Electrostatic Fields & Charges

EMPossible
https://empossible.net/

Governing Eqs.

Conditions

1. $\omega = 0$
2. Size $\ll \lambda$

$$\nabla \times \vec{E} = 0$$
$$\nabla \bullet \vec{D} = \rho_v$$
$$\vec{D} = [\varepsilon]\vec{E}$$

Electric Potential

Calculating $\vec{E}$ from V:

$$\vec{E} = -\nabla V$$

Calculating V from $\vec{E}$:

$$V_{AB} = V_B - V_A = \frac{W}{Q} = -\int_A^B \vec{E} \cdot d\vec{\ell}$$

Power & Energy

Energy in the Field:

$$W = \begin{cases} \frac{1}{2}\iiint_V \left(\vec{D} \bullet \vec{E}\right) dv & \text{general case} \\ \frac{1}{2}\iiint_V \varepsilon\left|\vec{E}\right|^2 dv & \text{for LHI} \end{cases}$$

Power in Conductors:

$$P = \begin{cases} \iiint_V \left(\vec{E} \bullet \vec{J}\right) dv & \text{general Joule's law} \\ \iiint_V \sigma\left|\vec{E}\right|^2 dv & \text{more common form} \end{cases}$$

Point Charges

Single Charge Q:

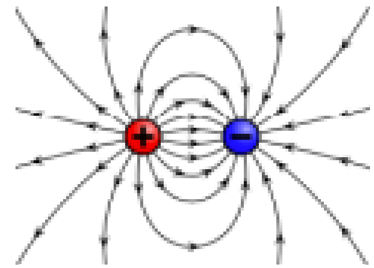
$$\vec{D} = \frac{Q}{4\pi}\frac{\hat{a}_R}{R^2} = \frac{Q}{4\pi}\frac{\vec{r} - \vec{r}_Q}{\left|\vec{r} - \vec{r}_Q\right|^3}$$

$$\vec{E} = \vec{D}/\varepsilon_0\varepsilon_r$$

$$V = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{Q}{\left|\vec{r} - \vec{r}_Q\right|}$$

Force on a Charge:

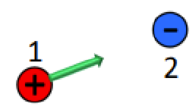
$$\vec{F} = Q\vec{E}$$

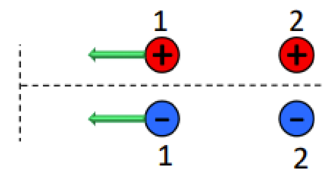


Coulomb's Law

$$\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0\varepsilon_r}\frac{\hat{a}_{12}}{R_{12}^2}$$

$$\vec{R}_{12} = \vec{r}_2 - \vec{r}_1$$





Multiple Point Charges:

$$\vec{D}_T = \sum_{i=1}^{N}\frac{Q_i}{4\pi}\frac{\vec{r} - \vec{r}_i}{\left|\vec{r} - \vec{r}_i\right|^3} \qquad V_T = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\sum_{i=1}^{N}\frac{Q_i}{\left|\vec{r} - \vec{r}_i\right|}$$

$$W_T = \frac{1}{2}\sum_{i=1}^{N} Q_i V_i$$

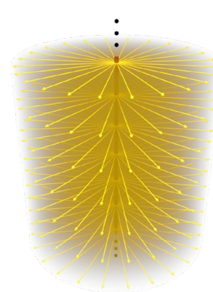
Charge Distributions

Quantity	Line ρ_ℓ (C/m)	Surface ρ_s (C/m^2)	Volume ρ_v (C/m^3)
Total Charge Q (C)	$\int_L \rho_\ell d\ell$	$\iint_S \rho_s ds$	$\iiint_V \rho_v dv$
Total Field $\vec{D}$ (C/m^2)	$\int_L \frac{\rho_\ell d\ell}{4\pi R^2}\hat{a}_R$	$\iint_S \frac{\rho_s ds}{4\pi R^2}\hat{a}_R$	$\iiint_V \frac{\rho_v dv}{4\pi R^2}\hat{a}_R$
Electric Potential V (V)	$\int_L \frac{\rho_\ell d\ell}{4\pi\varepsilon\left|\vec{r} - \vec{r}\,'\right|}$	$\iint_S \frac{\rho_s ds}{4\pi\varepsilon\left|\vec{r} - \vec{r}\,'\right|}$	$\iiint_V \frac{\rho_v dv}{4\pi\varepsilon\left|\vec{r} - \vec{r}\,'\right|}$
Energy W (J)	$\frac{1}{2}\int_L \rho_\ell V d\ell$	$\frac{1}{2}\iint_S \rho_s V ds$	$\frac{1}{2}\iiint_V \rho_v V dv$

Line Charge

$$Q = \rho_\ell L$$

$$\vec{D} = \frac{\rho_\ell}{4\pi\rho}\hat{a}_\rho$$



Surface Charge

$$Q = \rho_s S$$

$$\vec{D} = \frac{\rho_s}{2}\hat{a}_n$$