EE 4382/5306 – Antenna Engineering
Homework #1 – Due September 13, 2017

Antenna Parameters: A Practical Approach

Radiation Pattern and Beamwidth

1. - Plot in polar form the following functions for radiation patterns. You can help yourself with the diagram provided below.

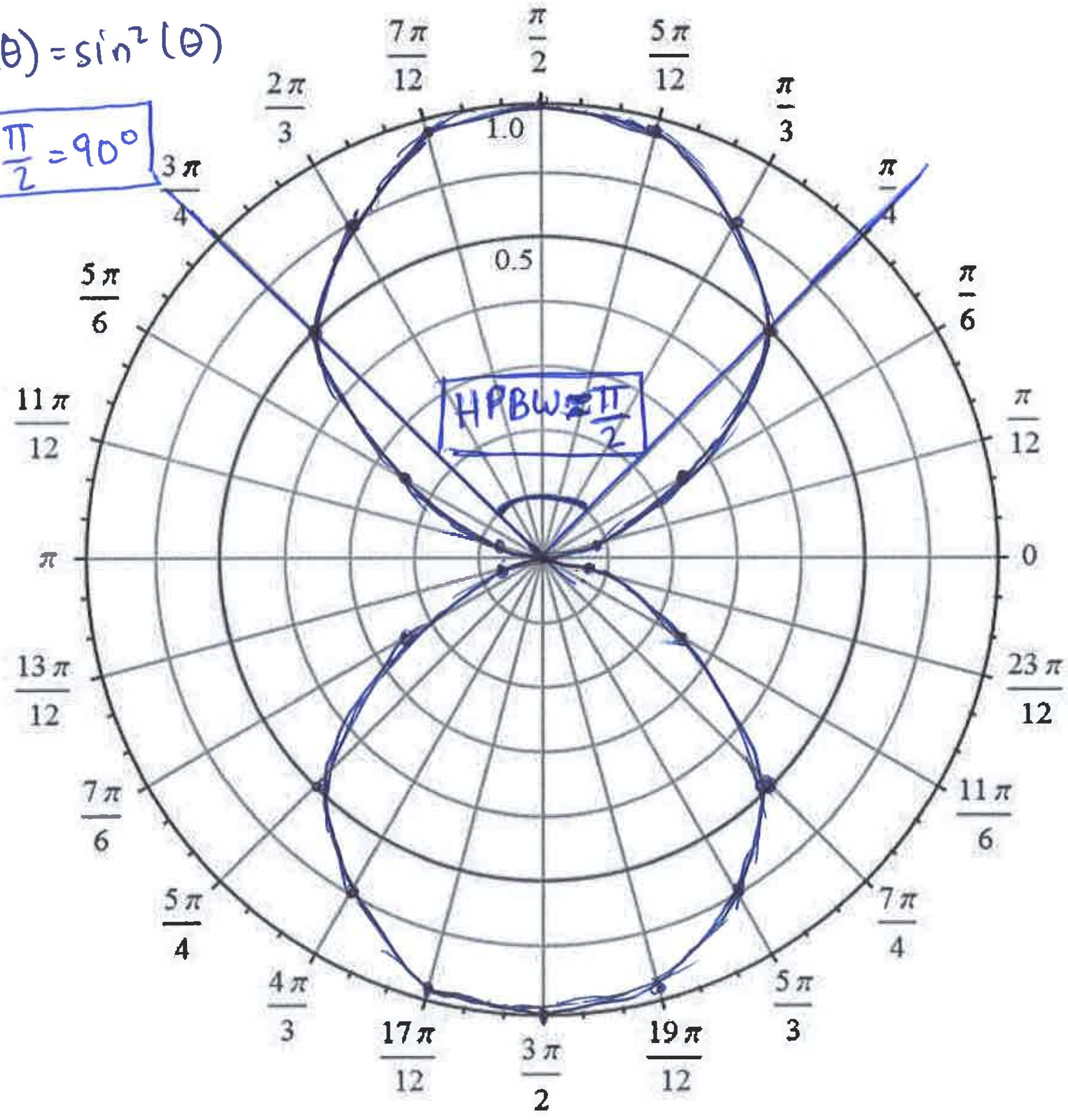
a. $U(\theta) = \sin^2(\theta)$
b. $U(\theta) = \sin(\theta)\cos(\theta)$ $\quad , 0 < \theta < 2\pi$
c. $U(\theta) = \cos^2(\theta)\sin(2\theta)$
d. $U(\theta) = \sin^2(\theta)\cos\left(\theta + \frac{\pi}{2}\right)$

Computer plot is also OK

2. – For the radiation patterns below, find also the half-power beamwidth, both in radians and degrees, and plot it on the top of part 1.

a. $U(\theta) = \sin^2(\theta)$

$\boxed{HPBW \approx \frac{\pi}{2} = 90^\circ}$



EE 4382/5306 – Antenna Engineering
Homework #1 – Due September 13, 2017

Antenna Parameters: A Practical Approach

Radiation Pattern and Beamwidth

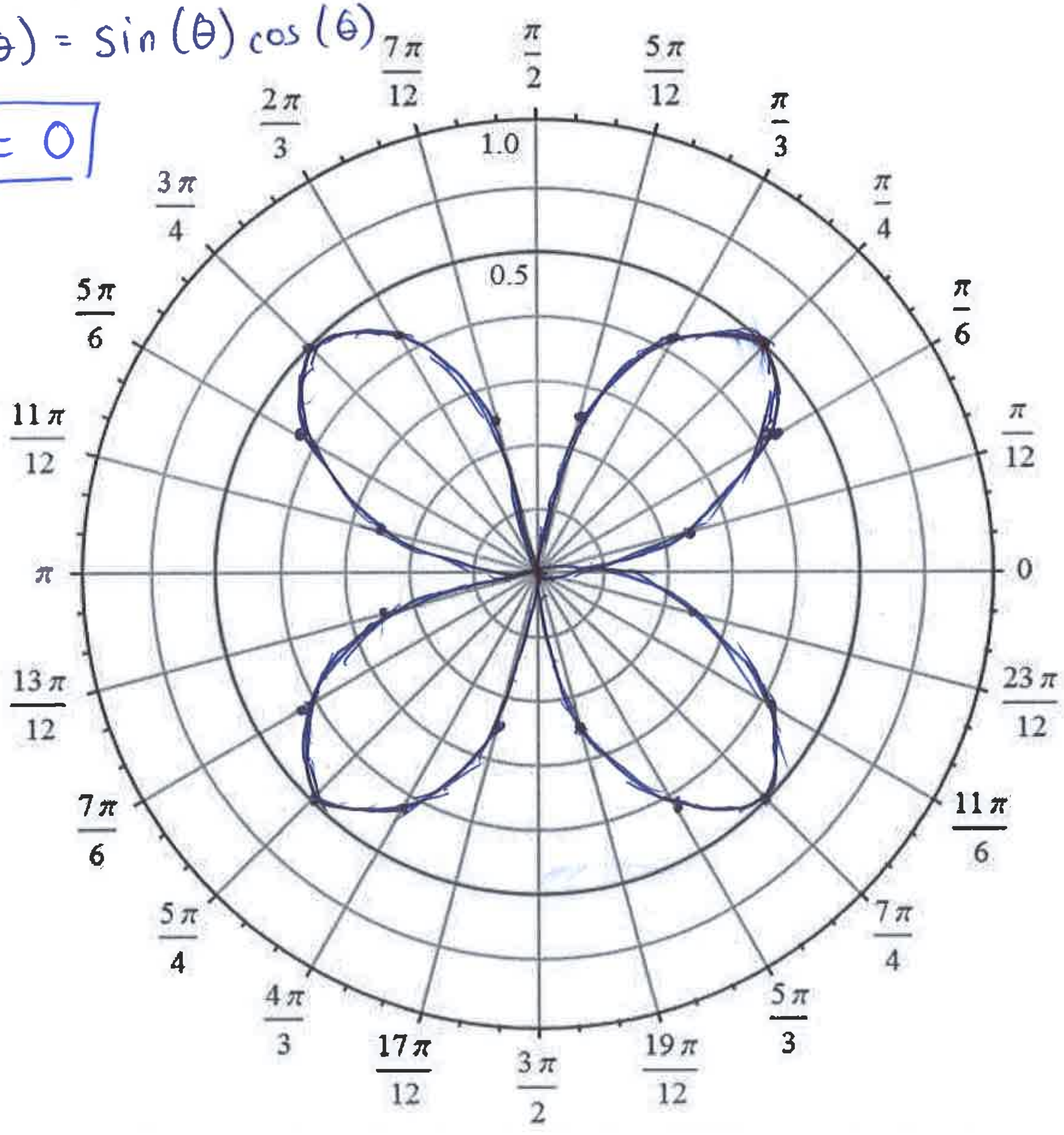
1. - Plot in polar form the following functions for radiation patterns. You can help yourself with the diagram provided below.

a. $U(\theta) = \sin^2(\theta)$
b. $U(\theta) = \sin(\theta)\cos(\theta)$ $\quad ,0 < \overset{\theta}{\cancel{\phi}} < 2\pi$
c. $U(\theta) = \cos^2(\theta)\sin(2\theta)$
d. $U(\theta) = \sin^2(\theta)\cos\left(\theta + \frac{\pi}{2}\right)$

2. – For the radiation patterns below, find also the half-power beamwidth, both in radians and degrees, and plot it on the top of part 1.

b. $U(\theta) = \sin(\theta)\cos(\theta)$

$\boxed{\text{HPBW} = 0}$



EE 4382/5306 – Antenna Engineering
Homework #1 – Due September 13, 2017

Antenna Parameters: A Practical Approach

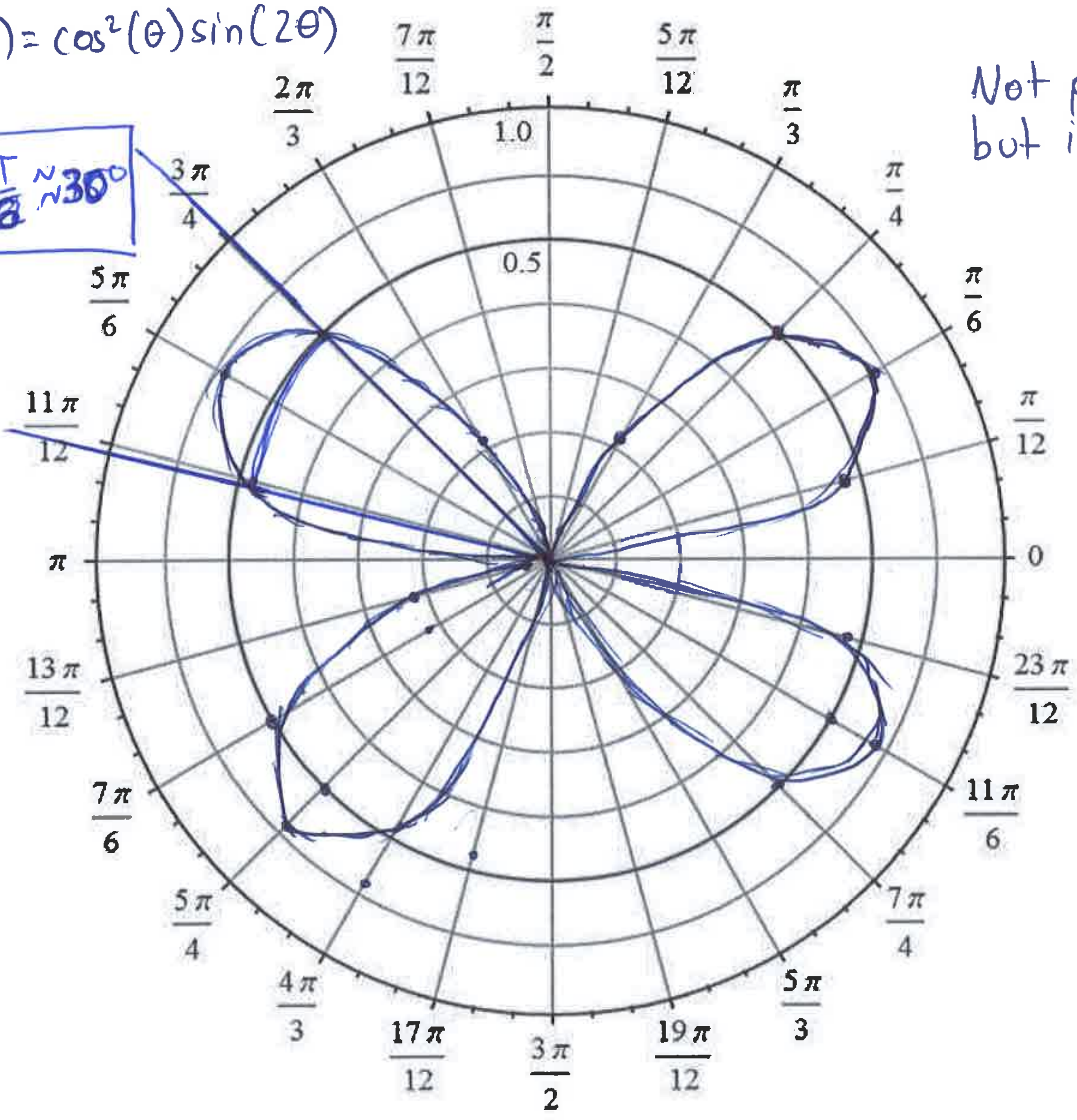
Radiation Pattern and Beamwidth

1. - Plot in polar form the following functions for radiation patterns. You can help yourself with the diagram provided below.

a. $U(\theta) = \sin^2(\theta)$
b. $U(\theta) = \sin(\theta)\cos(\theta)$, $0 < \overset{\theta}{\cancel{\phi}} < 2\pi$
c. $U(\theta) = \cos^2(\theta)\sin(2\theta)$
d. $U(\theta) = \sin^2(\theta)\cos\left(\theta + \frac{\pi}{2}\right)$

2. – For the radiation patterns below, find also the half-power beamwidth, both in radians and degrees, and plot it on the top of part 1.

c. $U(\theta) = \cos^2(\theta)\sin(2\theta)$



$\boxed{\text{HPBW} \approx \frac{\pi}{6} \approx 30^\circ}$

Not perfect but it's OK

EE 4382/5306 – Antenna Engineering
Homework #1 – Due September 13, 2017

Antenna Parameters: A Practical Approach

Radiation Pattern and Beamwidth

1. - Plot in polar form the following functions for radiation patterns. You can help yourself with the diagram provided below.

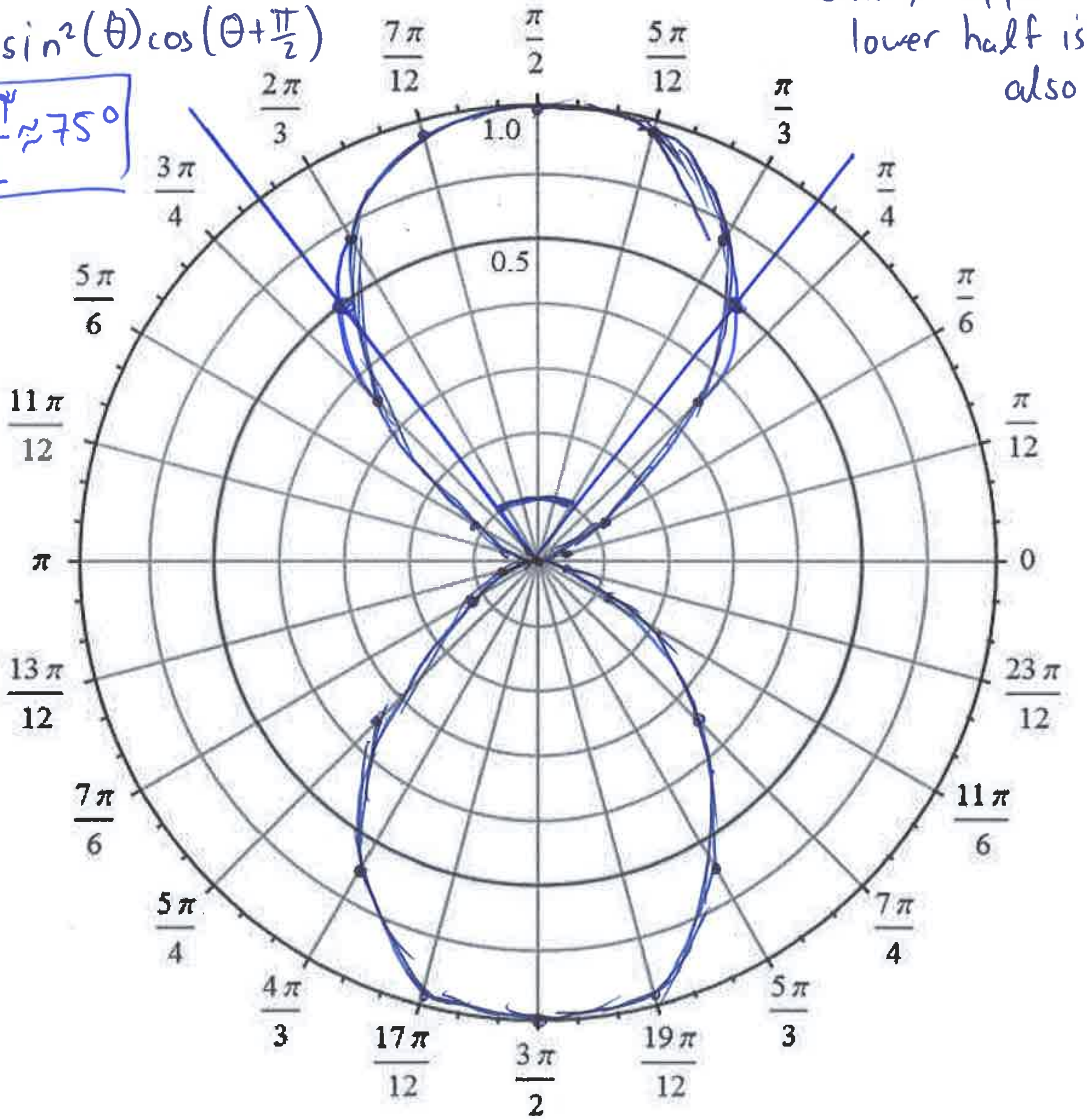
   a. $U(\theta) = \sin^2(\theta)$
   b. $U(\theta) = \sin(\theta)\cos(\theta)$ $\quad ,0 < \overset{\theta}{\cancel{\phi}} < 2\pi$
   c. $U(\theta) = \cos^2(\theta)\sin(2\theta)$
   d. $U(\theta) = \sin^2(\theta)\cos\left(\theta + \frac{\pi}{2}\right)$

2. – For the radiation patterns below, find also the half-power beamwidth, both in radians and degrees, and plot it on the top of part 1.

d. $U(\theta) = \sin^2(\theta)\cos\left(\theta + \frac{\pi}{2}\right)$

$\boxed{\text{HPBW} \approx \frac{5\pi}{12} \approx 75^\circ}$

Only upper or lower half is also OK

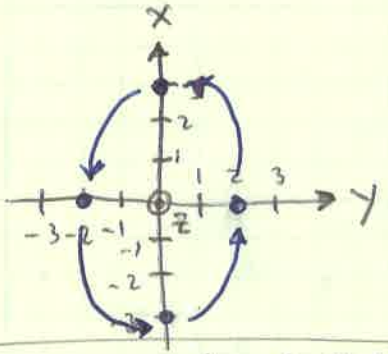


EE 4382-EE 5306 - Antenna Engineering Homework #1

POLARIZATION

a. $\vec{E}(z,t) = 3\hat{x}\cos(\omega t - kz) - 2\hat{y}\cos(\omega t - kz + \frac{\pi}{2})$

T	Ex	Ey
0	3	0
π/2	0	-2
π	-3	0
3π/2	0	2

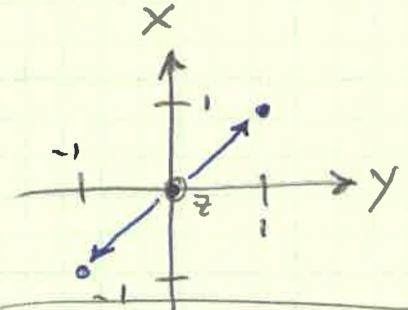


$\boxed{\text{Right-Hand Elliptical Polarization}}$

b. $\vec{E}(x,y,z,t) = (\hat{x}+\hat{y})\cos(\omega t - kz)$

$$\vec{E} = \hat{x}\cos(\omega t - kz) + \hat{y}\cos(\omega t - kz)$$

T	Ex	Ey
0	1	1
$\pi/2$	0	0
π	-1	-1
$3\pi/2$	0	0

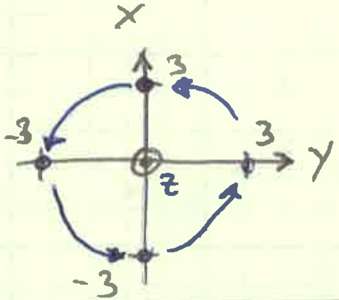


Linear Polarization

c. $\tilde{E}(z) = (3\hat{x} + 3e^{j\pi/2})\,e^{-jkz}$

$$\vec{E}(z,t) = 3\hat{x}\cos(\omega t - kz) + 3\hat{y}\cos(\omega t - kz + \pi/2)$$

T	Ex	Ey
0	3	0
$\pi/2$	0	-3
π	-3	0
$3\pi/2$	0	3

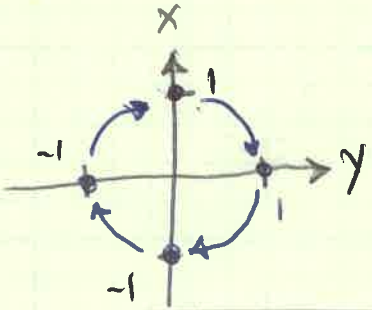


Right-Hand Circular Polarization

d. $\hat{E}(z) = (\hat{x} - j\hat{y})e^{-jkz}$

$$\vec{E}(z,t) = \hat{x}\cos(\omega t - kz) + \hat{y}\cos(\omega t - kz - \pi/2)$$

T	Ex	Ey
0	1	0
$\pi/2$	0	1
π	-1	0
$\frac{3\pi}{2}$	0	-1



Left-Hand Circular Polarization

EE 4382 - EE 5306 Homework #1

Antenna Design

a. $U(\theta,\phi) = \cos^n(\theta)$, $0 \le \theta \le \frac{\pi}{2}$, $0 \le \phi \le 2\pi$

$$\cos^n\left(\tfrac{1}{2}\text{HPBW}\right) = \tfrac{1}{2} \rightarrow 0.5 = \cos^n(5°)$$

$$[\cos(5°)]^n = (0.99619)^n$$

so we get

$$0.5 = (0.99619)^n$$

$$\log(0.5) = n \log(0.99619)$$

$$n = \frac{\log(0.5)}{\log(0.99619)} = \frac{-0.30103}{-0.00166} = \boxed{181.34}$$

a: so $\boxed{n = 181.34}$

b. $U(\theta,\phi) = \cos^{181.34}(\theta)$, max at $\theta = 0°$, $U_{max} = 1$

$$P_{rad} = \iint U(\theta,\phi)\sin\theta\, d\theta\, d\phi$$

$$= \int_0^{2\pi}\int_0^{\pi/2} \cos^{181.34}(\theta)\sin\theta\, d\theta\, d\phi$$

$$= 2\pi\left[-\frac{\cos^{182.34}(\theta)}{182.34}\right]_0^{\frac{\pi}{2}} = \frac{2\pi}{182.34} = 0.03446$$

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi}{0.03446} = 364.67$$

$$\boxed{D_0 = 364.67 = 25.62\,dB}$$

EE 4382-EE 5306 - Antenna Engineering

$R = 22{,}370$ miles $= 35{,}993$ Km $= 3.5993 \times 10^7$ m

$P_t = 8.0$ W

$f = 2$ GHz, $\lambda = 0.15$m

Polarization Mismatch $(\cos(8^\circ))^2 = 0.98$

Impedance - Matched

Isotropic (Directivity - 1)   Gain = 60 dB = 1×10^6

Lossless antenna

a) $$P_r = \frac{P_t}{4\pi R^2} = \frac{8.0}{4\pi(3.5993\times10^7)^2} = \boxed{4.9141\times10^{-16}\ W/m^2}$$

b) $$P_r = P_t\, e_{cdt}\, e_{cdr}\,(1-|\Gamma_t|^2)(1-|\Gamma_r|^2)\left(\frac{\lambda}{4\pi R}\right)^2 D_t D_r\, |\hat{\rho}_t\cdot\hat{\rho}_r|^2$$

$$P_r = (8.0)(1)(1)(1)(1)\left(\frac{0.15}{4\pi\,(3.5993\times10^7)}\right)^2(1)(1\times10^6)(0.98)$$

$$\boxed{P_r = 8.6227\times10^{-13}\ W}$$