0  0  0  0  0  0  0  0 ]
[  0 -0.5  0  0  0.25  0  0  0  0  0  0  0  0  0  0 ]
[  0  0 -0.5  0  0  0.25  0  0  0  0  0  0  0  0  0 ]
[  0.25  0  0 -0.5  0  0  0.25  0  0  0  0  0  0  0  0 ]
[  0  0.25  0  0 -0.5  0  0  0.25  0  0  0  0  0  0  0 ]
[  0  0  0.25  0  0 -0.5  0  0  0.25  0  0  0  0  0  0 ]
[  0  0  0  0.25  0  0 -0.5  0  0  0.25  0  0  0  0  0 ]
[  0  0  0  0  0.25  0  0 -0.5  0  0  0.25  0  0  0  0 ]
[  0  0  0  0  0  0.25  0  0 -0.5  0  0  0.25  0  0  0 ]
[  0  0  0  0  0  0  0.25  0  0 -0.5  0  0  0.25  0  0 ]
[  0  0  0  0  0  0  0  0.25  0  0 -0.5  0  0  0.25  0 ]
[  0  0  0  0  0  0  0  0  0.25  0  0 -0.5  0  0  0 ]
[  0  0  0  0  0  0  0  0  0  0.25  0  0 -0.5  0  0 ]
[  0  0  0  0  0  0  0  0  0  0  0.25  0  0 -0.5  0 ]
[  0  0  0  0  0  0  0  0  0  0  0  0.25  0  0 -0.5 ]
```

GRID SIZE: Nx = 5, Ny = 3
 RESOLUTION: dx = 1, dy = 2
 BC's: DIRICHLET

DX =

```
[  0  0.5  0  0  0  0  0  0  0  0  0  0  0  0  0 ]
[ -0.5  0  0.5  0  0  0  0  0  0  0  0  0  0  0  0 ]
[  0 -0.5  0  0.5  0  0  0  0  0  0  0  0  0  0  0 ]
[  0  0 -0.5  0  0.5  0  0  0  0  0  0  0  0  0  0 ]
[  0  0  0 -0.5  0  0  0  0  0  0  0  0  0  0  0 ]
[  0  0  0  0  0  0  0.5  0  0  0  0  0  0  0  0 ]
[  0  0  0  0  0  0 -0.5  0  0.5  0  0  0  0  0  0 ]
[  0  0  0  0  0  0  0 -0.5  0  0.5  0  0  0  0  0 ]
[  0  0  0  0  0  0  0  0 -0.5  0  0.5  0  0  0  0 ]
[  0  0  0  0  0  0  0  0  0 -0.5  0  0.5  0  0  0 ]
[  0  0  0  0  0  0  0  0  0  0 -0.5  0  0.5  0  0 ]
[  0  0  0  0  0  0  0  0  0  0  0 -0.5  0  0.5  0 ]
[  0  0  0  0  0  0  0  0  0  0  0  0 -0.5  0  0.5 ]
[  0  0  0  0  0  0  0  0  0  0  0  0  0 -0.5  0 ]
```

D2X =

```
[ -2  1  0  0  0  0  0  0  0  0  0  0  0  0  0 ]
[  1 -2  1  0  0  0  0  0  0  0  0  0  0  0  0 ]
[  0  1 -2  1  0  0  0  0  0  0  0  0  0  0  0 ]
[  0  0  1 -2  1  0  0  0  0  0  0  0  0  0  0 ]
[  0  0  0  1 -2  0  0  0  0  0  0  0  0  0  0 ]
[  0  0  0  0  0 -2  1  0  0  0  0  0  0  0  0 ]
[  0  0  0  0  0  0  1 -2  1  0  0  0  0  0  0 ]
[  0  0  0  0  0  0  0  1 -2  1  0  0  0  0  0 ]
[  0  0  0  0  0  0  0  0  1 -2  1  0  0  0  0 ]
[  0  0  0  0  0  0  0  0  0  1 -2  1  0  0  0 ]
[  0  0  0  0  0  0  0  0  0  0  1 -2  1  0  0 ]
[  0  0  0  0  0  0  0  0  0  0  0  1 -2  1  0 ]
[  0  0  0  0  0  0  0  0  0  0  0  0  1 -2  1 ]
[  0  0  0  0  0  0  0  0  0  0  0  0  0  1 -2 ]
```

DY =

```
[  0  0  0  0  0  0  0.25  0  0  0  0  0  0  0  0 ]
[  0  0  0  0  0  0  0  0.25  0  0  0  0  0  0  0 ]
[  0  0  0  0  0  0  0  0  0.25  0  0  0  0  0  0 ]
[  0  0  0  0  0  0  0  0  0  0.25  0  0  0  0  0 ]
[ -0.25  0  0  0  0  0  0  0  0  0  0.25  0  0  0  0 ]
[  0 -0.25  0  0  0  0  0  0  0  0  0  0.25  0  0  0 ]
[  0  0 -0.25  0  0  0  0  0  0  0  0  0  0.25  0  0 ]
[  0  0  0 -0.25  0  0  0  0  0  0  0  0  0  0.25  0 ]
[  0  0  0  0 -0.25  0  0  0  0  0  0  0  0  0  0.25 ]
[  0  0  0  0  0 -0.25  0  0  0  0  0  0  0  0  0 ]
[  0  0  0  0  0  0 -0.25  0  0  0  0  0  0  0  0 ]
[  0  0  0  0  0  0  0 -0.25  0  0  0  0  0  0  0 ]
[  0  0  0  0  0  0  0  0 -0.25  0  0  0  0  0  0 ]
[  0  0  0  0  0  0  0  0  0 -0.25  0  0  0  0  0 ]
```

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D2Y =

```
[ -0.5  0  0  0  0  0.25  0  0  0  0  0  0  0  0  0 ]
[  0 -0.5  0  0  0  0  0.25  0  0  0  0  0  0  0  0 ]
[  0  0 -0.5  0  0  0  0  0.25  0  0  0  0  0  0  0 ]
[  0  0  0 -0.5  0  0  0  0  0.25  0  0  0  0  0  0 ]
[  0  0  0  0 -0.5  0  0  0  0  0.25  0  0  0  0  0 ]
[ 0.25  0  0  0  0 -0.5  0  0  0  0  0.25  0  0  0  0 ]
[  0  0.25  0  0  0  0 -0.5  0  0  0  0  0.25  0  0  0 ]
[  0  0  0.25  0  0  0  0 -0.5  0  0  0  0  0.25  0  0 ]
[  0  0  0  0.25  0  0  0  0 -0.5  0  0  0  0  0.25  0 ]
[  0  0  0  0  0 0.25  0  0  0  0 -0.5  0  0  0  0.25 ]
[  0  0  0  0  0  0 0.25  0  0  0  0 -0.5  0  0  0 ]
[  0  0  0  0  0  0  0 0.25  0  0  0  0 -0.5  0  0 ]
[  0  0  0  0  0  0  0  0 0.25  0  0  0  0 -0.5  0 ]
[  0  0  0  0  0  0  0  0  0 0.25  0  0  0  0 -0.5 ]
[  0  0  0  0  0  0  0  0  0  0 0.25  0  0  0  -0.5 ]
```

GRID SIZE: Nx = 4, Ny = 4
 RESOLUTION: dx = 1, dy = 2
 BC's: DIRICHLET

DX =

```
[  0  0.5  0  0  0  0  0  0  0  0  0  0  0  0  0 ]
[ -0.5  0  0.5  0  0  0  0  0  0  0  0  0  0  0  0 ]
[  0 -0.5  0  0.5  0  0  0  0  0  0  0  0  0  0  0 ]
[  0  0 -0.5  0  0  0  0  0  0  0  0  0  0  0  0 ]
[  0  0  0  0  0  0.5  0  0  0  0  0  0  0  0  0 ]
[  0  0  0  0 -0.5  0  0.5  0  0  0  0  0  0  0  0 ]
[  0  0  0  0  0 -0.5  0  0.5  0  0  0  0  0  0  0 ]
[  0  0  0  0  0  0 -0.5  0  0.5  0  0  0  0  0  0 ]
[  0  0  0  0  0  0  0  0.5  0  0  0  0  0  0  0 ]
[  0  0  0  0  0  0  0  0  0.5  0  0  0  0  0  0 ]
[  0  0  0  0  0  0  0  0  0  0.5  0  0  0  0  0 ]
[  0  0  0  0  0  0  0  0  0  0  0.5  0  0  0  0 ]
[  0  0  0  0  0  0  0  0  0  0  0  0.5  0  0  0 ]
[  0  0  0  0  0  0  0  0  0  0  0  0  0.5  0  0 ]
[  0  0  0  0  0  0  0  0  0  0  0  0  0  0.5  0 ]
[  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0.5 ]
[  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0 ]
```

D2X =

```
[ -2  1  0  0  0  0  0  0  0  0  0  0  0  0  0 ]
[  1 -2  1  0  0  0  0  0  0  0  0  0  0  0  0 ]
[  0  1 -2  1  0  0  0  0  0  0  0  0  0  0  0 ]
[  0  0  1 -2  1  0  0  0  0  0  0  0  0  0  0 ]
[  0  0  0  1 -2  1  0  0  0  0  0  0  0  0  0 ]
[  0  0  0  0  1 -2  1  0  0  0  0  0  0  0  0 ]
[  0  0  0  0  0  1 -2  1  0  0  0  0  0  0  0 ]
[  0  0  0  0  0  0  1 -2  1  0  0  0  0  0  0 ]
[  0  0  0  0  0  0  0  1 -2  1  0  0  0  0  0 ]
[  0  0  0  0  0  0  0  0  1 -2  1  0  0  0  0 ]
[  0  0  0  0  0  0  0  0  0  1 -2  1  0  0  0 ]
[  0  0  0  0  0  0  0  0  0  0  1 -2  1  0  0 ]
[  0  0  0  0  0  0  0  0  0  0  0  1 -2  1  0 ]
[  0  0  0  0  0  0  0  0  0  0  0  0  1 -2  1 ]
[  0  0  0  0  0  0  0  0  0  0  0  0  0  1 -2 ]
```

DY =

```
[  0  0  0  0  0.25  0  0  0  0  0  0  0  0  0  0 ]
[  0  0  0  0  0  0.25  0  0  0  0  0  0  0  0  0 ]
[  0  0  0  0  0  0  0.25  0  0  0  0  0  0  0  0 ]
[  0  0  0  0  0  0  0  0.25  0  0  0  0  0  0  0 ]
[ -0.25  0  0  0  0  0  0  0  0.25  0  0  0  0  0  0 ]
[  0 -0.25  0  0  0  0  0  0  0  0.25  0  0  0  0  0 ]
[  0  0 -0.25  0  0  0  0  0  0  0  0.25  0  0  0  0 ]
[  0  0  0 -0.25  0  0  0  0  0  0  0  0.25  0  0  0 ]
[  0  0  0  0 -0.25  0  0  0  0  0  0  0  0.25  0  0 ]
[  0  0  0  0  0 -0.25  0  0  0  0  0  0  0  0.25  0 ]
[  0  0  0  0  0  0 -0.25  0  0  0  0  0  0  0  0.25 ]
[  0  0  0  0  0  0  0 -0.25  0  0  0  0  0  0  0.25 ]
[  0  0  0  0  0  0  0  0 -0.25  0  0  0  0  0  0.25 ]
[  0  0  0  0  0  0  0  0  0 -0.25  0  0  0  0  0 ]
[  0  0  0  0  0  0  0  0  0  0 -0.25  0  0  0  0 ]
[  0  0  0  0  0  0  0  0  0  0  0 -0.25  0  0  0 ]
```

D2Y =

```
[ -0.5  0  0  0  0.25  0  0  0  0  0  0  0  0  0  0  0 ]
[  0 -0.5  0  0  0  0.25  0  0  0  0  0  0  0  0  0  0 ]
[  0  0 -0.5  0  0  0  0.25  0  0  0  0  0  0  0  0  0 ]
[  0  0  0 -0.5  0  0  0  0.25  0  0  0  0  0  0  0  0 ]
[ 0.25  0  0  0 -0.5  0  0  0  0.25  0  0  0  0  0  0  0 ]
[  0  0.25  0  0  0 -0.5  0  0  0  0.25  0  0  0  0  0  0 ]
[  0  0  0.25  0  0  0 -0.5  0  0  0  0.25  0  0  0  0  0 ]
[  0  0  0  0.25  0  0  0 -0.5  0  0  0  0.25  0  0  0  0 ]
[  0  0  0  0  0.25  0  0  0 -0.5  0  0  0  0.25  0  0  0 ]
[  0  0  0  0  0  0.25  0  0  0 -0.5  0  0  0  0.25  0  0 ]
[  0  0  0  0  0  0  0.25  0  0  0 -0.5  0  0  0  0.25  0 ]
[  0  0  0  0  0  0  0  0.25  0  0  0 -0.5  0  0  0  0.25 ]
[  0  0  0  0  0  0  0  0  0.25  0  0  0 -0.5  0  0  0 ]
[  0  0  0  0  0  0  0  0  0  0.25  0  0  0 -0.5  0  0 ]
[  0  0  0  0  0  0  0  0  0  0  0.25  0  0  0 -0.5  0 ]
```

 ***** PERIODIC BOUNDARY CONDITIONS *****

GRID SIZE: Nx = 1, Ny = 3
 RESOLUTION: dx = 1, dy = 2
 BC's: PERIODIC

DX =

```
[ 0 0 0 ]
[ 0 0 0 ]
[ 0 0 0 ]
```

D2X =

```
[ 0 0 0 ]
[ 0 0 0 ]
[ 0 0 0 ]
```

DY =

```
[  0  0.25 -0.25 ]
[ -0.25  0  0.25 ]
[  0.25 -0.25  0 ]
```

D2Y =

```
[ -0.5  0.25  0.25 ]
[  0.25 -0.5  0.25 ]
[  0.25  0.25 -0.5 ]
```

GRID SIZE: Nx = 1, Ny = 5
 RESOLUTION: dx = 1, dy = 2
 BC's: PERIODIC

DX =

```
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
```

D2X =

```
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
```

DY =

```
[  0  0.25  0  0 -0.25 ]
[ -0.25  0  0.25  0  0 ]
[  0 -0.25  0  0.25  0 ]
[  0  0 -0.25  0  0.25 ]
[  0.25  0  0 -0.25  0 ]
```

D2Y =

```
[ -0.5  0.25  0  0  0.25 ]
[  0.25 -0.5  0.25  0  0 ]
[  0  0.25 -0.5  0.25  0 ]
[  0  0  0.25 -0.5  0.25 ]
[  0.25  0  0  0.25 -0.5 ]
```

GRID SIZE: Nx = 3, Ny = 1
 RESOLUTION: dx = 1, dy = 2
 BC's: PERIODIC

DX =

```
[ 0 0.5 -0.5 ]
[-0.5 0 0.5 ]
[ 0.5 -0.5 0 ]
```

D2X =

```
[-2 1 1 ]
[ 1 -2 1 ]
[ 1 1 -2 ]
```

DY =

```
[ 0 0 0 ]
[ 0 0 0 ]
[ 0 0 0 ]
```

D2Y =

```
[ 0 0 0 ]
[ 0 0 0 ]
[ 0 0 0 ]
```

GRID SIZE: Nx = 5, Ny = 1
 RESOLUTION: dx = 1, dy = 2
 BC's: PERIODIC

DX =

```
[ 0 0.5 0 0 -0.5 ]
[-0.5 0 0.5 0 0 ]
[ 0 -0.5 0 0.5 0 ]
[ 0 0 -0.5 0 0.5 ]
[ 0.5 0 0 -0.5 0 ]
```

D2X =

```
[-2 1 0 0 1 ]
[ 1 -2 1 0 0 ]
[ 0 1 -2 1 0 ]
[ 0 0 1 -2 1 ]
[ 1 0 0 1 -2 ]
```

DY =

```
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
```

D2Y =

```
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
```

GRID SIZE: Nx = 3, Ny = 3
 RESOLUTION: dx = 1, dy = 2
 BC's: PERIODIC

DX =

```
[ 0 0.5 -0.5 0 0 0 0 0 0 ]
[-0.5 0 0.5 0 0 0 0 0 0 ]
[ 0.5 -0.5 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0.5 -0.5 0 0 0 ]
[ 0 0 0 0 -0.5 0.5 0 0 0 ]
[ 0 0 0 0.5 -0.5 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0.5 -0.5 ]
[ 0 0 0 0 0 0 -0.5 0 0.5 ]
[ 0 0 0 0 0 0 0.5 -0.5 0 ]
```

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D2X =

```
[ -2  1  1  0  0  0  0  0  0  0 ]
[  1 -2  1  0  0  0  0  0  0  0 ]
[  1  1 -2  0  0  0  0  0  0  0 ]
[  0  0  0 -2  1  1  0  0  0  0 ]
[  0  0  0  1 -2  1  0  0  0  0 ]
[  0  0  0  1  1 -2  0  0  0  0 ]
[  0  0  0  0  0  0 -2  1  1  0 ]
[  0  0  0  0  0  0  1 -2  1  0 ]
[  0  0  0  0  0  0  1  1 -2  0 ]
```

DY =

```
[  0  0  0  0.25  0  0 -0.25  0  0  0 ]
[  0  0  0  0  0.25  0  0 -0.25  0  0 ]
[  0  0  0  0  0  0.25  0  0 -0.25  0 ]
[ -0.25  0  0  0  0  0  0.25  0  0  0 ]
[  0 -0.25  0  0  0  0  0  0.25  0  0 ]
[  0  0 -0.25  0  0  0  0  0  0.25  0 ]
[  0.25  0  0 -0.25  0  0  0  0  0  0 ]
[  0  0.25  0  0 -0.25  0  0  0  0  0 ]
[  0  0  0.25  0  0 -0.25  0  0  0  0 ]
```

D2Y =

```
[ -0.5  0  0  0.25  0  0  0.25  0  0  0 ]
[  0 -0.5  0  0  0.25  0  0  0.25  0  0 ]
[  0  0 -0.5  0  0  0.25  0  0  0.25  0 ]
[  0.25  0  0 -0.5  0  0  0.25  0  0  0 ]
[  0  0.25  0  0 -0.5  0  0  0.25  0  0 ]
[  0  0  0.25  0  0 -0.5  0  0  0.25  0 ]
[  0.25  0  0  0.25  0  0 -0.5  0  0  0 ]
[  0  0.25  0  0  0.25  0  0 -0.5  0  0 ]
[  0  0  0.25  0  0  0.25  0  0 -0.5  0 ]
```

 GRID SIZE: Nx = 3, Ny = 5
 RESOLUTION: dx = 1, dy = 2
 BC's: PERIODIC

DX =

```
[  0  0.5 -0.5  0  0  0  0  0  0  0  0  0  0  0  0 ]
[ -0.5  0  0.5  0  0  0  0  0  0  0  0  0  0  0  0 ]
[  0.5 -0.5  0  0  0  0  0  0  0  0  0  0  0  0  0 ]
[  0  0  0  0  0.5 -0.5  0  0  0  0  0  0  0  0  0 ]
[  0  0  0  0 -0.5  0.5  0  0  0  0  0  0  0  0  0 ]
[  0  0  0  0  0.5 -0.5  0  0  0  0.5 -0.5  0  0  0  0 ]
[  0  0  0  0  0  0  0 -0.5  0.5  0  0  0  0  0  0 ]
[  0  0  0  0  0  0  0.5 -0.5  0  0  0  0  0  0  0 ]
[  0  0  0  0  0  0  0  0  0  0.5 -0.5  0  0  0  0 ]
[  0  0  0  0  0  0  0  0  0  0.5 -0.5  0  0  0  0 ]
[  0  0  0  0  0  0  0  0  0  0  0  0.5 -0.5  0  0 ]
[  0  0  0  0  0  0  0  0  0  0  0  0  0.5 -0.5  0 ]
[  0  0  0  0  0  0  0  0  0  0  0  0  0  0.5 -0.5 ]
[  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0.5 ]
```

D2X =

```
[ -2  1  1  0  0  0  0  0  0  0  0  0  0  0  0 ]
[  1 -2  1  0  0  0  0  0  0  0  0  0  0  0  0 ]
[  1  1 -2  0  0  0  0  0  0  0  0  0  0  0  0 ]
[  0  0  0 -2  1  1  0  0  0  0  0  0  0  0  0 ]
[  0  0  0  1 -2  1  0  0  0  0  0  0  0  0  0 ]
[  0  0  0  1  1 -2  0  0  0  0  0  0  0  0  0 ]
[  0  0  0  0  0  0 -2  1  1  0  0  0  0  0  0 ]
[  0  0  0  0  0  0  1 -2  1  0  0  0  0  0  0 ]
[  0  0  0  0  0  0  1  1 -2  0  0  0  0  0  0 ]
[  0  0  0  0  0  0  0  0  0 -2  1  1  0  0  0 ]
[  0  0  0  0  0  0  0  0  0  1 -2  1  0  0  0 ]
[  0  0  0  0  0  0  0  0  0  1  1 -2  0  0  0 ]
[  0  0  0  0  0  0  0  0  0  0  0  0 -2  1  1 ]
[  0  0  0  0  0  0  0  0  0  0  0  0  1 -2  1 ]
[  0  0  0  0  0  0  0  0  0  0  0  0  1  1 -2 ]
```


DY =

```
[ 0 0 0 0.25 0 0 0 0 0 0 0 0 0 -0.25 0 0 ]
[ 0 0 0 0 0.25 0 0 0 0 0 0 0 0 0 -0.25 0 ]
[ 0 0 0 0 0 0.25 0 0 0 0 0 0 0 0 0 -0.25 ]
[ -0.25 0 0 0 0 0 0.25 0 0 0 0 0 0 0 0 0 ]
[ 0 -0.25 0 0 0 0 0 0.25 0 0 0 0 0 0 0 0 ]
[ 0 0 -0.25 0 0 0 0 0 0.25 0 0 0 0 0 0 0 ]
[ 0 0 0 -0.25 0 0 0 0 0 0.25 0 0 0 0 0 0 ]
[ 0 0 0 0 -0.25 0 0 0 0 0 0.25 0 0 0 0 0 ]
[ 0 0 0 0 0 -0.25 0 0 0 0 0 0.25 0 0 0 0 ]
[ 0 0 0 0 0 0 -0.25 0 0 0 0 0 0.25 0 0 0 ]
[ 0 0 0 0 0 0 0 -0.25 0 0 0 0 0 0.25 0 0 ]
[ 0 0 0 0 0 0 0 0 -0.25 0 0 0 0 0 0.25 0 ]
[ 0.25 0 0 0 0 0 0 0 0 -0.25 0 0 0 0 0 0 ]
[ 0 0.25 0 0 0 0 0 0 0 0 -0.25 0 0 0 0 0 ]
[ 0 0 0.25 0 0 0 0 0 0 0 0 -0.25 0 0 0 0 ]
```

D2Y =

```
[ -0.5 0 0 0.25 0 0 0 0 0 0 0 0.25 0 0 ]
[ 0 -0.5 0 0 0.25 0 0 0 0 0 0 0 0.25 0 ]
[ 0 0 -0.5 0 0 0.25 0 0 0 0 0 0 0 0.25 ]
[ 0.25 0 0 -0.5 0 0 0.25 0 0 0 0 0 0 0 ]
[ 0 0.25 0 0 -0.5 0 0 0.25 0 0 0 0 0 0 ]
[ 0 0 0.25 0 0 -0.5 0 0 0.25 0 0 0 0 0 ]
[ 0 0 0 0.25 0 0 -0.5 0 0 0.25 0 0 0 0 ]
[ 0 0 0 0 0.25 0 0 -0.5 0 0 0.25 0 0 0 ]
[ 0 0 0 0 0 0.25 0 0 -0.5 0 0 0.25 0 0 ]
[ 0 0 0 0 0 0 0.25 0 0 -0.5 0 0 0.25 0 ]
[ 0 0 0 0 0 0 0 0.25 0 0 -0.5 0 0 0.25 ]
[ 0.25 0 0 0 0 0 0 0 0.25 0 0 -0.5 0 0 ]
[ 0 0.25 0 0 0 0 0 0 0 0 0.25 0 -0.5 0 ]
[ 0 0 0.25 0 0 0 0 0 0 0 0 0.25 0 -0.5 ]
```

 GRID SIZE: Nx = 5, Ny = 3
 RESOLUTION: dx = 1, dy = 2
 BC's: PERIODIC

DX =

```
[ 0 0.5 0 0 -0.5 0 0 0 0 0 0 0 0 0 0 ]
[ -0.5 0 0.5 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 -0.5 0 0.5 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 -0.5 0 0.5 0 0 0 0 0 0 0 0 0 0 ]
[ 0.5 0 0 -0.5 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0.5 0 -0.5 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 -0.5 0 0.5 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 -0.5 0 0.5 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0.5 0 -0.5 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0.5 0 -0.5 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0.5 0 0 0 -0.5 ]
[ 0 0 0 0 0 0 0 0 0 0 -0.5 0 0.5 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 -0.5 0 0.5 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 -0.5 0 0.5 ]
[ 0 0 0 0 0 0 0 0 0 0.5 0 0 -0.5 0 ]
```

D2X =

```
[ -2 1 0 0 1 0 0 0 0 0 0 0 0 0 0 ]
[ 1 -2 1 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 1 -2 1 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 1 -2 1 0 0 0 0 0 0 0 0 0 0 ]
[ 1 0 0 1 -2 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 -2 1 0 0 1 0 0 0 0 0 ]
[ 0 0 0 0 0 0 1 -2 1 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 1 -2 1 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 1 -2 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 -2 1 0 0 1 ]
[ 0 0 0 0 0 0 0 0 0 0 1 -2 1 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 1 -2 1 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 1 -2 1 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 1 -2 ]
```

DY =

```
[ 0 0 0 0 0 0.25 0 0 0 0 -0.25 0 0 0 0 ]
[ 0 0 0 0 0 0 0.25 0 0 0 -0.25 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0.25 0 0 0 -0.25 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0.25 0 0 0 -0.25 0 0 ]
[ -0.25 0 0 0 0 0 0 0 0 0 0.25 0 0 0 0 ]
[ 0 -0.25 0 0 0 0 0 0 0 0 0 0.25 0 0 0 ]
[ 0 0 -0.25 0 0 0 0 0 0 0 0 0 0.25 0 0 ]
[ 0 0 0 -0.25 0 0 0 0 0 0 0 0 0 0.25 0 ]
[ 0 0 0 0 -0.25 0 0 0 0 0 0 0 0 0 0.25 ]
[ 0.25 0 0 0 0 0 -0.25 0 0 0 0 0 0 0 0 ]
[ 0 0.25 0 0 0 0 0 -0.25 0 0 0 0 0 0 0 ]
[ 0 0 0.25 0 0 0 0 -0.25 0 0 0 0 0 0 0 ]
[ 0 0 0 0.25 0 0 0 0 -0.25 0 0 0 0 0 0 ]
[ 0 0 0 0 0.25 0 0 0 0 -0.25 0 0 0 0 0 ]
[ 0 0 0 0 0 0.25 0 0 0 0 -0.25 0 0 0 0 ]
```

D2Y =

```
[ -0.5 0 0 0 0.25 0 0 0 0.25 0 0 0 0 0 ]
[ 0 -0.5 0 0 0 0.25 0 0 0 0.25 0 0 0 0 ]
[ 0 0 -0.5 0 0 0 0.25 0 0 0 0.25 0 0 0 ]
[ 0 0 0 -0.5 0 0 0 0.25 0 0 0 0.25 0 0 ]
[ 0 0 0 0 -0.5 0 0 0 0.25 0 0 0 0.25 0 ]
[ 0.25 0 0 0 0 -0.5 0 0 0 0.25 0 0 0 0 ]
[ 0 0.25 0 0 0 0 -0.5 0 0 0 0.25 0 0 0 ]
[ 0 0 0.25 0 0 0 0 -0.5 0 0 0 0.25 0 0 ]
[ 0 0 0 0.25 0 0 0 0 -0.5 0 0 0 0.25 0 ]
[ 0.25 0 0 0 0.25 0 0 0 -0.5 0 0 0 0 0 ]
[ 0 0.25 0 0 0 0.25 0 0 0 -0.5 0 0 0 0 ]
[ 0 0 0.25 0 0 0 0.25 0 0 0 -0.5 0 0 0 ]
[ 0 0 0 0.25 0 0 0 0.25 0 0 0 -0.5 0 0 ]
[ 0 0 0 0 0.25 0 0 0 0.25 0 0 0 -0.5 0 ]
[ 0 0 0 0 0 0.25 0 0 0 0.25 0 0 0 -0.5 ]
```

 GRID SIZE: Nx = 4, Ny = 4
 RESOLUTION: dx = 1, dy = 2
 BC's: PERIODIC

DX =

```
[ 0 0.5 0 -0.5 0 0 0 0 0 0 0 0 0 0 0 ]
[ -0.5 0 0.5 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 -0.5 0 0.5 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0.5 0 -0.5 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0.5 0 -0.5 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 -0.5 0 0.5 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0.5 0 -0.5 0 0 0 0 0 ]
[ 0 0 0 0 0 0.5 0 -0.5 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0.5 0 -0.5 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 -0.5 0 0.5 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 -0.5 0 0.5 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0.5 0 -0.5 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 -0.5 0 0.5 0 -0.5 ]
[ 0 0 0 0 0 0 0 0 0 0 0 -0.5 0 0.5 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 -0.5 0 0.5 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0.5 0 -0.5 ]
```

D2X =

```
[ -2 1 0 1 0 0 0 0 0 0 0 0 0 0 0 ]
[ 1 -2 1 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 1 -2 1 0 0 0 0 0 0 0 0 0 0 0 ]
[ 1 0 1 -2 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 -2 1 0 1 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 1 -2 1 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 1 -2 1 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 -2 1 0 1 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 1 -2 1 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 1 -2 1 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 1 -2 1 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 1 -2 1 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 1 -2 ]
```

DY =

```
[ 0 0 0 0 0.25 0 0 0 0 0 0 0 0 -0.25 0 0 0 0 ]
[ 0 0 0 0 0 0.25 0 0 0 0 0 0 0 0 -0.25 0 0 0 ]
[ 0 0 0 0 0 0 0.25 0 0 0 0 0 0 0 0 -0.25 0 0 ]
[ -0.25 0 0 0 0 0 0 0.25 0 0.25 0 0 0 0 0 0 0 0 ]
[ 0 -0.25 0 0 0 0 0 0 0 0 0.25 0 0 0 0 0 0 0 ]
[ 0 0 -0.25 0 0 0 0 0 0 0 0 0.25 0 0 0 0 0 0 ]
[ 0 0 0 -0.25 0 0 0 0 0 0 0 0 0.25 0 0 0 0 0 ]
[ 0 0 0 0 -0.25 0 0 0 0 0 0 0 0 0.25 0 0 0 0 ]
[ 0 0 0 0 0 -0.25 0 0 0 0 0 0 0 0 0.25 0 0 0 ]
[ 0 0 0 0 0 0 -0.25 0 0 0 0 0 0 0 0 0.25 0 0 ]
[ 0.25 0 0 0 0 0 0 0 0 0 -0.25 0 0 0 0 0 0 0 0 ]
[ 0 0.25 0 0 0 0 0 0 0 0 0 -0.25 0 0 0 0 0 0 ]
[ 0 0 0.25 0 0 0 0 0 0 0 0 -0.25 0 0 0 0 0 0 ]
[ 0 0 0 0.25 0 0 0 0 0 0 0 0 -0.25 0 0 0 0 0 ]
```

D2Y =

```
[ -0.5 0 0 0 0.25 0 0 0 0 0 0 0.25 0 0 0 0 ]
[ 0 -0.5 0 0 0 0.25 0 0 0 0 0 0 0.25 0 0 0 ]
[ 0 0 -0.5 0 0 0.25 0 0 0 0 0 0 0 0.25 0 0 ]
[ 0.25 0 0 0 -0.5 0 0 0.25 0 0 0 0 0 0 0 0 ]
[ 0 0.25 0 0 0 -0.5 0 0 0.25 0 0 0 0 0 0 0 ]
[ 0 0 0.25 0 0 0 -0.5 0 0 0.25 0 0 0 0 0 0 ]
[ 0 0 0 0.25 0 0 0 -0.5 0 0 0.25 0 0 0 0 0 ]
[ 0 0 0 0 0.25 0 0 0 -0.5 0 0 0.25 0 0 0 0 ]
[ 0 0 0 0 0 0.25 0 0 0 -0.5 0 0 0.25 0 0 0 ]
[ 0.25 0 0 0 0 0 0 0.25 0 0 -0.5 0 0 0 0 0 ]
[ 0 0.25 0 0 0 0 0 0 0.25 0 0 -0.5 0 0 0 0 ]
[ 0 0 0.25 0 0 0 0 0 0 0.25 0 0 -0.5 0 0 0 ]
[ 0 0 0 0.25 0 0 0 0 0 0 0.25 0 0 -0.5 0 0 ]
```

 ***** NEUMANN BOUNDARY CONDITIONS *****

GRID SIZE: Nx = 1, Ny = 3
 RESOLUTION: dx = 1, dy = 2
 BC's: NEUMANN

DX =

```
[ 0 0 0 ]
[ 0 0 0 ]
[ 0 0 0 ]
```

D2X =

```
[ 0 0 0 ]
[ 0 0 0 ]
[ 0 0 0 ]
```

DY =

```
[ -0.5 0.5 0 ]
[ -0.25 0 0.25 ]
[ 0 -0.5 0.5 ]
```

D2Y =

```
[ 0 0 0 ]
[ 0.25 -0.5 0.25 ]
[ 0 0 0 ]
```

 GRID SIZE: Nx = 1, Ny = 5
 RESOLUTION: dx = 1, dy = 2
 BC's: NEUMANN

DX =

```
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
```

D2X =

```
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
```

DY =

```
[ -0.5  0.5  0  0  0 ]
[ -0.25  0  0.25  0  0 ]
[  0 -0.25  0  0.25  0 ]
[  0  0 -0.25  0  0.25 ]
[  0  0  0 -0.5  0.5 ]
```

D2Y =

```
[  0  0  0  0  0 ]
[ 0.25 -0.5  0.25  0  0 ]
[  0  0.25 -0.5  0.25  0 ]
[  0  0  0.25 -0.5  0.25 ]
[  0  0  0  0  0 ]
```

 GRID SIZE: Nx = 3, Ny = 1
 RESOLUTION: dx = 1, dy = 2
 BC's: NEUMANN

DX =

```
[ -1  1  0 ]
[ -0.5  0  0.5 ]
[  0 -1  1 ]
```

D2X =

```
[ 0 0 0 ]
[ 1 -2 1 ]
[ 0 0 0 ]
```

DY =

```
[ 0 0 0 ]
[ 0 0 0 ]
[ 0 0 0 ]
```

D2Y =

```
[ 0 0 0 ]
[ 0 0 0 ]
[ 0 0 0 ]
```

 GRID SIZE: Nx = 5, Ny = 1
 RESOLUTION: dx = 1, dy = 2
 BC's: NEUMANN

DX =

```
[ -1  1  0  0  0 ]
[ -0.5  0  0.5  0  0 ]
[  0 -0.5  0  0.5  0 ]
[  0  0 -0.5  0  0.5 ]
[  0  0  0 -1  1 ]
```

D2X =

```
[ 0 0 0 0 0 ]
[ 1 -2 1 0 0 ]
[ 0 1 -2 1 0 ]
[ 0 0 1 -2 1 ]
[ 0 0 0 0 0 ]
```

DY =

```
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
[ 0 0 0 0 0 ]
```

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D2Y =

```
[ 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 ]
```

GRID SIZE: Nx = 3, Ny = 3
 RESOLUTION: dx = 1, dy = 2
 BC's: NEUMANN

DX =

```
[ -1  1  0  0  0  0  0  0  0  0 ]
[ -0.5 0  0.5 0  0  0  0  0  0 ]
[  0 -1  1  0  0  0  0  0  0 ]
[  0  0  0 -1  1  0  0  0  0 ]
[  0  0  0 -0.5  0  0.5  0  0  0 ]
[  0  0  0  0 -1  1  0  0  0 ]
[  0  0  0  0  0  0 -1  1  0 ]
[  0  0  0  0  0  0 -0.5  0  0.5 ]
[  0  0  0  0  0  0  0 -1  1 ]
```

D2X =

```
[ 0 0 0 0 0 0 0 0 0 ]
[ 1 -2 1 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 1 -2 1 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 1 -2 1 ]
[ 0 0 0 0 0 0 0 0 0 ]
```

DY =

```
[ -0.5  0  0  0.5  0  0  0  0  0 ]
[  0 -0.5  0  0  0.5  0  0  0  0 ]
[  0  0 -0.5  0  0  0.5  0  0  0 ]
[ -0.25  0  0  0  0  0  0.25  0  0 ]
[  0 -0.25  0  0  0  0  0  0.25  0 ]
[  0  0 -0.25  0  0  0  0  0  0.25 ]
[  0  0  0 -0.5  0  0  0.5  0  0 ]
[  0  0  0  0 -0.5  0  0  0.5  0 ]
[  0  0  0  0  0 -0.5  0  0  0.5 ]
```

D2Y =

```
[ 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 ]
[ 0.25 0 0 -0.5 0 0 0.25 0 0 ]
[ 0 0.25 0 0 -0.5 0 0 0.25 0 ]
[ 0 0 0.25 0 0 -0.5 0 0 0.25 ]
[ 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 ]
```

GRID SIZE: Nx = 3, Ny = 5
 RESOLUTION: dx = 1, dy = 2
 BC's: NEUMANN

DX =

```
[ -1  1  0  0  0  0  0  0  0  0  0  0  0  0  0 ]
[ -0.5 0  0.5 0  0  0  0  0  0  0  0  0  0  0 ]
[  0 -1  1  0  0  0  0  0  0  0  0  0  0  0 ]
[  0  0  0 -1  1  0  0  0  0  0  0  0  0  0 ]
[  0  0  0 -0.5  0  0.5  0  0  0  0  0  0  0 ]
[  0  0  0  0 -1  1  0  0  0  0  0  0  0 ]
[  0  0  0  0  0  0 -1  1  0  0  0  0  0 ]
[  0  0  0  0  0  0  0 -0.5  0  0.5  0  0  0 ]
[  0  0  0  0  0  0  0 -1  1  0  0  0  0 ]
[  0  0  0  0  0  0  0  0 -1  1  0  0  0 ]
[  0  0  0  0  0  0  0  0  0 -0.5  0  0.5  0 ]
[  0  0  0  0  0  0  0  0  0  0 -1  1  0 ]
[  0  0  0  0  0  0  0  0  0  0  0 -0.5  0  0.5 ]
[  0  0  0  0  0  0  0  0  0  0  0  0 -1  1 ]
```

D2X =

```
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 1 -2 1 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 1 -2 1 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 1 -2 1 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 1 -2 1 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 1 -2 1 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
```

DY =

```
[ -0.5 0 0 0.5 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 -0.5 0 0 0.5 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 -0.5 0 0 0.5 0 0 0 0 0 0 0 0 0 ]
[ -0.25 0 0 0 0 0.25 0 0 0 0 0 0 0 0 0 ]
[ 0 -0.25 0 0 0 0 0.25 0 0 0 0 0 0 0 0 ]
[ 0 0 -0.25 0 0 0 0 0.25 0 0 0 0 0 0 0 ]
[ 0 0 0 -0.25 0 0 0 0 0.25 0 0 0 0 0 0 ]
[ 0 0 0 0 -0.25 0 0 0 0 0.25 0 0 0 0 0 ]
[ 0 0 0 0 0 -0.25 0 0 0 0 0.25 0 0 0 0 ]
[ 0 0 0 0 0 0 -0.25 0 0 0 0 0.25 0 0 0 ]
[ 0 0 0 0 0 0 0 -0.25 0 0 0 0 0.25 0 0 ]
[ 0 0 0 0 0 0 0 0 -0.25 0 0 0 0 0.25 0 ]
[ 0 0 0 0 0 0 0 0 0 -0.5 0 0 0.5 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 -0.5 0 0 0.5 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 -0.5 0 0 0.5 ]
```

D2Y =

```
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0.25 0.25 0 -0.5 0 0.25 0 0 0 0 0 0 0 0 0 ]
[ 0 0.25 0 0 -0.5 0 0.25 0 0 0 0 0 0 0 0 ]
[ 0 0 0.25 0 0 -0.5 0 0.25 0 0 0 0 0 0 0 ]
[ 0 0 0 0.25 0 0 -0.5 0 0.25 0 0 0 0 0 0 ]
[ 0 0 0 0 0.25 0 0 -0.5 0 0.25 0 0 0 0 0 ]
[ 0 0 0 0 0 0.25 0 0 -0.5 0 0.25 0 0 0 0 ]
[ 0 0 0 0 0 0 0.25 0 0 -0.5 0 0.25 0 0 0 ]
[ 0 0 0 0 0 0 0 0.25 0 0 -0.5 0 0.25 0 0 ]
[ 0 0 0 0 0 0 0 0 0.25 0 0 -0.5 0 0.25 0 ]
[ 0 0 0 0 0 0 0 0 0 0.25 0 0 -0.5 0 0.25 ]
[ 0 0 0 0 0 0 0 0 0 0 0.25 0 0 -0.5 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0.25 0 0 -0.5 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0.25 0 0 ]
```

GRID SIZE: Nx = 5, Ny = 3
 RESOLUTION: dx = 1, dy = 2
 BC's: NEUMANN

DX =

```
[ -1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ -0.5 0 0.5 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 -0.5 0 0.5 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 -0.5 0 0.5 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 -1 1 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 -1 1 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 -0.5 0.5 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 -0.5 0.5 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 -1 1 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 -1 1 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 -1 1 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 -0.5 0.5 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 -0.5 0.5 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 -0.5 0.5 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.5 0.5 ]
```

D2X =

```
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 1 -2 1 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 1 -2 1 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 1 -2 1 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 1 -2 1 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 1 -2 1 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 1 -2 1 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 1 -2 1 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 1 -2 1 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 1 -2 1 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 1 -2 1 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 ]
```

DY =

```
[ -0.5 0 0 0 0 0.5 0 0 0 0 0 0 0 0 0 ]
[ 0 -0.5 0 0 0 0 0.5 0 0 0 0 0 0 0 0 ]
[ 0 0 -0.5 0 0 0 0 0.5 0 0 0 0 0 0 0 ]
[ 0 0 0 -0.5 0 0 0 0 0.5 0 0 0 0 0 0 ]
[ -0.25 0 0 0 -0.5 0 0 0 0.5 0 0 0 0 0 0 ]
[ 0 -0.25 0 0 0 0 0 0 0 0.25 0 0 0 0 0 ]
[ 0 0 -0.25 0 0 0 0 0 0 0 0.25 0 0 0 0 ]
[ 0 0 0 -0.25 0 0 0 0 0 0 0 0.25 0 0 0 ]
[ 0 0 0 0 -0.25 0 0 0 0 0 0 0 0.25 0 0 ]
[ 0 0 0 0 0 -0.5 0 0 0.5 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 -0.5 0 0 0.5 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 -0.5 0 0 0 0.5 0 0 0 ]
[ 0 0 0 0 0 0 0 0 -0.5 0 0 0 0.5 0 0 ]
[ 0 0 0 0 0 0 0 0 0 -0.5 0 0 0 0.5 0 ]
[ 0 0 0 0 0 0 0 0 0 0 -0.5 0 0 0 0.5 ]
```

D2Y =

```
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0.25 0 0 0 0 -0.5 0 0 0.25 0 0 0 0 0 0 ]
[ 0 0.25 0 0 0 0 -0.5 0 0 0.25 0 0 0 0 0 ]
[ 0 0 0.25 0 0 0 -0.5 0 0 0.25 0 0 0 0 0 ]
[ 0 0 0 0.25 0 0 0 -0.5 0 0 0.25 0 0 0 0 ]
[ 0 0 0 0 0.25 0 0 0 0 -0.5 0 0 0.25 0 0 ]
[ 0 0 0 0 0 0.25 0 0 0 0 0 -0.5 0 0.25 0 ]
[ 0 0 0 0 0 0 0.25 0 0 0 0 0 0 -0.5 0 ]
[ 0 0 0 0 0 0 0 0.25 0 0 0 0 0 0 -0.5 ]
[ 0 0 0 0 0 0 0 0 0.25 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0.25 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0.25 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0.25 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0.25 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0.25 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.25 ]
```

GRID SIZE: Nx = 4, Ny = 4
 RESOLUTION: dx = 1, dy = 2
 BC's: NEUMANN

DX =

```
[ -1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ -0.5 0 0.5 0 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 -0.5 0 0.5 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 -1 1 0 0 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 -1 1 0 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 -0.5 0.5 0 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 -0.5 0.5 0 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 -1 1 0 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 -1 1 0 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 -0.5 0.5 0 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 -0.5 0.5 0 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 -1 1 0 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 -1 1 0 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.5 0.5 ]
[ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 1 ]
```

