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EE 5337

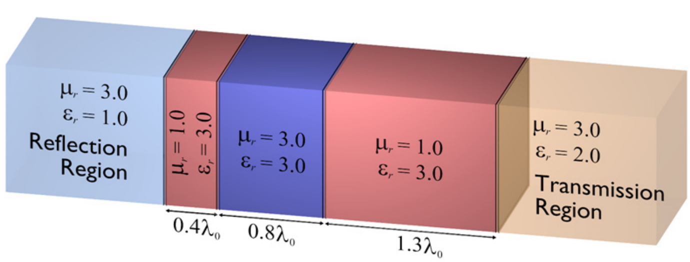
Computational Electromagnetics

Lecture #5

Transfer Matrix Method Using Scattering Matrices

Outline

- Review
- Calculating reflected and transmitted power
- Simplifications for 1D transfer matrix method
- Notes on implementation
- Parameter Sweeps



Review

Two Paths to Combined Solution

4×4 Matrix

$$\frac{\partial}{\partial z'}\begin{bmatrix} E_x \\ E_y \\ \tilde{H}_x \\ \tilde{H}_y \end{bmatrix} = \begin{bmatrix} -j\left(\tilde{k}_y \frac{\mu_{yz}}{\mu_{zz}} + \tilde{k}_x \frac{\varepsilon_{zx}}{\varepsilon_{zz}}\right) & j\tilde{k}_x\left(\frac{\mu_{yz}}{\mu_{zz}} - \frac{\varepsilon_{zy}}{\varepsilon_{zz}}\right) & \left(\frac{\tilde{k}_x\tilde{k}_y}{\varepsilon_{zz}} + \mu_{yx} - \frac{\mu_{yz}\mu_{zx}}{\mu_{zz}}\right) & \left(-\tilde{k}\frac{\hat{k}_x^2}{\varepsilon_{zz}} + \mu_{yy} - \frac{\mu_{yz}\mu_{zy}}{\mu_{zz}}\right) \\ j\tilde{k}_y\left(\frac{\mu_{xz}}{\mu_{zz}} - \frac{\varepsilon_{zx}}{\varepsilon_{zz}}\right) & -j\left(\tilde{k}_x \frac{\mu_{xz}}{\mu_{zz}} + \tilde{k}_y \frac{\varepsilon_{zy}}{\varepsilon_{zz}}\right) & \left(\frac{\tilde{k}_y^2}{\varepsilon_{zz}} - \mu_{xx} + \frac{\mu_{xz}\mu_{zx}}{\mu_{zz}}\right) & \left(-\frac{\tilde{k}_x\tilde{k}_y}{\varepsilon_{zz}} - \mu_{xy} + \frac{\mu_{xz}\mu_{zy}}{\mu_{zz}}\right) \\ \left(\frac{\tilde{k}_x\tilde{k}_y}{\mu_{zz}} + \varepsilon_{yx} - \frac{\varepsilon_{yz}\varepsilon_{zx}}{\varepsilon_{zz}}\right) & \left(-\frac{\tilde{k}_x^2}{\mu_{zz}} + \varepsilon_{yy} - \frac{\varepsilon_{yz}\varepsilon_{zy}}{\varepsilon_{zz}}\right) & -j\left(\tilde{k}_y \frac{\varepsilon_{yz}}{\varepsilon_{zz}} + \tilde{k}_x \frac{\mu_{zx}}{\mu_{zz}}\right) & j\tilde{k}_x\left(\frac{\varepsilon_{yz}}{\varepsilon_{zz}} - \frac{\mu_{zy}}{\mu_{zz}}\right) \\ \left(\frac{\tilde{k}_y^2}{\mu_{zz}} - \varepsilon_{xx} + \frac{\varepsilon_{xz}\varepsilon_{zx}}{\varepsilon_{zz}}\right) & \left(-\frac{\tilde{k}_x\tilde{k}_y}{\mu_{zz}} - \varepsilon_{xy} + \frac{\varepsilon_{xz}\varepsilon_{zy}}{\varepsilon_{zz}}\right) & j\tilde{k}_y\left(\frac{\varepsilon_{xz}}{\varepsilon_{zz}} - \frac{\mu_{zx}}{\mu_{zz}}\right) & -j\left(\tilde{k}_x \frac{\varepsilon_{xz}}{\varepsilon_{zz}} + \tilde{k}_y \frac{\mu_{zy}}{\mu_{zz}}\right) \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ \tilde{H}_x \\ \tilde{H}_y \end{bmatrix}$$

Anisotropic

Sort Eigen-Modes

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_E^+ & \mathbf{W}_E^- \\ \mathbf{W}_H^+ & \mathbf{W}_H^- \end{bmatrix} \begin{matrix} E_x \\ E_y \\ \tilde{H}_x \\ \tilde{H}_y \end{matrix}$$

$$e^{\lambda z'} = \begin{bmatrix} e^{\lambda^+ z'} & \mathbf{0} \\ \mathbf{0} & e^{-\lambda^- z'} \end{bmatrix}$$

Maxwell's Equations

$$\nabla \times \vec{E} = k_0 \left[\mu_r\right] \vec{\tilde{H}}$$

$$\nabla \times \vec{\tilde{H}} = k_0 \left[\varepsilon_r\right] \vec{E}$$

Field Solution

$$\boldsymbol{\psi}(z') = \begin{bmatrix} \mathbf{W}_E^+ & \mathbf{W}_E^- \\ \mathbf{V}_H^+ & \mathbf{V}_H^- \end{bmatrix} \begin{bmatrix} e^{\lambda^+ z'} & \mathbf{0} \\ \mathbf{0} & e^{-\lambda^- z'} \end{bmatrix} \begin{bmatrix} \mathbf{c}^+ \\ \mathbf{c}^- \end{bmatrix}$$

$$\boldsymbol{\psi}(z') = \begin{bmatrix} \mathbf{W} & \mathbf{W} \\ \mathbf{V} & -\mathbf{V} \end{bmatrix} \begin{bmatrix} e^{\lambda z'} & \mathbf{0} \\ \mathbf{0} & e^{-\lambda z'} \end{bmatrix} \begin{bmatrix} \mathbf{c}^+ \\ \mathbf{c}^- \end{bmatrix}$$

Isotropic or diagonally anisotropic

2×2 Matrices

$$\mathbf{P} = \frac{1}{\varepsilon_r}\begin{bmatrix} \tilde{k}_x\tilde{k}_y & \mu_r\varepsilon_r - \tilde{k}_x^2 \\ \tilde{k}_y^2 - \mu_r\varepsilon_r & -\tilde{k}_x\tilde{k}_y \end{bmatrix}$$

$$\mathbf{Q} = \frac{1}{\mu_r}\begin{bmatrix} \tilde{k}_x\tilde{k}_y & \mu_r\varepsilon_r - \tilde{k}_x^2 \\ \tilde{k}_y^2 - \mu_r\varepsilon_r & -\tilde{k}_x\tilde{k}_y \end{bmatrix}$$

PQ Method

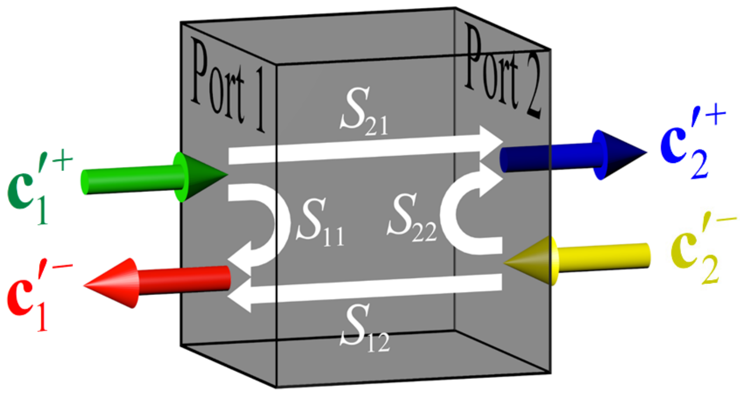
No sorting! ☺

Definition of A Scattering Matrix

$$\begin{bmatrix} \mathbf{c}_1'^{-} \\ \mathbf{c}_2'^{+} \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{c}_1'^{+} \\ \mathbf{c}_2'^{-} \end{bmatrix}$$

$\mathbf{S}_{11} \equiv$ reflection

$\mathbf{S}_{21} \equiv$ transmission

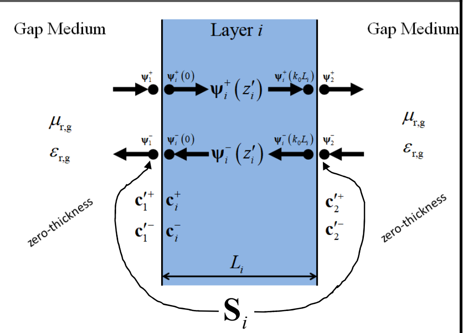


This is consistent with network theory and experimental convention.

Scattering Matrix for a Single Layer

The scattering matrix $\mathbf{S}_i$ of the i^{th} layer is still defined as:

$$\begin{bmatrix} \mathbf{c}_1'^{-} \\ \mathbf{c}_2'^{+} \end{bmatrix} = \mathbf{S}^{(i)} \begin{bmatrix} \mathbf{c}_1'^{+} \\ \mathbf{c}_2'^{-} \end{bmatrix} \qquad \mathbf{S}^{(i)} = \begin{bmatrix} \mathbf{S}_{11}^{(i)} & \mathbf{S}_{12}^{(i)} \\ \mathbf{S}_{21}^{(i)} & \mathbf{S}_{22}^{(i)} \end{bmatrix}$$



But the equations to calculate the elements reduce to

$$\mathbf{S}_{11}^{(i)} = \left(\mathbf{A}_i - \mathbf{X}_i\mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{X}_i\mathbf{B}_i\right)^{-1}\left(\mathbf{X}_i\mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{X}_i\mathbf{A}_i - \mathbf{B}_i\right)$$

$$\mathbf{S}_{12}^{(i)} = \left(\mathbf{A}_i - \mathbf{X}_i\mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{X}_i\mathbf{B}_i\right)^{-1}\mathbf{X}_i\left(\mathbf{A}_i - \mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{B}_i\right)$$

$$\mathbf{A}_i = \mathbf{W}_i^{-1}\mathbf{W}_g + \mathbf{V}_i^{-1}\mathbf{V}_g$$

$$\mathbf{B}_i = \mathbf{W}_i^{-1}\mathbf{W}_g - \mathbf{V}_i^{-1}\mathbf{V}_g$$

$$\mathbf{S}_{21}^{(i)} = \mathbf{S}_{12}^{(i)}$$

$$\mathbf{S}_{22}^{(i)} = \mathbf{S}_{11}^{(i)}$$

- Layers are symmetric so the scattering matrix elements have redundancy.
- Scattering matrix equations are simplified.
- Fewer calculations.
- Less memory storage.

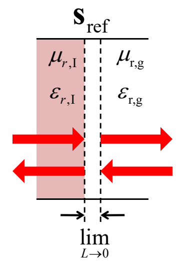
$$\mathbf{X}_i = e^{\boldsymbol{\lambda}_i k_0 L_i}$$

Reflection/Transmission Side Scattering Matrices

The reflection-side scattering matrix is

$$\mathbf{S}_{11}^{(\text{ref})} = -\mathbf{A}_{\text{ref}}^{-1}\mathbf{B}_{\text{ref}}$$
$$\mathbf{S}_{12}^{(\text{ref})} = 2\mathbf{A}_{\text{ref}}^{-1}$$
$$\mathbf{S}_{21}^{(\text{ref})} = 0.5\left(\mathbf{A}_{\text{ref}} - \mathbf{B}_{\text{ref}}\mathbf{A}_{\text{ref}}^{-1}\mathbf{B}_{\text{ref}}\right)$$
$$\mathbf{S}_{22}^{(\text{ref})} = \mathbf{B}_{\text{ref}}\mathbf{A}_{\text{ref}}^{-1}$$

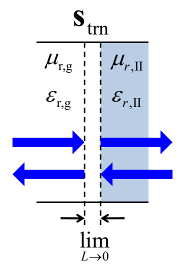
$$\mathbf{A}_{\text{ref}} = \mathbf{W}_{\text{g}}^{-1}\mathbf{W}_{\text{ref}} + \mathbf{V}_{\text{g}}^{-1}\mathbf{V}_{\text{ref}}$$
$$\mathbf{B}_{\text{ref}} = \mathbf{W}_{\text{g}}^{-1}\mathbf{W}_{\text{ref}} - \mathbf{V}_{\text{g}}^{-1}\mathbf{V}_{\text{ref}}$$



The transmission-side scattering matrix is

$$\mathbf{S}_{11}^{(\text{trn})} = \mathbf{B}_{\text{trn}}\mathbf{A}_{\text{trn}}^{-1}$$
$$\mathbf{S}_{12}^{(\text{trn})} = 0.5\left(\mathbf{A}_{\text{trn}} - \mathbf{B}_{\text{trn}}\mathbf{A}_{\text{trn}}^{-1}\mathbf{B}_{\text{trn}}\right)$$
$$\mathbf{S}_{21}^{(\text{trn})} = 2\mathbf{A}_{\text{trn}}^{-1}$$
$$\mathbf{S}_{22}^{(\text{trn})} = -\mathbf{A}_{\text{trn}}^{-1}\mathbf{B}_{\text{trn}}$$

$$\mathbf{A}_{\text{trn}} = \mathbf{W}_{\text{g}}^{-1}\mathbf{W}_{\text{trn}} + \mathbf{V}_{\text{g}}^{-1}\mathbf{V}_{\text{trn}}$$
$$\mathbf{B}_{\text{trn}} = \mathbf{W}_{\text{g}}^{-1}\mathbf{W}_{\text{trn}} - \mathbf{V}_{\text{g}}^{-1}\mathbf{V}_{\text{trn}}$$



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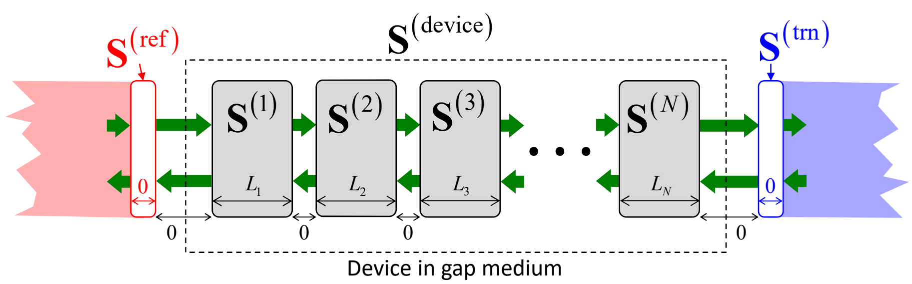
Summary of Using Scattering Matrices



$$\mathbf{S}^{(\text{global})} = \mathbf{S}^{(\text{ref})} \otimes \underbrace{\left[\mathbf{S}^{(1)} \otimes \mathbf{S}^{(2)} \otimes \cdots \otimes \mathbf{S}^{(N)}\right]}_{\mathbf{S}^{(\text{device})}} \otimes \mathbf{S}^{(\text{trn})}$$

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Redheffer Star Product

Two scattering matrices may be combined into a single scattering matrix using Redheffer's star product.

$$\mathbf{S}^{(AB)} = \mathbf{S}^{(A)} \otimes \mathbf{S}^{(B)} \qquad \mathbf{S}^{(A)} = \begin{bmatrix} \mathbf{S}_{11}^{(\mathrm{A})} & \mathbf{S}_{12}^{(\mathrm{A})} \\ \mathbf{S}_{21}^{(\mathrm{A})} & \mathbf{S}_{22}^{(\mathrm{A})} \end{bmatrix} \qquad \mathbf{S}^{(B)} = \begin{bmatrix} \mathbf{S}_{11}^{(\mathrm{B})} & \mathbf{S}_{12}^{(\mathrm{B})} \\ \mathbf{S}_{21}^{(\mathrm{B})} & \mathbf{S}_{22}^{(\mathrm{B})} \end{bmatrix}$$

The combined scattering matrix is then

$$\mathbf{S}^{(AB)} = \begin{bmatrix} \mathbf{S}_{11}^{(\mathrm{AB})} & \mathbf{S}_{12}^{(\mathrm{AB})} \\ \mathbf{S}_{21}^{(\mathrm{AB})} & \mathbf{S}_{22}^{(\mathrm{AB})} \end{bmatrix}$$

$$\mathbf{S}_{11}^{(\mathrm{AB})} = \mathbf{S}_{11}^{(\mathrm{A})} + \mathbf{S}_{12}^{(\mathrm{A})} \left[\mathbf{I} - \mathbf{S}_{11}^{(\mathrm{B})} \mathbf{S}_{22}^{(\mathrm{A})} \right]^{-1} \mathbf{S}_{11}^{(\mathrm{B})} \mathbf{S}_{21}^{(\mathrm{A})}$$

$$\mathbf{S}_{12}^{(\mathrm{AB})} = \mathbf{S}_{12}^{(\mathrm{A})} \left[\mathbf{I} - \mathbf{S}_{11}^{(\mathrm{B})} \mathbf{S}_{22}^{(\mathrm{A})} \right]^{-1} \mathbf{S}_{12}^{(\mathrm{B})}$$

$$\mathbf{S}_{21}^{(\mathrm{AB})} = \mathbf{S}_{21}^{(\mathrm{B})} \left[\mathbf{I} - \mathbf{S}_{22}^{(\mathrm{A})} \mathbf{S}_{11}^{(\mathrm{B})} \right]^{-1} \mathbf{S}_{21}^{(\mathrm{A})}$$

$$\mathbf{S}_{22}^{(\mathrm{AB})} = \mathbf{S}_{22}^{(\mathrm{B})} + \mathbf{S}_{21}^{(\mathrm{B})} \left[\mathbf{I} - \mathbf{S}_{22}^{(\mathrm{A})} \mathbf{S}_{11}^{(\mathrm{B})} \right]^{-1} \mathbf{S}_{22}^{(\mathrm{A})} \mathbf{S}_{12}^{(\mathrm{B})}$$

R. Redheffer, "Difference equations and functional equations in transmission-line theory," *Modern Mathematics for the Engineer*, Vol. 12, pp. 282-337, McGraw-Hill, New York, 1961.

Calculating Transmitted and Reflected Power

Recall How to Calculate Source Parameters

Incident Wave Vector

$$\vec{k}_{\text{inc}} = k_0 n_{\text{inc}} \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix}$$

Surface Normal

$$\hat{n} = \hat{a}_z = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

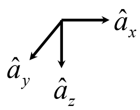
Unit Vectors in Direction of TE & TM

$$\hat{a}_{\text{TE}} = \begin{cases} \hat{a}_y & \theta = 0^\circ \\ \dfrac{\hat{n}\times\vec{k}_{\text{inc}}}{\left|\hat{n}\times\vec{k}_{\text{inc}}\right|} & \theta \neq 0^\circ \end{cases}$$

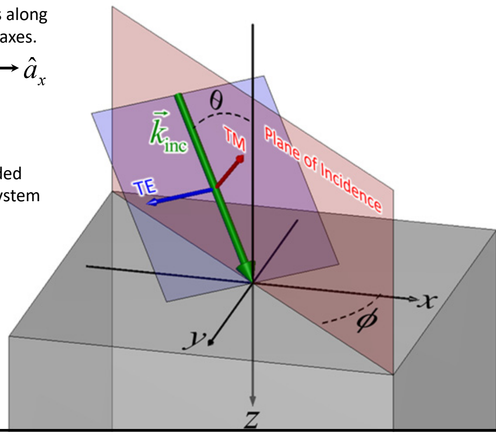
Can be any direction in the x-y plane

$$\hat{a}_{\text{TM}} = \frac{\hat{a}_{\text{TE}}\times\vec{k}_{\text{inc}}}{\left|\hat{a}_{\text{TE}}\times\vec{k}_{\text{inc}}\right|}$$

Unit vectors along x, y, and z axes.



Right-handed coordinate system



Composite Polarization Vector

$$\vec{P} = p_{\text{TE}}\hat{a}_{\text{TE}} + p_{\text{TM}}\hat{a}_{\text{TM}}$$

In CEM, we usually make

$$\left|\vec{P}\right| = 1$$

Solution Using Scattering Matrices

The external fields (i.e. incident wave, reflected wave, transmitted wave) are related through the global transfer matrix.

$$\begin{bmatrix} \mathbf{c}_{\text{ref}} \\ \mathbf{c}_{\text{trn}} \end{bmatrix} = \mathbf{S}^{(\text{global})} \begin{bmatrix} \mathbf{c}_{\text{inc}} \\ \mathbf{0} \end{bmatrix} \qquad \mathbf{c}_{\text{inc}} = \mathbf{W}_{\text{ref}}^{-1} \begin{bmatrix} E_{x,\text{inc}} \\ E_{y,\text{inc}} \end{bmatrix}$$

We get $E_{x,\text{inc}}$ and $E_{y,\text{inc}}$ from the polarization vector P.

Note that $E_{z,\text{inc}}$ is not needed.

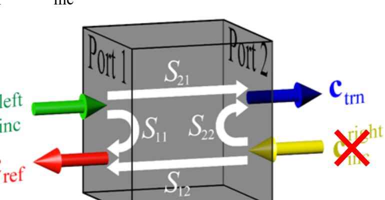
$$\begin{bmatrix} E_{x,\text{inc}} \\ E_{y,\text{inc}} \\ E_{z,\text{inc}} \end{bmatrix} = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}$$

This matrix equation can be solved to calculate the mode coefficients of the reflected and transmitted fields.

$$\begin{bmatrix} \mathbf{c}_{\text{ref}} \\ \mathbf{c}_{\text{trn}} \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{11}^{(\text{global})} & \mathbf{S}_{12}^{(\text{global})} \\ \mathbf{S}_{21}^{(\text{global})} & \mathbf{S}_{22}^{(\text{global})} \end{bmatrix} \begin{bmatrix} \mathbf{c}_{\text{inc}} \\ \mathbf{0} \end{bmatrix} \Rightarrow \begin{array}{l} \mathbf{c}_{\text{ref}} = \mathbf{S}_{11}^{(\text{global})}\mathbf{c}_{\text{inc}} \\ \mathbf{c}_{\text{trn}} = \mathbf{S}_{21}^{(\text{global})}\mathbf{c}_{\text{inc}} \end{array}$$

$\mathbf{c}_{\text{inc}}^{\text{right}}$ not typically used

$\mathbf{c}_{\text{inc}} = \mathbf{c}_{\text{inc}}^{\text{left}}$



Calculation of Transmitted and Reflected Fields

The procedure described thus far calculated $\mathbf{c}_{\text{ref}}$ and $\mathbf{c}_{\text{trn}}$.

The transmitted and reflected fields are then

$$\begin{bmatrix} E_x^{\text{ref}} \\ E_y^{\text{ref}} \end{bmatrix} = \mathbf{W}_{\text{ref}}\mathbf{c}_{\text{ref}} = \mathbf{W}_{\text{ref}}\mathbf{S}_{11}^{(\text{global})}\mathbf{c}_{\text{inc}} = \mathbf{W}_{\text{ref}}\mathbf{S}_{11}^{(\text{global})}\mathbf{W}_{\text{ref}}^{-1}\begin{bmatrix} E_x^{\text{inc}} \\ E_y^{\text{inc}} \end{bmatrix}$$

$$\begin{bmatrix} E_x^{\text{trn}} \\ E_y^{\text{trn}} \end{bmatrix} = \mathbf{W}_{\text{trn}}\mathbf{c}_{\text{trn}} = \mathbf{W}_{\text{trn}}\mathbf{S}_{21}^{(\text{global})}\mathbf{c}_{\text{inc}} = \mathbf{W}_{\text{trn}}\mathbf{S}_{21}^{(\text{global})}\mathbf{W}_{\text{ref}}^{-1}\begin{bmatrix} E_x^{\text{inc}} \\ E_y^{\text{inc}} \end{bmatrix}$$

Calculation of the Longitudinal Components

We are still missing the longitudinal field component E_z on the reflection and transmission sides.

These are calculated using Maxwell's divergence equation.

$$\nabla \bullet \vec{E} = 0$$

$$\frac{\partial}{\partial x}\left(E_{0,x}e^{j\vec{k}\bullet\vec{r}}\right) + \frac{\partial}{\partial y}\left(E_{0,y}e^{j\vec{k}\bullet\vec{r}}\right) + \frac{\partial}{\partial z}\left(E_{0,z}e^{j\vec{k}\bullet\vec{r}}\right) = 0$$

$$jk_x E_{0,x}e^{j\vec{k}\bullet\vec{r}} + jk_y E_{0,y}e^{j\vec{k}\bullet\vec{r}} + jk_z E_{0,z}e^{j\vec{k}\bullet\vec{r}} = 0$$

$$k_x E_{0,x} + k_y E_{0,y} + k_z E_{0,z} = 0$$

$$k_z E_{0,z} = -k_x E_{0,x} - k_y E_{0,y}$$

$$E_{0,z} = -\frac{k_x E_{0,x} + k_y E_{0,y}}{k_z}$$

$$E_z^{\text{ref}} = -\frac{\tilde{k}_x E_x^{\text{ref}} + \tilde{k}_y E_y^{\text{ref}}}{\tilde{k}_z^{\text{ref}}}$$

$$E_z^{\text{trn}} = -\frac{\tilde{k}_x E_x^{\text{trn}} + \tilde{k}_y E_y^{\text{trn}}}{\tilde{k}_z^{\text{trn}}}$$

Note:
$\nabla \bullet \left(\varepsilon \vec{E}\right) = 0$ reduces to
$\nabla \bullet \vec{E} = 0$ when ε is homogeneous.

Calculation of Power Flow

Reflectance is defined as the fraction of power reflected from a device.

$$R = \frac{\left|\vec{E}_{\text{ref}}\right|^2}{\left|\vec{E}_{\text{inc}}\right|^2} \qquad \left|\vec{E}\right|^2 = \left|E_x\right|^2 + \left|E_y\right|^2 + \left|E_z\right|^2$$

Transmittance is defined as the fraction of power transmitted through a device.

$$T = \frac{\left|\vec{E}_{\text{trn}}\right|^2}{\left|\vec{E}_{\text{inc}}\right|^2} \frac{\text{Re}\left[k_z^{\text{trn}} / \mu_{\text{r,trn}}\right]}{\text{Re}\left[k_z^{\text{inc}} / \mu_{\text{r,inc}}\right]}$$

Note: We will derive these formulas in Lecture 7.

It is always good practice to check for conservation of power.

$$R + T \rightarrow \begin{cases} < 1 & \text{materials have loss} \\ = 1 & \text{materials have no loss and no gain} \\ > 1 & \text{materials have gain} \end{cases}$$

Note: Recall
$A + R + T = 1$

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Reflectance and Transmittance on a Decibel Scale

Decibel Scale

$$P_{\text{dB}} = 20 \log_{10}(A)$$ How to calculate decibels from an amplitude quantity A.

$$P_{\text{dB}} = 10 \log_{10}(P)$$ How to calculate decibels from a power quantity P.

$P = A^2 \qquad P_{\text{dB}} = 10 \log_{10}(A^2) = 20 \log_{10}(A)$

Reflectance and Transmittance

Reflectance and transmittance are power quantities, so

$$R_{\text{dB}} = 10 \log_{10}(R)$$

$$T_{\text{dB}} = 10 \log_{10}(T)$$

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Simplifications for 1D Transfer Matrix Method

Analytical Expressions for W and λ

The dispersion relation with a normalized wave vector is

$$\mu_r \varepsilon_r = \tilde{k}_x^2 + \tilde{k}_y^2 + \tilde{k}_z^2$$

Using this relation, we can simplify the matrix equation for $\boldsymbol{\Omega}^2$.

$$\boldsymbol{\Omega}^2 = \mathbf{PQ} = \frac{1}{\mu_r \varepsilon_r} \begin{bmatrix} \tilde{k}_x \tilde{k}_y & \mu_r \varepsilon_r - \tilde{k}_x^2 \\ \tilde{k}_y^2 - \mu_r \varepsilon_r & -\tilde{k}_x \tilde{k}_y \end{bmatrix} \begin{bmatrix} \tilde{k}_x \tilde{k}_y & \mu_r \varepsilon_r - \tilde{k}_x^2 \\ \tilde{k}_y^2 - \mu_r \varepsilon_r & -\tilde{k}_x \tilde{k}_y \end{bmatrix} = \begin{bmatrix} -\tilde{k}_z^2 & 0 \\ 0 & -\tilde{k}_z^2 \end{bmatrix} = -\tilde{k}_z^2 \mathbf{I}$$

$$\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{identity matrix}$$

A lot of algebra

Since $\boldsymbol{\Omega}^2$ is a diagonal matrix, we can conclude that

$$\mathbf{W} = \mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \;\Rightarrow\; \boldsymbol{\lambda} = \begin{bmatrix} j\tilde{k}_z & 0 \\ 0 & j\tilde{k}_z \end{bmatrix} = j\tilde{k}_z \mathbf{I} \;\Rightarrow\; e^{\boldsymbol{\lambda} z'} = \begin{bmatrix} e^{j\tilde{k}_z z'} & 0 \\ 0 & e^{j\tilde{k}_z z'} \end{bmatrix}$$

$$\boldsymbol{\lambda}^2 = \boldsymbol{\Omega}^2$$

For isotropic materials and diagonally anisotropic materials, we don't actually have to solve the eigen-value problem to obtain the eigen-modes! ☺

Simplifications for TMM in LHI Media

In LHI media,

$$\mathbf{W}_i = \mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{\Omega}_i = j\tilde{k}_{z,i}\mathbf{I} \qquad \mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ identity matrix}$$

Now we do not actually have to calculate λ because

$$\mathbf{\lambda}_i = \mathbf{\Omega}_i$$

Given all of this, the eigen-vectors for the magnetic fields can be calculated as

$$\mathbf{V}_i = \mathbf{Q}_i\mathbf{W}_i\mathbf{\lambda}_i^{-1} = \underline{\mathbf{Q}_i\mathbf{\Omega}_i^{-1}}$$

When calculating scattering matrices, the intermediate matrices $\mathbf{A}_i$ and $\mathbf{B}_i$ reduce to

$$\mathbf{A}_i = \mathbf{W}_i^{-1}\mathbf{W}_g + \mathbf{V}_i^{-1}\mathbf{V}_g = \underline{\mathbf{I} + \mathbf{V}_i^{-1}\mathbf{V}_g}$$

$$\mathbf{B}_i = \mathbf{W}_i^{-1}\mathbf{W}_g - \mathbf{V}_i^{-1}\mathbf{V}_g = \underline{\mathbf{I} - \mathbf{V}_i^{-1}\mathbf{V}_g}$$

The fields and mode coefficients are now related through

$$\mathbf{c}_{\text{inc}} = \mathbf{W}_{\text{ref}}^{-1}\begin{bmatrix} P_x \\ P_y \end{bmatrix} = \underline{\begin{bmatrix} P_x \\ P_y \end{bmatrix}} \qquad \begin{bmatrix} E_x^{\text{ref}} \\ E_y^{\text{ref}} \end{bmatrix} = \mathbf{W}_{\text{ref}}\mathbf{S}_{11}\mathbf{c}_{\text{inc}} = \underline{\mathbf{S}_{11}\mathbf{c}_{\text{inc}}} \qquad \begin{bmatrix} E_x^{\text{trn}} \\ E_y^{\text{trn}} \end{bmatrix} = \mathbf{W}_{\text{trn}}\mathbf{S}_{21}\mathbf{c}_{\text{inc}} = \underline{\mathbf{S}_{21}\mathbf{c}_{\text{inc}}}$$

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Simplified External S-Matrices in LHI Media

The reflection-side scattering matrix is

$$\mathbf{S}_{11}^{(\text{ref})} = -\mathbf{A}_{\text{ref}}^{-1}\mathbf{B}_{\text{ref}}$$

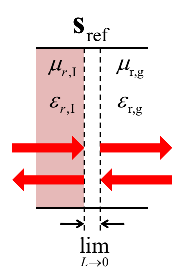
$$\mathbf{S}_{12}^{(\text{ref})} = 2\mathbf{A}_{\text{ref}}^{-1}$$

$$\mathbf{S}_{21}^{(\text{ref})} = 0.5\left(\mathbf{A}_{\text{ref}} - \mathbf{B}_{\text{ref}}\mathbf{A}_{\text{ref}}^{-1}\mathbf{B}_{\text{ref}}\right)$$

$$\mathbf{S}_{22}^{(\text{ref})} = \mathbf{B}_{\text{ref}}\mathbf{A}_{\text{ref}}^{-1}$$

$$\mathbf{A}_{\text{ref}} = \mathbf{I} + \mathbf{V}_g^{-1}\mathbf{V}_{\text{ref}}$$

$$\mathbf{B}_{\text{ref}} = \mathbf{I} - \mathbf{V}_g^{-1}\mathbf{V}_{\text{ref}}$$



The transmission-side scattering matrix is

$$\mathbf{S}_{11}^{(\text{trn})} = \mathbf{B}_{\text{trn}}\mathbf{A}_{\text{trn}}^{-1}$$

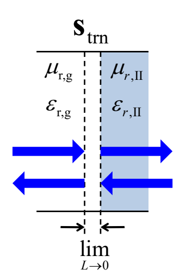
$$\mathbf{S}_{12}^{(\text{trn})} = 0.5\left(\mathbf{A}_{\text{trn}} - \mathbf{B}_{\text{trn}}\mathbf{A}_{\text{trn}}^{-1}\mathbf{B}_{\text{trn}}\right)$$

$$\mathbf{S}_{21}^{(\text{trn})} = 2\mathbf{A}_{\text{trn}}^{-1}$$

$$\mathbf{S}_{22}^{(\text{trn})} = -\mathbf{A}_{\text{trn}}^{-1}\mathbf{B}_{\text{trn}}$$

$$\mathbf{A}_{\text{trn}} = \mathbf{I} + \mathbf{V}_g^{-1}\mathbf{V}_{\text{trn}}$$

$$\mathbf{B}_{\text{trn}} = \mathbf{I} - \mathbf{V}_g^{-1}\mathbf{V}_{\text{trn}}$$



Lecture 5b — Slide 20

Notes on Implementation

Outline

CEM

- Step 0 – Define problem

human does this

computer does the rest

- Step 1 – Dashboard
- Step 2 – Describe device layers
- Step 3 – Compute wave vector components
- Step 4 – Compute gap medium parameters
- Step 5 – Initialize global scattering matrix
- Step 6 – Main loop through layers
- Step 7 – Compute reflection side scattering matrix
- Step 8 – Compute transmission side scattering matrix
- Step 9 – Update global scattering matrix
- Step 10 – Compute source
- Step 11 – Compute reflected and transmitted fields
- Step 12 – Compute reflectance and transmittance
- Step 13 – Verify conservation of power

Step 6: Iterate through layers

- Compute $\mathbf{P}$ and $\mathbf{Q}$
- Compute eigen-modes
- Compute layer scattering matrix
- Update global scattering matrix

Storing the Problem

How is a device described and stored for TMM?

We don't use a grid for this method*!*

Store the permittivity for each layer in a 1D array.
Store the permeability for each layer in a 1D array.
Store the thickness of each layer in a 1D array.

```
ER = [ 2.50 , 3.50 , 2.00 ];
UR = [ 1.00 , 1.00 , 1.00 ];
L  = [ 0.25 , 0.75 , 0.89 ];
```

Input arrays for three layers

We will also need the external materials, and source parameters.

`er1`, `er2`, `ur1`, `ur2`, `theta`, `phi`, `pte`, `ptm`, and `lam0`

Storing Scattering Matrices

We often talk about the scattering matrix $\mathbf{S}$ as a single matrix.

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{bmatrix}$$

However, we very rarely ally use the scattering matrix $\mathbf{S}$ this way.
We usually use the individual terms $\mathbf{S}_{11}$, $\mathbf{S}_{12}$, $\mathbf{S}_{21}$, and $\mathbf{S}_{22}$ separately.

So, scattering matrices are actually best stored as the four separate components of the scattering matrix.

$$\cancel{\mathbf{S} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{21} & \mathbf{S}_{22} \end{bmatrix}} \quad \Rightarrow \quad \mathbf{S}_{11},\ \mathbf{S}_{12},\ \mathbf{S}_{21},\ \text{and } \mathbf{S}_{22}$$

Initializing the Global Scattering Matrix

Before we iterate through all the layers, we must initialize the global scattering matrix as the scattering matrix of "nothing."

What are the ideal properties of nothing?

1. Transmits 100% of power with no phase change.

$$\mathbf{S}_{12}^{(\text{global})} = \mathbf{S}_{21}^{(\text{global})} = \mathbf{I}$$

2. Does not reflect.

$$\mathbf{S}_{11}^{(\text{global})} = \mathbf{S}_{22}^{(\text{global})} = \mathbf{0}$$

We therefore initialize our global scattering matrix as

$$\mathbf{S}^{(\text{global})} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}$$

This is NOT an identity matrix*!*
Look at the position of the $\mathbf{0}$'s and $\mathbf{I}$'s.

Lecture 5b Slide 25

Calculating the Parameters of the Gap Media

Our analytical solution for a homogeneous gap medium is

$$\mathbf{Q}_{\text{g}} = \frac{1}{\mu_{\text{r,g}}} \begin{bmatrix} \tilde{k}_x \tilde{k}_y & \mu_{\text{r,g}}\varepsilon_{\text{r,g}} - \tilde{k}_x^2 \\ \tilde{k}_y^2 - \mu_{\text{r,g}}\varepsilon_{\text{r,g}} & -\tilde{k}_x \tilde{k}_y \end{bmatrix}$$

$$\tilde{k}_{z,\text{g}}^2 = \mu_{\text{r,g}}\varepsilon_{\text{r,g}} - \tilde{k}_x^2 - \tilde{k}_y^2$$

$$\mathbf{W}_{\text{g}} = \mathbf{I}$$

$$\boldsymbol{\lambda}_{\text{g}} = j\tilde{k}_{z,\text{g}}\mathbf{I}$$

$$\mathbf{V}_{\text{g}} = \mathbf{Q}_{\text{g}}\mathbf{W}_{\text{g}}\boldsymbol{\lambda}_{\text{g}}^{-1}$$

We are free to choose any $\mu_{\text{r,g}}$ and $\varepsilon_{\text{r,g}}$ that we wish. We also wish to avoid the case of $k_{z,\text{g}} = 0$. For convenience, we choose

$$\mu_{\text{r,g}} = 1.0 \quad \text{and} \quad \varepsilon_{\text{r,g}} = 1 + \tilde{k}_x^2 + \tilde{k}_y^2$$

We then have

$$\mathbf{Q}_{\text{g}} = \begin{bmatrix} \tilde{k}_x \tilde{k}_y & 1 + \tilde{k}_y^2 \\ -\left(1 + \tilde{k}_x^2\right) & -\tilde{k}_x \tilde{k}_y \end{bmatrix}$$

$$\mathbf{W}_{\text{g}} = \mathbf{I}$$

$$\mathbf{V}_{\text{g}} = -j\mathbf{Q}_{\text{g}}$$

$\mathbf{W}$ not even used in TMM.

Lecture 5b Slide 26

Calculating $\mathbf{X}_i = \exp(\Omega_i k_0 L_i)$

Recall the correct answer:

$$\mathbf{X}_i = e^{\mathbf{\Omega}_i k_0 L_i} = \begin{bmatrix} e^{j\tilde{k}_z k_0 L_i} & 0 \\ 0 & e^{j\tilde{k}_z k_0 L_i} \end{bmatrix}$$

It is incorrect to use the function `exp()` because this calculates a point-by-point exponential, not a matrix exponential.

WRONG

```
X = exp(OMEGA*k0*L);
```

→

```
X =
   0.0135 + 0.9999i   1.0000
   1.0000             0.0135 + 0.9999i
```

Approach #1: `expm()`

```
X = expm(OMEGA*k0*L);

X =
   0.0135 + 0.9999i        0
        0             0.0135 + 0.9999i
```

Approach #2: `diag()`

```
X = diag(exp(diag(OMEGA)*k0*L));

X =
   0.0135 + 0.9999i        0
        0             0.0135 + 0.9999i
```


Efficient Calculation of Layer S-Matrices

There are redundant calculations in the equations for the scattering matrix elements.

$$\mathbf{S}_{11}^{(i)} = \mathbf{S}_{22}^{(i)} = \left(\mathbf{A}_i - \mathbf{X}_i\mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{X}_i\mathbf{B}_i\right)^{-1}\left(\mathbf{X}_i\mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{X}_i\mathbf{A}_i - \mathbf{B}_i\right)$$

$$\mathbf{S}_{12}^{(i)} = \mathbf{S}_{21}^{(i)} = \left(\mathbf{A}_i - \mathbf{X}_i\mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{X}_i\mathbf{B}_i\right)^{-1}\mathbf{X}_i\left(\mathbf{A}_i - \mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{B}_i\right)$$

These are more efficiently calculated as

$$\mathbf{A}_i = \mathbf{I} + \mathbf{V}_i^{-1}\mathbf{V}_g$$

$$\mathbf{B}_i = \mathbf{I} - \mathbf{V}_i^{-1}\mathbf{V}_g$$

$$\mathbf{X}_i = e^{\lambda_i k_0 L_i}$$

$$\mathbf{D} = \mathbf{A}_i - \mathbf{X}_i\mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{X}_i\mathbf{B}_i$$

$$\mathbf{S}_{11}^{(i)} = \mathbf{S}_{22}^{(i)} = \mathbf{D}^{-1}\left(\mathbf{X}_i\mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{X}_i\mathbf{A}_i - \mathbf{B}_i\right)$$

$$\mathbf{S}_{12}^{(i)} = \mathbf{S}_{21}^{(i)} = \mathbf{D}^{-1}\mathbf{X}_i\left(\mathbf{A}_i - \mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{B}_i\right)$$

Efficient Star Product

After observing the equations to implement the Redheffer star product, we see there are some common terms. Calculating these multiple times is inefficient so we calculate them only once using intermediate parameters.

$$\mathbf{S}^{(AB)} = \mathbf{S}^{(A)} \otimes \mathbf{S}^{(B)}$$

$$\mathbf{S}_{11}^{(AB)} = \mathbf{S}_{11}^{(A)} + \mathbf{S}_{12}^{(A)} \left[\mathbf{I} - \mathbf{S}_{11}^{(B)}\mathbf{S}_{22}^{(A)}\right]^{-1} \mathbf{S}_{11}^{(B)}\mathbf{S}_{21}^{(A)}$$

$$\mathbf{S}_{12}^{(AB)} = \mathbf{S}_{12}^{(A)} \left[\mathbf{I} - \mathbf{S}_{11}^{(B)}\mathbf{S}_{22}^{(A)}\right]^{-1} \mathbf{S}_{12}^{(B)}$$

$$\mathbf{S}_{21}^{(AB)} = \mathbf{S}_{21}^{(B)} \left[\mathbf{I} - \mathbf{S}_{22}^{(A)}\mathbf{S}_{11}^{(B)}\right]^{-1} \mathbf{S}_{21}^{(A)}$$

$$\mathbf{S}_{22}^{(AB)} = \mathbf{S}_{22}^{(B)} + \mathbf{S}_{21}^{(B)} \left[\mathbf{I} - \mathbf{S}_{22}^{(A)}\mathbf{S}_{11}^{(B)}\right]^{-1} \mathbf{S}_{22}^{(A)}\mathbf{S}_{12}^{(B)}$$

$$\mathbf{D} = \mathbf{S}_{12}^{(A)} \left[\mathbf{I} - \mathbf{S}_{11}^{(B)}\mathbf{S}_{22}^{(A)}\right]^{-1}$$

$$\mathbf{F} = \mathbf{S}_{21}^{(B)} \left[\mathbf{I} - \mathbf{S}_{22}^{(A)}\mathbf{S}_{11}^{(B)}\right]^{-1}$$

$$\mathbf{S}_{11}^{(AB)} = \mathbf{S}_{11}^{(A)} + \mathbf{D}\mathbf{S}_{11}^{(B)}\mathbf{S}_{21}^{(A)}$$

$$\mathbf{S}_{12}^{(AB)} = \mathbf{D}\mathbf{S}_{12}^{(B)}$$

$$\mathbf{S}_{21}^{(AB)} = \mathbf{F}\mathbf{S}_{21}^{(A)}$$

$$\mathbf{S}_{22}^{(AB)} = \mathbf{S}_{22}^{(B)} + \mathbf{F}\mathbf{S}_{22}^{(A)}\mathbf{S}_{12}^{(B)}$$

Using the Star Product as an Update

Very often we update our global scattering matrix using a star product.

When we use this equation as an update, we MUST pay close attention to the order that we implement the equations so that we don't accidentally overwrite a value that we need.

$$\mathbf{S}^{(\text{global})} = \mathbf{S}^{(i)} \otimes \mathbf{S}^{(\text{global})}$$

$$\mathbf{D} = \mathbf{S}_{12}^{(i)} \left[\mathbf{I} - \mathbf{S}_{11}^{(\text{global})}\mathbf{S}_{22}^{(i)}\right]^{-1}$$

$$\mathbf{F} = \mathbf{S}_{21}^{(\text{global})} \left[\mathbf{I} - \mathbf{S}_{22}^{(i)}\mathbf{S}_{11}^{(\text{global})}\right]^{-1}$$

reverse order (↑)

$$\mathbf{S}_{22}^{(\text{global})} = \mathbf{S}_{22}^{(\text{global})} + \mathbf{F}\mathbf{S}_{22}^{(i)}\mathbf{S}_{12}^{(\text{global})}$$

$$\mathbf{S}_{21}^{(\text{global})} = \mathbf{F}\mathbf{S}_{21}^{(i)}$$

$$\mathbf{S}_{12}^{(\text{global})} = \mathbf{D}\mathbf{S}_{12}^{(\text{global})}$$

$$\mathbf{S}_{11}^{(\text{global})} = \mathbf{S}_{11}^{(i)} + \mathbf{D}\mathbf{S}_{11}^{(\text{global})}\mathbf{S}_{21}^{(i)}$$

$$\mathbf{S}^{(\text{global})} = \mathbf{S}^{(\text{global})} \otimes \mathbf{S}^{(i)}$$

$$\mathbf{D} = \mathbf{S}_{12}^{(\text{global})} \left[\mathbf{I} - \mathbf{S}_{11}^{(i)}\mathbf{S}_{22}^{(\text{global})}\right]^{-1}$$

$$\mathbf{F} = \mathbf{S}_{21}^{(i)} \left[\mathbf{I} - \mathbf{S}_{22}^{(\text{global})}\mathbf{S}_{11}^{(i)}\right]^{-1}$$

standard order (↓)

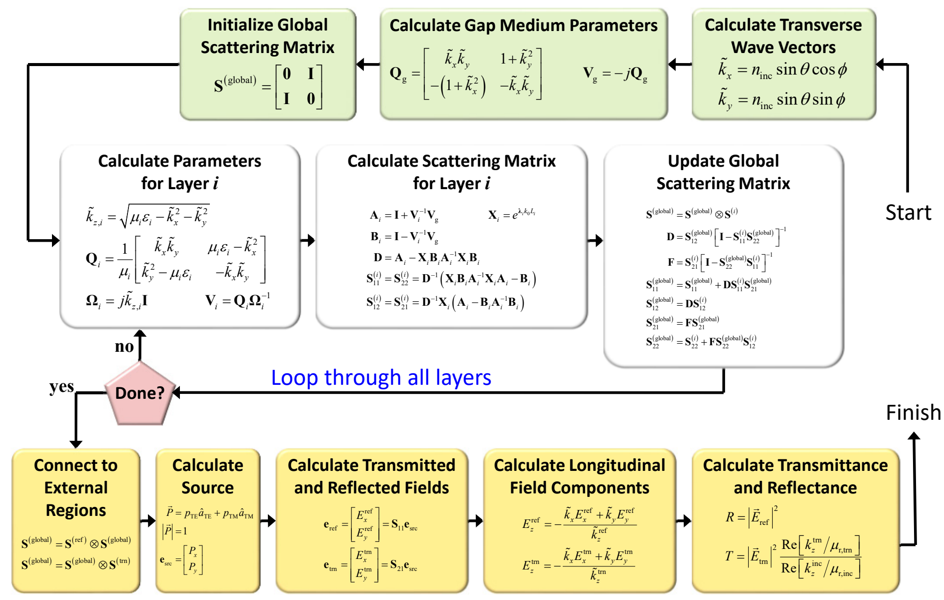
$$\mathbf{S}_{11}^{(\text{global})} = \mathbf{S}_{11}^{(\text{global})} + \mathbf{D}\mathbf{S}_{11}^{(i)}\mathbf{S}_{21}^{(\text{global})}$$

$$\mathbf{S}_{12}^{(\text{global})} = \mathbf{D}\mathbf{S}_{12}^{(i)}$$

$$\mathbf{S}_{21}^{(\text{global})} = \mathbf{F}\mathbf{S}_{21}^{(\text{global})}$$

$$\mathbf{S}_{22}^{(\text{global})} = \mathbf{S}_{22}^{(i)} + \mathbf{F}\mathbf{S}_{22}^{(\text{global})}\mathbf{S}_{12}^{(i)}$$

Block Diagram of TMM Using S-Matrices



How to Handle Zero Number of Layers

Follow the block diagram!!

Setup your loop this way…

```
NLAY = length(L);
for nlay = 1 : NLAY
    ...
end
```

For zero layers:

```
ER = [];
UR = [];
L  = [];
```

If NLAY $= 0$, then the loop will not execute and the global scattering matrix will remain as it was initialized.

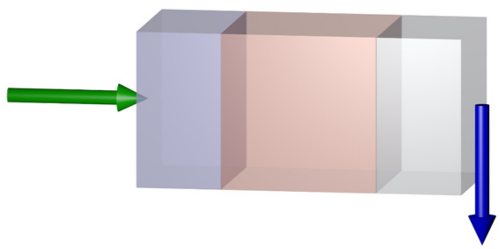
$$\mathbf{S}^{(\text{global})} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}$$

Can TMM Fail?

Yes!

The TMM can fail to give an answer and behave numerically strange any time $k_z = 0$. This happens at a critical angle when the transmitted wave is at or very near its cutoff.

We fixed this problem in the gap medium, but this can also happen in any of the layers or in the transmission region.



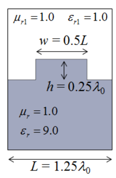
This happens in any layer where

$$\mu_r \varepsilon_r = \tilde{k}_x^2 + \tilde{k}_y^2$$

Parameter Sweeps

What is a Parameter Sweep?

So far, we have learned to simulate a single device at a single frequency, or wavelength.

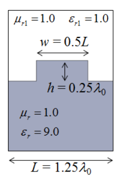


Sim →

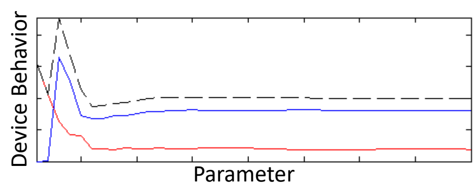
R = 81%
T = 19%

Parameter sweeps are perhaps the most powerful tool in the analysis arsenal.

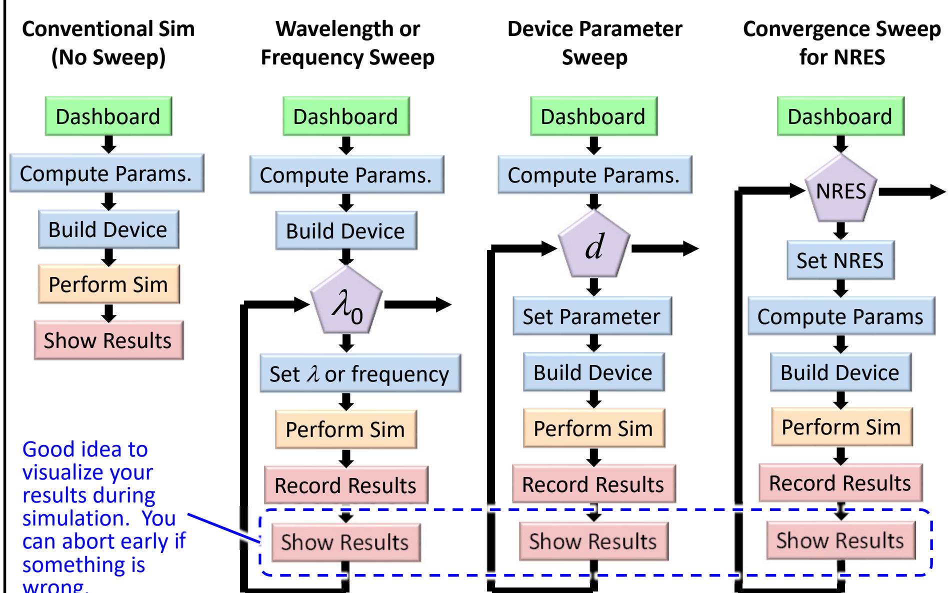
Suppose we calculate this data as we continuously change one or more parameters? This is called a parameter sweep.



Sim →



Block Diagrams of Common Parameter Sweeps



Make a Generic TMM Function

A great way to simplify programming your parameter sweeps is to first make a generic function out of your TMM code.

The basic TMM simulation will take as input arguments:

Source: λ_0, θ, ϕ, polarization, etc.
Device: `UR`, `ER`, `L`, etc.

Given these input arguments, your TMM function will simulate the device and calculate reflectance, transmittance, fields, etc.

It may return `REF`, `TRN`, or whatever else you wish.

Example Header for a Generic TMM Function

These comments are displayed at the command prompt by typing

`>>help tmm1d`

It is always a good idea to include a help section at the start of your codes.

```
function DAT = tmm1d(DEV,SRC)
% TMM1D     One-Dimensional Transfer Matrix Method
%
% DAT = tmm1d(DEV,SRC);
%
% INPUT ARGUMENTS
% ================
% DEV       Device Parameters
%   .er1    relative permittivity in reflection region
%   .ur1    relative permeability in reflection region
%   .er2    relative permittivity in transmission region
%   .ur2    relative permeability in transmission region
%   .ER     array containing permittivity of each layer
%   .UR     array containing permeability of each layer
%   .L      array containing thickness of each layer
%
% SRC       Source Parameters
%   .lam0   free space wavelength
%   .theta  elevation angle of incidence (radians)
%   .phi    azimuthal angle of incidence (radians)
%   .ate    amplitude of TE polarization
%   .atm    amplitude of TM polarization
%
% OUTPUT ARGUMENTS
% ================
% DAT       Output Data
%   .REF    Reflectance
%   .TRN    Transmittance
```

What Steps are Performed by TMM1D()

- Step 0 – Define problem
- Step 1 – Dashboard
- Step 2 – Describe device layers
- Step 3 – Compute wave vector components
- Step 4 – Compute gap medium parameters
- Step 5 – Initialize global scattering matrix
- Step 6 – Main loop through layers
- Step 7 – Compute reflection side scattering matrix
- Step 8 – Compute transmission side scattering matrix
- Step 9 – Update global scattering matrix
- Step 10 – Compute source
- Step 11 – Compute reflected and transmitted fields
- Step 12 – Compute reflectance and transmittance
- Step 13 – Verify conservation of power

human does this

computer does the rest

Step 6: Iterate through layers

- Compute **P** and **Q**
- Compute eigen-modes
- Compute layer scattering matrix
- Update global scattering matrix

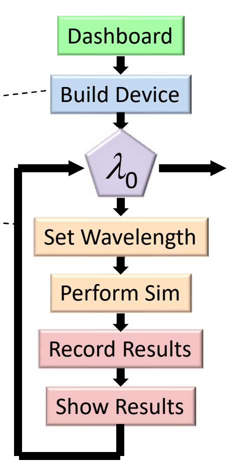
`tmm1d()`

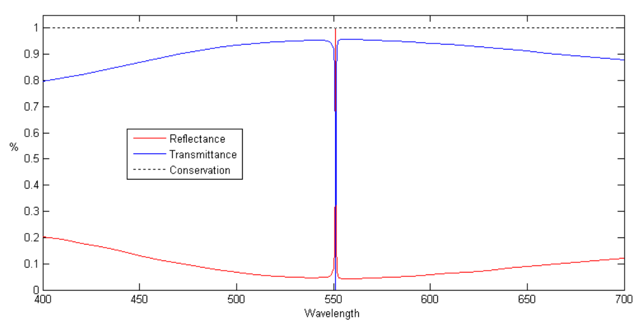
Wavelength or Frequency Parameter Sweep

By far, the most common parameter sweep is calculating the device behavior as a function of frequency or wavelength.

```
UR = [ 1 1 1 ];
ER = [ 2.5 6.0 2.0 ];
L  = [ 0.5 0.78 0.25 ];
```

```
for nlam = 1 : NLAM
    SRC.lam0 = LAMBDA(nlam);
    DAT = tmm1d(DEV,SRC);
    REF(nlam) = DAT.REF;
end
```





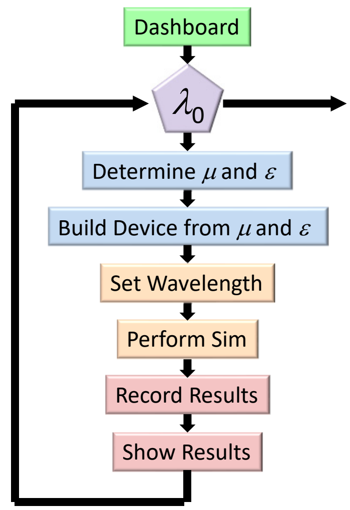
Incorporating Material Dispersion in a Parameter Sweep

Sometimes the material properties change significantly as a function of frequency, or wavelength.

This is called dispersion.

Dispersion can be incorporated into your parameter sweep by:

(1) Calculate the material properties at the given wavelength or frequency.
(2) Rebuild the device each iteration with the material properties that were just calculated.



Bad Vs. Good Parameter Sweeps