## Directivity

$$E = \begin{cases} 1 & 0 < \theta \le 45° \\ 0 & 45° < \theta \le 90° \\ \frac{1}{2} & 90° < \theta \le 180° \end{cases}$$

$$U = \frac{r^2 E^2}{2\eta} = \frac{r^2|E|^2}{\eta}, \qquad U_{\max} = \frac{r^2}{\eta} = \frac{1}{120\pi}$$

$$P_{\text{rad}} = \frac{r^2}{\eta}\int_0^{2\pi} d\phi \left[\int_0^{45°} \sin\theta\, d\theta + \int_{90°}^{180°} \frac{1}{4}\sin\theta\, d\theta\right]$$

$$= \frac{r^2}{\eta}[2\pi]\left[-\cos\theta|_0^{45°} + \frac{1}{4}(-\cos\theta)|_{90°}^{180°}\right]$$

$$= \frac{2r^2\pi}{\eta}\left[-\cos 45° + \cos 0° - \frac{1}{4}\cos 180° + \frac{1}{4}\cos 90°\right]$$

$$P_{\text{rad}} = 0.54289\frac{2\pi r^2}{\eta}$$

$$D = \frac{4\pi U_{\max}}{P_{\text{rad}}} = \frac{4\pi\left(\frac{r^2}{\eta}\right)}{0.54289(2\pi)r^2/\eta} = 3.684$$

## Polarization

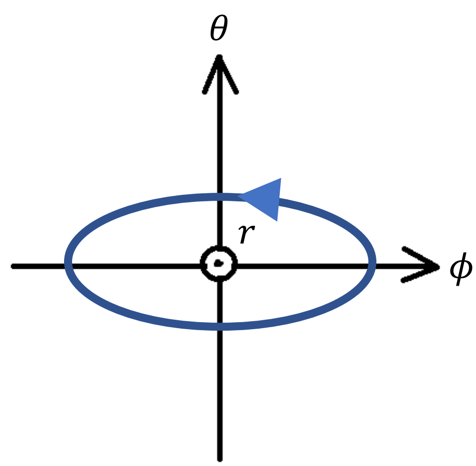
The receiving antenna has wave function of the form

$$\underline{E}_a = E_0(j\hat{a}_\theta + 2\hat{a}_\phi)f_0(\theta_0,\phi_0)\frac{e^{-jkr}}{r} = E_0\underbrace{\left(\frac{j\hat{a}_\theta + 2\hat{a}_\phi}{\sqrt{5}}\right)}_{\hat{\rho}_a}\sqrt{5}f_0(\theta_0,\phi_0)\frac{e^{-jkr}}{r}$$

$$\hat{\rho}_a = \left(\frac{j\hat{a}_\theta + 2\hat{a}_\phi}{\sqrt{5}}\right)$$

$$\text{Re}\,(\text{E}_\text{a}) = \hat{a}_\theta \sin(\omega t - kr) + 2\hat{a}_\phi \cos(\omega t - kr)$$

| $T$ | $E_\theta$ | $E_\phi$ |
|---|---|---|
| 0 | 0 | 2 |
| $\pi/2$ | 1 | 0 |
| $\pi$ | 0 | $-2$ |
| $3\pi/2$ | $-1$ | 0 |



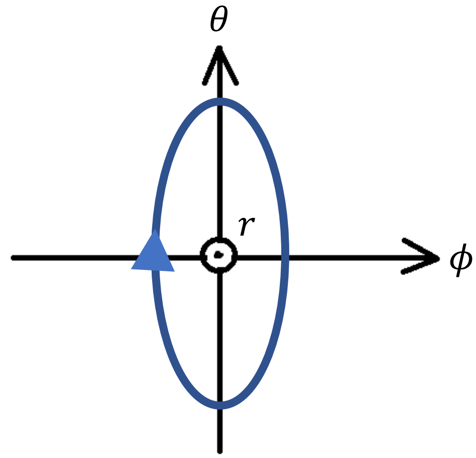

**Polarization is Right Hand Elliptical**

The transmitting antenna has wave function of the form

$$\underline{E}_w = E_1(2\hat{a}_\theta + j\hat{a}_\phi) f_1(\theta_0, \phi_0) \frac{e^{+jkr}}{r}$$

$$= E_1 \underbrace{\left( \frac{2\hat{a}_\theta + j\hat{a}_\phi}{\sqrt{5}} \right)}_{\hat{\rho}_w} \sqrt{5} f_1(\theta_0, \phi_0) \frac{e^{+jkr}}{r}$$

$$\hat{\rho}_w = \left( \frac{2\hat{a}_\theta + j\hat{a}_\phi}{\sqrt{5}} \right)$$




$$\mathrm{Re}\,(\mathrm{E_w}) = 2\hat{a}_\theta \cos(\omega t - kr) + \hat{a}_\phi \sin(\omega t - kr)$$

| $T$ | $E_\theta$ | $E_\phi$ |
|---|---|---|
| $0$ | $2$ | $0$ |
| $\pi/2$ | $0$ | $1$ |
| $\pi$ | $-2$ | $0$ |
| $3\pi/2$ | $0$ | $-1$ |

**Polarization is Left-Hand Elliptical if viewed in the same direction of propagation as the receiving antenna, Right-Hand Elliptical if viewed in transmitting direction**

$$\mathrm{PLF} = |\hat{\rho}_a \cdot \hat{\rho}_w|^2 = \left| \left( \frac{j\hat{a}_\theta + 2\hat{a}_\phi}{\sqrt{5}} \right) \cdot \left( \frac{2\hat{a}_\theta + j\hat{a}_\phi}{\sqrt{5}} \right) \right|^2 = \left| \frac{2j + j2}{\sqrt{25}} \right|^2 = \left| \frac{4j}{\sqrt{25}} \right|^2$$

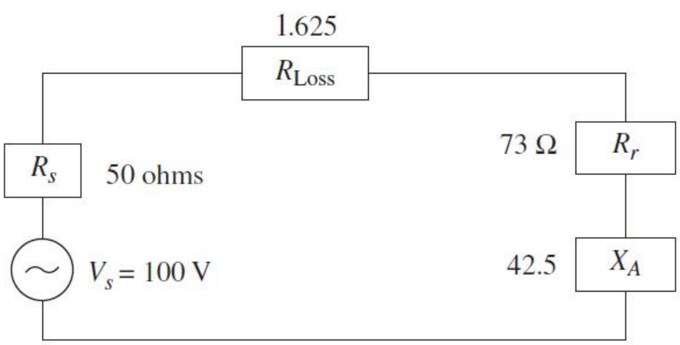
$$\boxed{\mathrm{PLF} = \frac{16}{25} = 0.64 = -1.938 \text{ dB}}$$

## Antenna Impedance, Input Impedance, and Power

$f = 150$ MHz, $\lambda = 2$ m

$\Rightarrow$ 1 m dipole is $\frac{\lambda}{2}$ in electrical length

$\Rightarrow R_r = 73$ ohms, $Z_{in} = 73 + j42.5$ ohms



(a) $I_{\text{ant}} = \dfrac{V_s}{50 + 73 + 1.625 + j42.5} = 0.759\angle -18.83°\ A$

(b) $P_{\text{dissip}} = P_{\text{Loss}} = \dfrac{1}{2}|I_{\text{ant}}|^2 \cdot R_{\text{Loss}} = 469\ \text{mW}$

(c) $P_{\text{rad}} = \dfrac{1}{2}|I_{\text{ant}}|^2 \cdot R_r = 21.05\ W$

(d) $E_{cd} = \dfrac{R_r}{R_r + R_{\text{Loss}}} = \dfrac{73}{73 + 1.625} = 97.8\%$