Computational Science:
Computational Methods in Engineering

Introduction to Optimization

Outline

- Introduction
- The Merit Function

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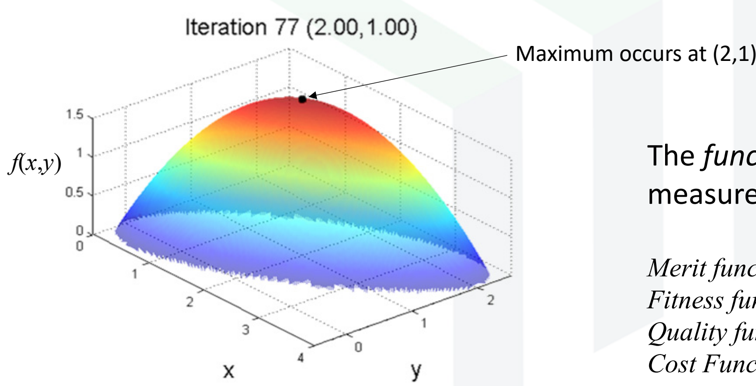
Introduction

What is Optimization?

Optimization is simply finding the minimum or a maximum of a function. In this regard, it is similar to root finding.



The *function* is usually chosen to be a measure of how "good" something is.

Merit function
Fitness function
Quality function
Cost Function
…

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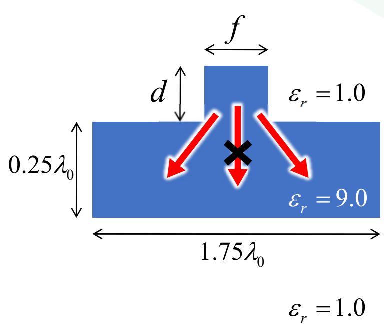
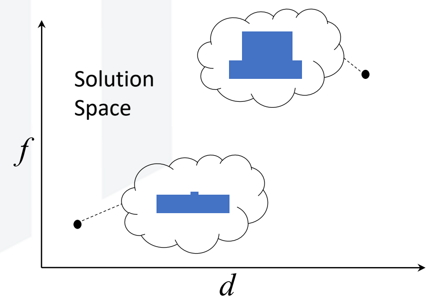
A Simple Example

Suppose it is needed to choose *f* and *d* so as to prevent diffraction into the zero-order transmitted mode for a normally incident wave.

What values of *f* and *d* do this?





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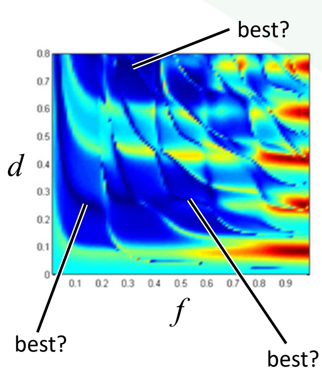
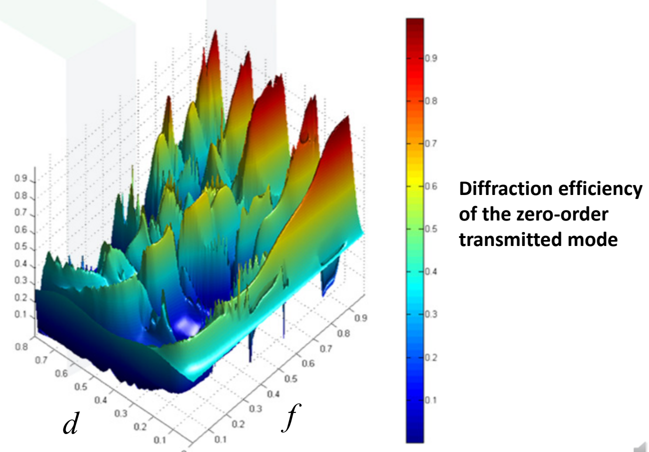
Global Best Vs. Local Best

The solution space is often sprinkled among many possible solutions (local extrema). It is the primary goal of optimization to find the absolute best solution (global extremum). Without having some apriori knowledge of the solution, however, it is usually impossible to determine if the solution is a global best solution.





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Common Optimization Algorithms

- Direct Methods
  - Complete Search
  - Gradient Methods

Only guaranteed method for finding the global extrema.

Converges very quickly to a local extremum. No global search.

- Stochastic Optimization
  - Particle Swarm Optimization
  - Genetic Algorithms
  - Simulated Annealing

Searches globally. Usually finds a good solution. No guarantee it is a global best solution.

Random search to converge to a local best solution.

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Notes on Optimization

- Stochastic methods are used most effectively when very little is known about the solution space. That is, when the engineer has no idea what the best design will look like.
- Unless something is known about the solution space, it is not possible to certify that the global best solution has been found. Engineers are usually satisfied with "good enough" solutions.
- Only an exhaustive complete search can guarantee a global best solution has been found unless something else is known.
- Direct methods converge very fast, but can only find local best solutions.
- It's much more about the merit function instead of the algorithm.

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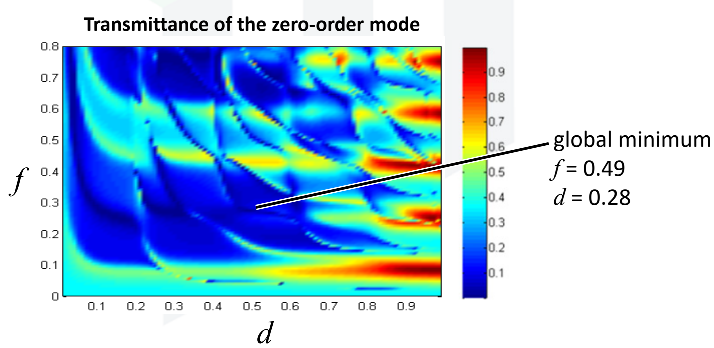
The Merit Function

The Merit Function

A single number is needed that quantifies how "good" a particular solution is. This is called the *merit function*. The optimization can attempt to minimize or maximize this merit function. In this case, the merit function is the diffraction efficiency in the zero-order transmitted mode and it is desired figure out values of f and d that minimize it.

$$M(f,d) = T_0 \equiv \text{transmittance of zero-order mode}$$



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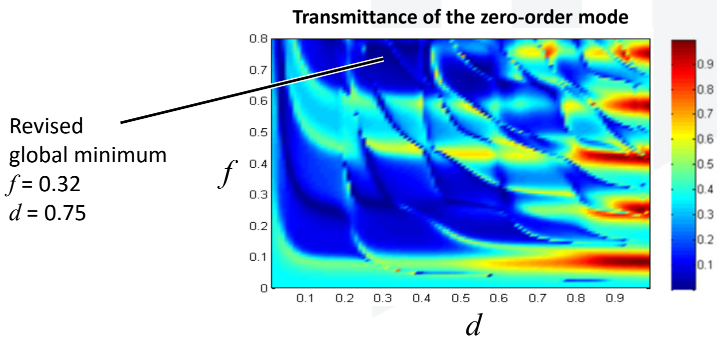
Rethink the Merit Function!!!

In the previous case, the global best was a very narrow region. This probably isn't feasible when fabrication tolerances are considered. It also assumes the model is perfectly accurate. It is often good practice to include some type of "bandwidth" in the merit function to pick a minimum that is also broad and robust.

$$M\left(f,d\right)=T_0 \div W_0 \qquad W_0 \equiv \text{width of extrema}$$



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Multiple Considerations

The merit function can be challenging to formulate when multiple things must be considered.

There is no cookbook way of doing this. It is up to the ingenuity of the engineer to arrive at this.

A common approach is to form a product where teach term is a separate consideration.

$$M=\frac{A_1\cdot A_2\cdot A_3\cdots A_M}{B_1\cdot B_2\cdot B_3\cdots B_N}$$

Parameters to maximize (minimize)

Parameters to minimize (maximize)

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Incorporating Relative Importance

If the considerations are not of equal importance, a way is needed to enhance or suppress their impact on the merit function.

It is usually not possible to just scale them by a constant.

$$M = aA_1 \cdot bA_2 \cdot cA_3 = abc\left(A_1 \cdot A_2 \cdot A_3\right)$$

There are some effective ways of doing this.

$$M = A_1^{\alpha} \cdot A_2^{\beta} \cdot A_3^{\gamma}$$ exponents

$$M = A_1 \cdot \log\left(A_2\right) \cdot A_3$$ logarithms

$$M = A_1 \cdot \left(1 + A_2\right) \cdot A_3$$ adding constants

$$M = A_1^{\alpha} \cdot \left(1 + A_2\right)^{\beta} \cdot \log\left(1 + A_3\right)$$ hybrids

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Example (1 of 2)

Suppose the design of an antenna is to be optimized.

It is probably the bandwidth B and efficiency E that are of most concern. Based on this, a merit function could be designed as

$$M = B \cdot E$$

Are B and E equally important?

The *Shannon capacity theorem* sets a limit on the bit rate C of data given the bandwidth B of the channel and the signal-to-noise ratio SNR.

$$C = B \log_2\left(1 + \text{SNR}\right)$$

This shows that bandwidth and efficiency are not equally important.

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Example (2 of 2)

A better merit function inspired by the *Shannon capacity theorem* is

$$M = B\log_2\left(1+E\right)$$

Maybe size L is also a consideration. Perhaps it is desired that data rate be as high as possible while using an antenna as small as possible. A new merit function that considers size could be

$$M = \frac{B}{L}\log_2\left(1+E\right)$$

This merit function, however, would approach infinity as L approaches zero, leading to a false solution.

This problem can be fixed by adding a constant to L.

$$M = \frac{B}{1+L}\log_2\left(1+E\right)$$

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