MICROWAVE ENGINEERING

Read Supplemental Chapter 2, Electrical Properties of Materials.

Problem #1: Complex Refractive Index

The complex refractive index $\tilde{n}$ and complex dielectric constant $\tilde{\varepsilon}_r$ are defined as

$$\tilde{n} = n + j\kappa \tag{1}$$

$$\tilde{\varepsilon}_r = \varepsilon_r' + j\varepsilon_r'' \tag{2}$$

Assuming the magnetic response $\tilde{\mu}_r$ of a material is negligible, the complex refractive index is related to the complex dielectric function through

$$\tilde{n}^2 = \tilde{\varepsilon}_r \tag{3}$$

Derive the following equations for the real and imaginary parts of the complex refractive index in terms of the real and imaginary parts of the dielectric constant. Show each step and justify the sign of the roots.

$$n = \sqrt{\frac{1}{2}\left(\sqrt{\varepsilon_r'^2 + \varepsilon_r''^2} + \varepsilon_r'\right)} \qquad \kappa = \sqrt{\frac{1}{2}\left(\sqrt{\varepsilon_r'^2 + \varepsilon_r''^2} - \varepsilon_r'\right)} \tag{4}$$

Problem #2: Lorentz Model for Dielectrics

At room temperature, the dielectric response of water can be modeled in the microwave spectrum with the following Lorentz parameters:

$$\omega_p = 5.1\times10^{13}\ \text{s}^{-1} \qquad \Gamma = 1.6\times10^{14}\ \text{s}^{-1}$$

$$\omega_0 = 5.8\times10^{12}\ \text{s}^{-1} \qquad \varepsilon_{r,\infty} = 4.0$$

(a) Using a computer, plot both the real and imaginary parts of the dielectric function from 300 MHz to 3.0 THz on the same axes. Use a log scale for the frequency axis. Mark where 2.45 GHz is on the plot.
(b) Calculate the complex dielectric constant at 2.45 GHz, the frequency of a microwave oven.
(c) Based on the plot, where might you expect water to have the highest absorption?
(d) Based on your answers above, why do microwave ovens operate at 2.45 GHz?

Problem #3: Drude Model for Metals

Metallic magnesium (Mg) has a free electron density of 8.61×10^{28} m^{-3} and a DC conductivity of 2.26×10^{7} Ω^{-1}m^{-1}. Assuming the electromagnetic properties follow the Drude model, calculate the following parameters:

a. Plasma frequency f_p in hertz
b. Momentum scattering time τ
c. Complex dielectric constant at λ_0 = 500 nm.
d. Complex refractive index at λ_0 = 500 nm.
e. Reflectance from air at normal incidence at λ_0 = 500 nm.