Linear Antenna Arrays

Three-Element Isotropic Array

6.3. Method I

(a) Derive the array factor;

$$AF = -e^{jkd\cos\theta} - j + e^{-jkd\cos\theta} = -2j\sin(kd\cos\theta) - j$$

$$AF = 2\sin(kd\cos\theta) + 1$$

$$AF = 2\sin(\pi\cos\theta) + 1$$

(b) $2\sin(\pi\cos\theta) = -1$

$$kd\cos\theta = \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}, \frac{-5\pi}{6}, \frac{-13\pi}{6}, \ldots, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6} = x$$

$$\text{For } x = \lambda/2 \Rightarrow \theta_n = \cos^{-1}\left(\frac{x}{\pi}\right)$$

$$x = -\frac{\pi}{6} \rightarrow \quad \theta_1 = 99.59°$$

$$x = -\frac{5\pi}{6} \rightarrow \quad \theta_2 = 146.44°$$

Method II

Uniform array with $\beta = -\pi/2$

(a) $$AF = \frac{\sin\frac{N\psi}{2}}{N\sin\frac{\psi}{2}} = \frac{\sin\frac{3}{2}\left[\pi\cos\theta - \frac{\pi}{2}\right]}{3\sin\frac{1}{2}\left[\pi\cos\theta - \frac{\pi}{2}\right]}$$

(b) $$\theta_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2n}{N}\pi\right)\right] \quad n = 1, 2$$

$$= \cos^{-1}\left[\frac{1}{\pi}\left(\frac{\pi}{2} \pm \frac{2\pi}{3}n\right)\right] \quad n \neq 3, 6, 9$$

$$n = 1; \quad \cos^{-1}\left[-\frac{1}{6}\right] = 99.59°$$

$$n = 2; \quad \cos^{-1}\left[-\frac{5}{6}\right] = 146.44°$$

Linea Array Design

(a) $D_0 = 4N\left(\frac{d}{\lambda}\right)$

$$20 = 10\,\log_{10}D_0\ \text{(dimensionless)}\ \Rightarrow D_0\ \text{(dimensionless)}\ = 10^2 = 100$$

$$100 = 4N\left(\frac{\lambda}{4\lambda}\right) = N \Rightarrow N = 100$$

(b) $L = 99\left(\frac{\lambda}{4}\right) = \frac{99}{4}\lambda = 24.175\lambda$

(c) $\Theta_{3\text{dB}} = \Theta_h = 2\cos^{-1}\left(1 - \frac{1.391\lambda}{Nd\pi}\right)|_{n=100} = 2\cos^{-1}\left(1 - \frac{1.391\lambda}{\pi\left(\frac{\lambda}{4}\right)100}\right)$

$$= 2\cos^{-1}\left(1 - \frac{1.391(4)}{100\pi}\right) = 2\cos^{-1}(1 - 0.01771) = 2\cos^{-1}(0.98228)$$

$$\Theta_h = 2(10.799^\circ) = 21.598^\circ \simeq 21.6^\circ$$

(d) Sidelobe (dB) $\simeq -13.5$ dB

(e) $\beta = \pm kd = \pm\frac{2\pi}{\lambda}\left(\frac{\lambda}{4}\right) = \pm\frac{\pi}{2} = \pm 90^\circ$

Linear Array Design

$D_0 = 9.5545 = 10\,\log_{10}D_0$(dimensionless)

D_0(dimensionless) $= 10^{0.95545} = 9.025$

(a) End-Fire Hansen-Woodyard design.

(b) $D_0(\text{dimensionless}) = 1.805\left[4N\left(\frac{d}{\lambda}\right)\right]$; Table 6.8.

$$\frac{d}{\lambda} = \frac{1}{4}\left(\frac{N-1}{N}\right)$$

$$D_0 = 9.025 = 1.805\left[4N\frac{1}{4}\left(\frac{N-1}{N}\right)\right] = 1.805(N-1)$$

$$N - 1 = 9.025/1.805 = 5$$

$$\boxed{N = 6}$$

(c) $d = \frac{\lambda}{4}\left(\frac{N-1}{N}\right) = \frac{\lambda}{4}\left(\frac{6-1}{6}\right) = \lambda\left(\frac{5}{24}\right) = 0.20833\lambda$

$$\boxed{d = 0.20833\lambda}$$

(d) $\beta = -\left(kd + \frac{2.92}{N}\right) = -\left[\frac{2\pi}{\lambda}(0.20833\lambda) + \frac{2.92}{6}\right]$ (6-239)

$$= -\left[2\pi(0.20833) + \frac{2.92}{6}\right] = -(1.30898 + 0.487)$$

$$\boxed{\beta = -1.79565\ \text{rads}\ = -102.883^\circ}$$

Approximate polar plot of problem #3 (not normalized)