

Problem #1: Wave Equation

Starting with Maxwell's curl equations

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}, \quad (1)$$

$$\nabla \times \vec{H} = j\omega\varepsilon\vec{E}, \quad (2)$$

derive the wave equation for the electric field in a linear, homogeneous, and isotropic (LHI) medium.

$$\nabla^2 \vec{E} + (k_0 n)^2 \vec{E} = 0 \quad (3)$$

In this equation, n is refractive index and k_0 is the free space wave number. These are defined as

$$n = \sqrt{\mu_r \varepsilon_r} \quad (4)$$

$$k_0 = 2\pi/\lambda_0 \quad (5)$$

Problem #2: Dispersion Relation

Given that the solution to the wave equation is a plane wave

$$\vec{E}(\vec{r}) = \vec{E}_0 e^{j\vec{k} \cdot \vec{r}}, \quad (6)$$

substitute this solution back into the wave equation to derive the dispersion relation for LHI media given in Eq. (7).

$$k_x^2 + k_y^2 + k_z^2 = (k_0 n)^2 \quad (7)$$

Problem #3: Inverse of a Block 2x2 Matrix

Using proper matrix algebra rules, derive the following expression for the inverse of a 2x2 block matrix. Remember that \mathbf{A} , \mathbf{B} , \mathbf{C} , and \mathbf{D} are matrices themselves.

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{DX}^{-1} & -\mathbf{BX}^{-1} \\ -\mathbf{CX}^{-1} & \mathbf{AX}^{-1} \end{bmatrix} \quad \mathbf{X} = \mathbf{AD} - \mathbf{BC} \quad (8)$$

Problem #4: Block Matrix Division

Using proper matrix algebra rules, simplify the following block matrix expression where \mathbf{W}_i , \mathbf{W}_j , \mathbf{V}_i and \mathbf{V}_j are themselves matrices.

$$\begin{bmatrix} \mathbf{W}_i & \mathbf{W}_j \\ \mathbf{V}_i & -\mathbf{V}_j \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{W}_j & \mathbf{W}_j \\ \mathbf{V}_j & -\mathbf{V}_j \end{bmatrix}, \quad (9)$$

Your final answer should be

$$\frac{1}{2} \begin{bmatrix} \mathbf{A}_{ij} & \mathbf{B}_{ij} \\ \mathbf{B}_{ij} & \mathbf{A}_{ij} \end{bmatrix} \quad \text{where} \quad \begin{aligned} \mathbf{A}_{ij} &= \mathbf{W}_i^{-1} \mathbf{W}_j + \mathbf{V}_i^{-1} \mathbf{V}_j \\ \mathbf{B}_{ij} &= \mathbf{W}_i^{-1} \mathbf{W}_j - \mathbf{V}_i^{-1} \mathbf{V}_j \end{aligned} \quad (10)$$