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Electromagnetics:
Electromagnetic Field Theory

# Terms & Definitions

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# Outline

- Maxwell's Equations
- Terms and Definitions
- Types of Electric Current
- Duality

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# Maxwell's Equations

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## Forms of Maxwell's Equations

| | Integral Form | Differential Form |
|---|---|---|
| Time-Domain | **Most general form**<br>$\oiint_S \vec{D} \bullet d\vec{s} = \iiint_V \rho_v dv$<br>$\oiint_S \vec{B} \bullet d\vec{s} = 0$<br>$\oint_L \vec{E} \bullet d\vec{\ell} = -\iint_S \left[ \frac{\partial \vec{B}}{\partial t} \right] \bullet d\vec{s}$<br>$\oint_L \vec{H} \bullet d\vec{\ell} = \iint_S \left[ \vec{J} + \frac{\partial \vec{D}}{\partial t} \right] \bullet d\vec{s}$ | $\nabla \bullet \vec{D} = \rho_v$<br>$\nabla \bullet \vec{B} = 0$<br>$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$<br>$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ |
| Frequency-Domain | $\oiint_S \vec{D} \bullet d\vec{s} = \iiint_V \rho_v dv$<br>$\oiint_S \vec{B} \bullet d\vec{s} = 0$<br>$\oint_L \vec{E} \bullet d\vec{\ell} = -\iint_S \left[ j\omega \vec{B} \right] \bullet d\vec{s}$<br>$\oint_L \vec{H} \bullet d\vec{\ell} = \iint_S \left[ \vec{J} + j\omega \vec{D} \right] \bullet d\vec{s}$ | **Most common form**<br>$\nabla \bullet \vec{D} = \rho_v$<br>$\nabla \bullet \vec{B} = 0$<br>$\nabla \times \vec{E} = -j\omega \vec{B}$<br>$\nabla \times \vec{H} = \vec{J} + j\omega \vec{D}$ |

**Constitutive Relations:** $\vec{D} = \varepsilon \vec{E}$ $\vec{B} = \mu \vec{H}$

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## Notes About Maxwell's Equations

- The time-domain integral form is the most general.
- The frequency-domain differential form is the most common.
- Maxwell's equations are rarely directly useful. We derive useful equations from them.
- Maxwell's equations do not directly describe how electromagnetic fields interact with materials. This information comes from the constitutive relations.
- Electric fields do not directly experience permeability.
- Magnetic fields do not directly experience permittivity.

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# Terms and Definitions

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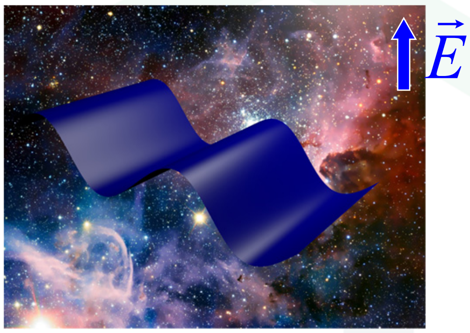
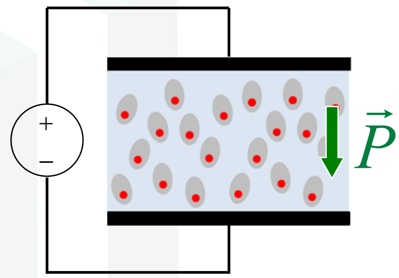
# Two Forms of Electric Energy

Electric energy can exist in at least two different forms: $\vec{E}$ and $\vec{P}$




The mere fact that stars can be seen and radio waves can be transmitted through space means that electric energy can exist in a vacuum.

From capacitors, it is known that electric energy can be stored inside of a material when the material becomes polarized.

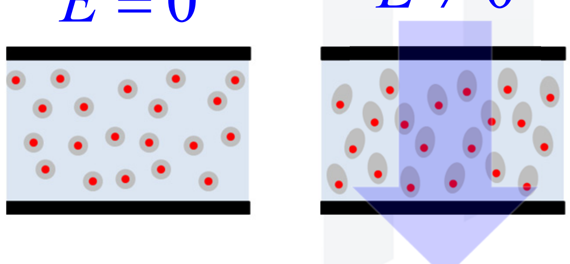


# Electric Field Intensity, $\vec{E}$ (V/m)

Think of the electric field intensity $\vec{E}$ as the form of electric energy that exists in the vacuum of space.

The electric field intensity $\vec{E}$ is most closely related to voltage $V$ and force $\vec{F}$. It is the initial "push" on a dielectric medium.

$\vec{E} = 0$ $\quad\quad$ $\vec{E} \neq 0$



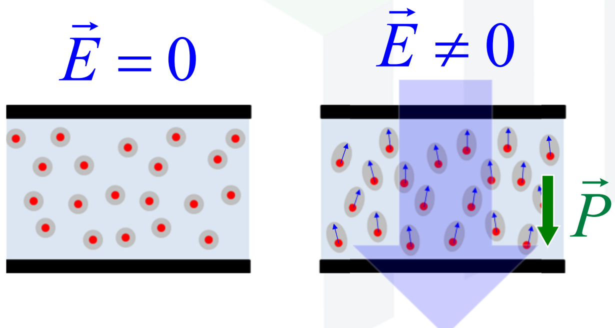
Force on a charge, $\vec{F}$

$\vec{F} = Q\vec{E}$

Protons and electrons have opposite charge so they are forced in opposite directions in the presence of an applied electric field.

# Electric Polarization, $\vec{P}$ (C/m$^2$)

All charged particles experience a force when an electric field is applied. Electrons and protons have opposite charge so they are pushed in opposite directions. This stretches the electron orbitals, creating electric dipoles that induce their own electric fields. A material said to be *electrically polarized* when these dipoles are present.




Electric Polarization, $\vec{P}$

$\vec{P} = \varepsilon_0 \chi_e \vec{E}$

The electric susceptibility $\chi_e$ is a measure of how easily a material can be polarized due to an applied electric field.

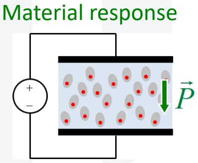


# Electric Flux Density, $\vec{D}$ (C/m$^2$)

The electric field induced by dipoles within a material combines with external electric fields to produce an overall electric flux. The combined electric flux is the electric flux density $\vec{D}$.

$$\vec{D} = \underbrace{\varepsilon_0 \vec{E}}_{\text{Vacuum response}} + \underbrace{\vec{P}}_{\text{Material response}}$$

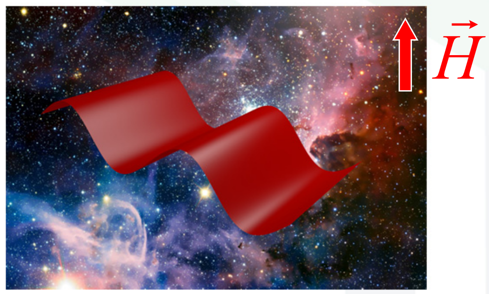
Note: $\varepsilon_0$ is the free space permittivity and multiplies $\vec{E}$ so that $\varepsilon_0 \vec{E}$ has the same units as $\vec{P}$.

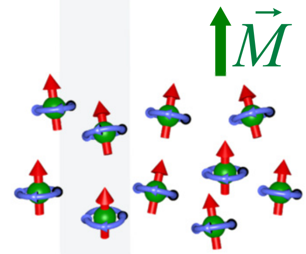




The electric flux density $\vec{D}$ is most closely related to charge.

# Two Forms of Magnetic Energy

Magnetic energy can exist in at least two different forms: $\vec{H}$ and $\vec{M}$




The mere fact that stars can be seen and radio waves can be transmitted through space means that magnetic energy can exist in a vacuum.

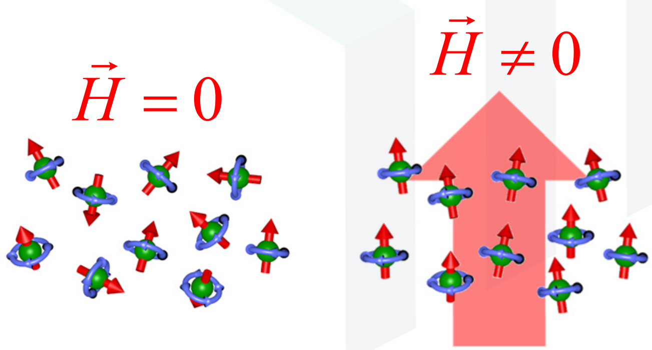
From inductors, it is known that magnetic energy can be stored inside of a material when the material becomes magnetized.



# Magnetic Field Intensity, $\vec{H}$ (A/m)

Think of the magnetic field intensity $\vec{H}$ as the form of magnetic energy that exists in the vacuum of space.

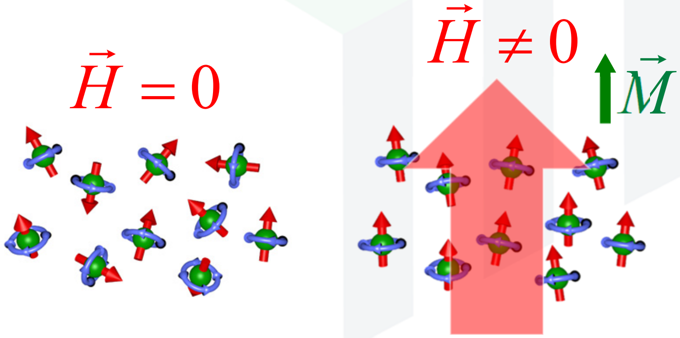
The magnetic field intensity $\vec{H}$ is most closely related to electric current $I$. It is the initial "torque" put on a magnetic dipole.




Magnetic dipoles naturally occur due to circulating charge at the atomic scale. An external magnetic field will act to align the magnetic dipoles.

## Magnetic Polarization, $\vec{M}$ (T or Wb/m$^2$)

When a magnetic field is applied to a material, it puts a mechanical torque on all magnetic dipoles. The dipoles tend to align with the external magnetic field. The magnetic dipoles induce their own magnetic fields. A material said to be *magnetized* when these dipoles are aligned.




Magnetic Polarization, $\vec{M}$

$$\vec{M} = \mu_0 \chi_m \vec{H}$$

The magnetic susceptibility $\chi_m$ is a measure of how easily a magnetic dipole is tilted due to an applied magnetic field.

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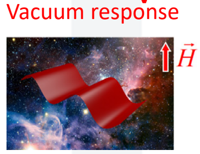
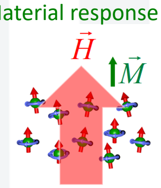
## Magnetic Flux Density, $\vec{B}$ (T or Wb/m$^2$)

The magnetic field induced by magnetic dipoles within a material combine with the external magnetic field to produce an overall magnetic flux. The combined magnetic flux is the magnetic flux density $\vec{B}$.

$$\vec{B} = \underbrace{\mu_0 \vec{H}}_{\text{Vacuum response}} + \underbrace{\vec{M}}_{\text{Material response}}$$

Note: $\mu_0$ is the free space permeability and multiples $\vec{H}$ so that $\mu_0 \vec{H}$ has the same units as $\vec{M}$.




The magnetic flux density $\vec{B}$ is most closely related to force $\vec{F}$.
Compass needles align in the direction of the $\vec{B}$ field.

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# Magnetic Polarization Vs. Magnetization

$$\vec{B} = \mu_0 \vec{H} + \vec{M}$$

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{\mathrm{M}}$$

Magnetic Polarization $\vec{M}$ (Wb/m$^2$ or T)

$$\vec{M} = \mu_0 \vec{\mathrm{M}}$$

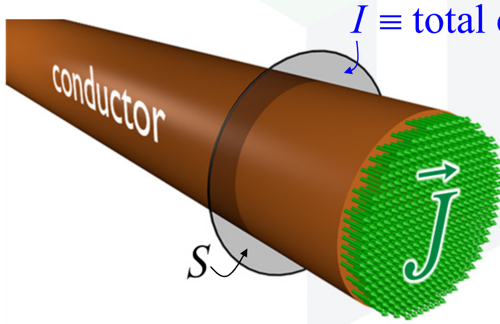
Magnetization $\vec{\mathrm{M}}$ (A/m)



# Electric Current Density, $\vec{J}$ (A/m$^2$)

The electric current density $\vec{J}$ is the electric current $I$ per unit area $S$. At low frequencies, current utilizes the entire volume of a conductor so the current is distributed over the entire cross section of a conductor. At high frequencies, the skin effect must be considered.

$I \equiv$ total enclosed current




$$I = \iint_S \vec{J} \bullet d\vec{s}$$

## Volume Charge Density, $\rho_v$ (C/m$^3$)

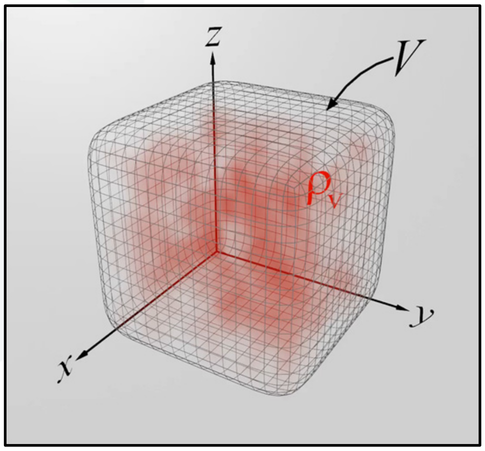
The volume charge density $\rho_v$ is the electric charge $Q$ per unit volume $V$. Charge is distributed throughout a volume.

Total Enclosed Charge $Q$
(nonuniform charge distribution)

$$Q = \iiint_V \rho_v \, dv$$

Total Enclosed Charge $Q$
(uniform charge distribution)

$$Q = \rho_v V$$

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## Electric Charge, $Q$ (C)

- The fundamental unit of charge is the Coulomb (C).
- One Coulomb is equal to the amount of charge that flows from 1 A for one second.
- The charge of an electron is $1.60 \times 10^{-19}$ C.
- There are $6.24 \times 10^{18}$ electrons in one Coulomb.
- ~5000 C are released by a AA battery during its life, but only ~15 C are released during a lightning bolt.
- Two objects charged with 1 C can levitate 900,000 tons one meter above the ground. That is about 10 aircraft carriers.

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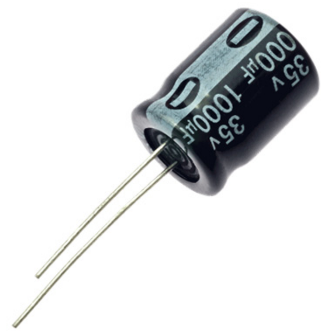
## Permittivity, $\varepsilon$ (F/m)

The permittivity is a measure of how well a medium stores electric energy.

It can be thought of as a measure of how much interaction an electric field has with a material.

Permittivity is most closely related to capacitance.




$$\varepsilon = \varepsilon_0 \varepsilon_r$$

$$\varepsilon_0 = 8.8541878176 \times 10^{-12} \text{ F/m}$$

$$\varepsilon_r \geq 1 \ (\text{no units})$$

$\varepsilon \equiv$ permittivity

$\varepsilon_0 \equiv$ vacuum permittivity

$\varepsilon_r \equiv$ relative permittivity (dielectric constant)

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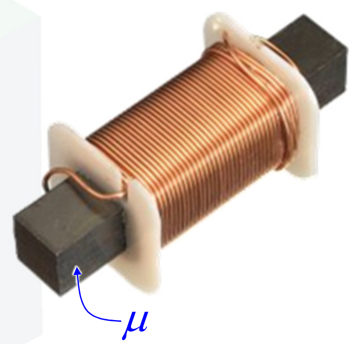
## Permeability, $\mu$ (H/m)

The permeability is a measure of how well a medium stores magnetic energy.

It can be thought of as a measure of how much interaction a magnetic field has with a material.

Permeability is most closely related to inductance.




$$\mu = \mu_0 \mu_r$$

$$\mu_0 = 1.2566370614 \times 10^{-6} \text{ H/m}$$

$$\mu_r \geq 1 \ (\text{no units})$$

$\mu \equiv$ permeability

$\mu_0 \equiv$ vacuum permeability

$\mu_r \equiv$ relative permeability

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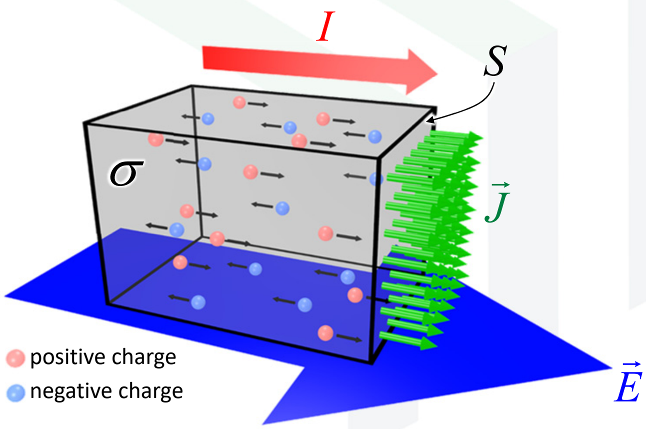
## Conductivity, $\sigma$ (1/ Ω·m)

The conductivity describes the degree to which a material conducts electricity.




Resistivity

$\rho\ (\Omega \cdot \mathrm{m}) = 1/\sigma$

Ohm's Law for EM

$\vec{J} = \sigma \vec{E}$

Total Current

$I = JS$

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# Types of Electric Current

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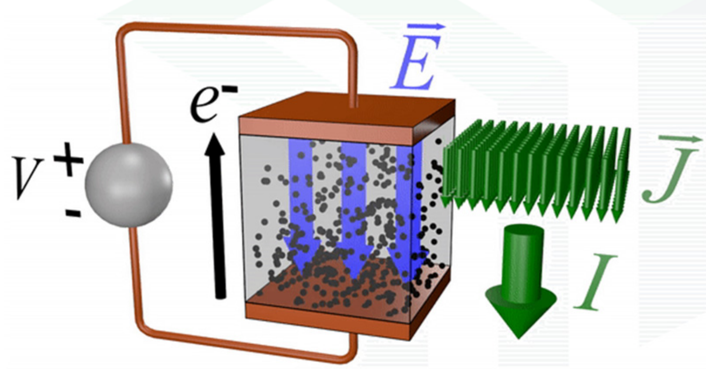
# Types of Electrical Current

- Conduction Current, $\vec{J}_\sigma$
  - Flow of free charges through conductors.
  - Traditional kind of current studied in circuit theory.
- Convection Current, $\vec{J}_\epsilon$
  - Flow of free charges through insulators.
- Displacement Current, $\vec{J}_D$
  - Momentary movement of bound charges in a dielectric.
- Total Current: $\vec{J}_\mathrm{T} = \vec{J}_\sigma + \vec{J}_\varepsilon + \vec{J}_D$



# Conduction Current, $\vec{J}_\sigma$

Conduction current $\vec{J}_\sigma$ is the flow of free charges through a conductor.

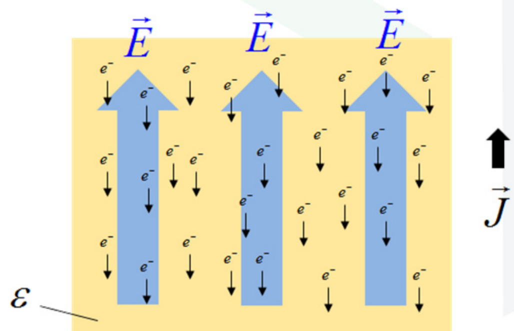



Ohm's law for electromagnetics

$$\vec{J}_\sigma = \sigma \vec{E}$$

# Convection Current, $\vec{J}_\varepsilon$

Convection current $\vec{J}_\varepsilon$ is the flow of free charges through an insulator. This current does not obey Ohm's law since no conductors are involved. For convection current to exist, the insulators must be charged.




Convection Current

$$\vec{J}_\varepsilon = \rho_v \vec{v}$$

Continuity of Current Equation

$$\nabla \bullet \vec{J} = -\frac{\partial \rho_v}{\partial t}$$

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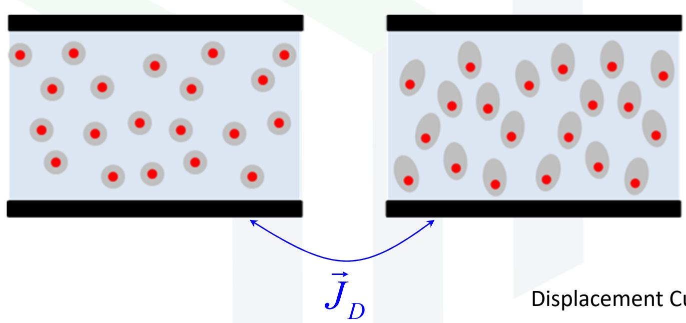
# Displacement Current, $\vec{J}_D$

There exists a short-lived net movement of charge as a material changes its state of polarization.




Displacement Current

$$\vec{J}_D = \frac{\partial \vec{D}}{\partial t}$$

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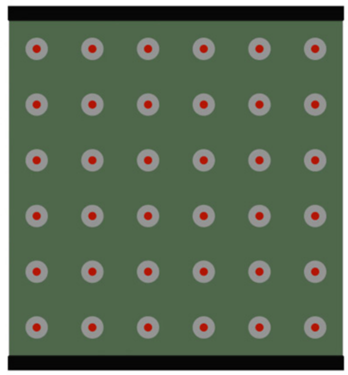
# Displacement Current, $\vec{J}_D$

There exists a short-lived net movement of charge as a material changes its state of polarization.

Displacement Current

$$\vec{J}_D = \frac{\partial \vec{D}}{\partial t}$$



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# Total Current, $\vec{J}_\mathrm{T}$

The total current $\vec{J}_\mathrm{T}$ is the sum of conduction current, convection current, and displacement current.

$$\vec{J}_\mathrm{T} = \underbrace{\vec{J}_\sigma + \vec{J}_\varepsilon}_{\substack{\text{Current due to} \\ \text{free charges} \\ \vec{J}}} + \underbrace{\vec{J}_D}_{\substack{\text{Current due to} \\ \text{bound charges} \\ \frac{\partial \vec{D}}{\partial t}}}$$

This is the $\vec{J}$ term in Maxwell's equations. It describes only the movement of *free* charges.

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# Duality





## Symmetric Form of Maxwell's Equations

To illustrate and to utilize duality, two non-physical terms are added to Maxwell's equations. These are the magnetic charge density $\rho_{\mathrm{m}}$ and the magnetic current density $\vec{J}_{\mathrm{m}}$.

$$\nabla \bullet \vec{D} = \rho_{\mathrm{e}}$$

$$\nabla \bullet \vec{B} = \rho_{\mathrm{m}}$$

$$\nabla \times \vec{E} = -\left[ \vec{J}_{\mathrm{m}} + \frac{\partial \vec{B}}{\partial t} \right]$$

$$\nabla \times \vec{H} = +\left[ \vec{J}_{\mathrm{e}} + \frac{\partial \vec{D}}{\partial t} \right]$$

$$\vec{D} = \varepsilon \vec{E}$$

$$\vec{B} = \mu \vec{H}$$

Aside: The nonphysical terms have applications in analysis. For example, interfaces with high dielectric contrast can sometimes be approximated as a perfect magnetic conductor.

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# Duality of the Parameters

The duality of the parameters can now be presented.

$$\nabla \bullet \vec{D} = \rho_{\mathrm{e}} \qquad \nabla \times \vec{E} = -\left[ \vec{J}_{\mathrm{m}} + \frac{\partial \vec{B}}{\partial t} \right]$$

$$\nabla \bullet \vec{B} = \rho_{\mathrm{m}} \qquad \nabla \times \vec{H} = +\left[ \vec{J}_{\mathrm{e}} + \frac{\partial \vec{D}}{\partial t} \right]$$

| Electric Quantity | Magnetic Quantity |
|---|---|
| $\vec{E}$ (V/m), electric field intensity | $\vec{H}$ (A/m), magnetic field intensity |
| $\vec{D}$ (C/m$^2$), electric flux density | $\vec{B}$ (Wb/m$^2$), magnetic flux density |
| $\vec{J}_{\mathrm{e}}$ (A/m$^2$), electric current density | $\vec{J}_{\mathrm{m}}$ (Wb/s·m$^2$), magnetic current density |
| $\rho_{\mathrm{e}}$ (C/m$^3$), electric charge density | $\rho_{\mathrm{m}}$ (Wb/m$^3$), magnetic charge density |
| $\varepsilon$ (F/m), permittivity | $\mu$ (H/m), permeability |



# Notes on Duality

- **Self-Check for Derivations** -- Analogous equations can be derived for electric and magnetic quantities. Aside from a sign, the equations should be the same after a change of variables. This is a great self check!
- **Design Freedom** – Permittivity and permeability $\varepsilon$ and $\mu$ can be swapped and the device will behave the same, but with the roles of $\vec{E}$ and $\vec{H}$ swapped as well.