Scattering Analysis of 3D Anisotropic Devices Using

Finite-Difference Frequency-Domain

BENEFITS & APPLICATIONS

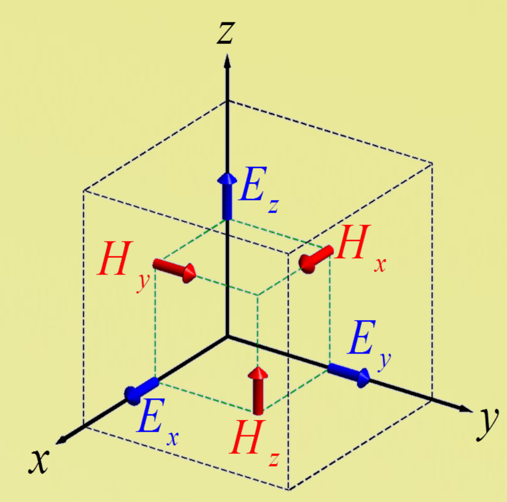
- Simple to implement.
- Excellent for field visualization.
- Obtains a rigorous solution to Maxwell's equations.

DRAWBACKS

- Typically uses an unstructured grid which is inefficient for simple or curved geometries.
- Does not scale well to three dimensions.
- Literature on the method is sparse.

YEE GRID

- Yee grid is inherently divergence free.
- Efficient arrangement for approximating Maxwell's curl equations with finite-differences.
- Physical boundary conditions are naturally satisfied.



MAXWELL'S EQUATIONS WITH A UPML

$$\nabla \times \vec{H} = k_0[\varepsilon_r][s]\vec{E}$$

$$\nabla \times \vec{E} = k_0[\mu_r][s]\vec{\tilde{H}}$$

$$[\varepsilon_r'] = [\varepsilon_r][s]$$

$$[\mu_r'] = [\mu_r][s]$$

$$\vec{\tilde{H}} = -j\sqrt{\frac{\mu_0}{\varepsilon_0}}\vec{H}$$

UNIAXIAL PML (UPML)

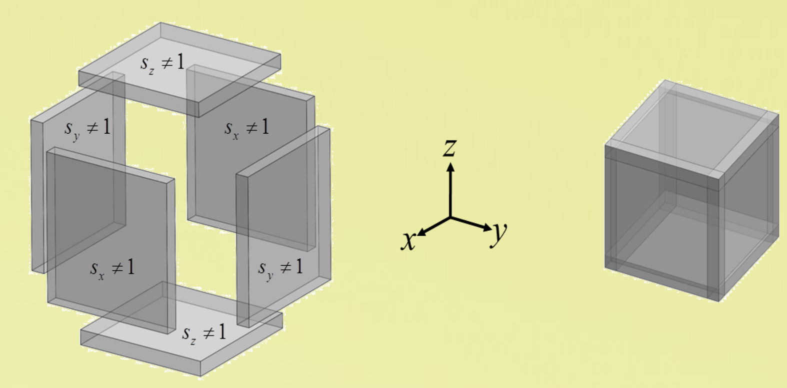
$$[s] = \begin{bmatrix} \frac{s_y s_z}{s_x} & 0 & 0 \\ 0 & \frac{s_x s_z}{s_y} & 0 \\ 0 & 0 & \frac{s_x s_y}{s_z} \end{bmatrix}$$

$$s_x(x) = s_{x0}(x)\left[1 + \frac{\eta_0}{jk_0}\sigma_x^*(x)\right]$$

$$s_y(y) = s_{y0}(y)\left[1 + \frac{\eta_0}{jk_0}\sigma_y^*(y)\right]$$

$$s_z(z) = s_{z0}(z)\left[1 + \frac{\eta_0}{jk_0}\sigma_z^*(z)\right]$$

$$s_{z0}(z) = 1 + s_{\max}\left(\frac{z}{L}\right)^p \qquad \sigma_z^*(z) = \sigma_{\max}^* \sin^2\left(\frac{\pi z}{2L}\right)$$



FINITE-DIFFERENCE APPROXIMATION OF MAXWELL'S EQUATIONS

$$\frac{\tilde{H}_z^{i,j,k} - \tilde{H}_z^{i,j-1,k}}{\Delta y'} - \frac{\tilde{H}_y^{i,j,k} - \tilde{H}_y^{i,j,k-1}}{\Delta z'} = \varepsilon_{xx}'^{i,j,k}E_x^{i,j,k} + \frac{\varepsilon_{xy}'^{i,j,k}E_y^{i,j,k} + \varepsilon_{xy}'^{i,j-1,k}E_y^{i,j-1,k} + \varepsilon_{xy}'^{i+1,j-1,k}E_y^{i+1,j-1,k} + \varepsilon_{xy}'^{i+1,j,k}E_y^{i+1,j,k}}{4} + \frac{\varepsilon_{xz}'^{i,j,k}E_z^{i,j,k} + \varepsilon_{xz}'^{i,j,k-1}E_z^{i,j,k-1} + \varepsilon_{xz}'^{i+1,j,k-1}E_z^{i+1,j,k-1} + \varepsilon_{xz}'^{i+1,j,k}E_z^{i+1,j,k}}{4}$$

$$\frac{\tilde{H}_x^{i,j,k} - \tilde{H}_x^{i,j,k-1}}{\Delta z'} - \frac{\tilde{H}_z^{i,j,k} - \tilde{H}_z^{i-1,j,k}}{\Delta x'} = \frac{\varepsilon_{yx}'^{i,j,k}E_x^{i,j,k} + \varepsilon_{yx}'^{i,j+1,k}E_x^{i,j+1,k} + \varepsilon_{yx}'^{i-1,j+1,k}E_x^{i-1,j+1,k} + \varepsilon_{yx}'^{i-1,j,k}E_x^{i-1,j,k}}{4} + \varepsilon_{yy}'^{i,j,k}E_y^{i,j,k} + \frac{\varepsilon_{yz}'^{i,j,k}E_z^{i,j,k} + \varepsilon_{yz}'^{i,j,k-1}E_z^{i,j,k-1} + \varepsilon_{yz}'^{i,j+1,k-1}E_z^{i,j+1,k-1} + \varepsilon_{yz}'^{i,j+1,k}E_z^{i,j+1,k}}{4}$$

$$\frac{\tilde{H}_y^{i,j,k} - \tilde{H}_y^{i-1,j,k}}{\Delta x'} - \frac{\tilde{H}_x^{i,j,k} - \tilde{H}_x^{i,j-1,k}}{\Delta y'} = \frac{\varepsilon_{zx}'^{i,j,k}E_x^{i,j,k} + \varepsilon_{zx}'^{i-1,j,k}E_x^{i-1,j,k} + \varepsilon_{zx}'^{i-1,j,k+1}E_x^{i-1,j,k+1} + \varepsilon_{zx}'^{i,j,k+1}E_x^{i,j,k+1}}{4} + \frac{\varepsilon_{zy}'^{i,j,k}E_y^{i,j,k} + \varepsilon_{zy}'^{i,j-1,k}E_y^{i,j-1,k} + \varepsilon_{zy}'^{i,j-1,k+1}E_y^{i,j-1,k+1} + \varepsilon_{zy}'^{i,j,k+1}E_y^{i,j,k+1}}{4} + \varepsilon_{zz}'^{i,j,k}E_z^{i,j,k}$$

$$\frac{E_z^{i,j+1,k} - E_z^{i,j,k}}{\Delta y'} - \frac{E_y^{i,j,k+1} - E_y^{i,j,k}}{\Delta z'} = \mu_{xx}'^{i,j,k}\tilde{H}_x^{i,j,k} + \frac{\mu_{xy}'^{i,j,k}\tilde{H}_y^{i,j,k} + \mu_{xy}'^{i-1,j,k}\tilde{H}_y^{i-1,j,k} + \mu_{xy}'^{i,j+1,k}\tilde{H}_y^{i,j+1,k} + \mu_{xy}'^{i-1,j+1,k}\tilde{H}_y^{i-1,j+1,k}}{4} + \frac{\mu_{xz}'^{i,j,k}\tilde{H}_z^{i,j,k} + \mu_{xz}'^{i,j,k+1}\tilde{H}_z^{i,j,k+1} + \mu_{xz}'^{i-1,j,k+1}\tilde{H}_z^{i-1,j,k+1} + \mu_{xz}'^{i-1,j,k}\tilde{H}_z^{i-1,j,k}}{4}$$

$$\frac{E_x^{i,j,k+1} - E_x^{i,j,k}}{\Delta z'} - \frac{E_z^{i+1,j,k} - E_z^{i,j,k}}{\Delta x'} = \frac{\mu_{yx}'^{i,j,k}\tilde{H}_x^{i,j,k} + \mu_{yx}'^{i+1,j,k}\tilde{H}_x^{i+1,j,k} + \mu_{yx}'^{i,j-1,k}\tilde{H}_x^{i,j-1,k} + \mu_{yx}'^{i+1,j-1,k}\tilde{H}_x^{i+1,j-1,k}}{4} + \mu_{yy}'^{i,j,k}\tilde{H}_y^{i,j,k} + \frac{\mu_{yz}'^{i,j,k}\tilde{H}_z^{i,j,k} + \mu_{yz}'^{i,j,k+1}\tilde{H}_z^{i,j,k+1} + \mu_{yz}'^{i,j-1,k+1}\tilde{H}_z^{i,j-1,k+1} + \mu_{yz}'^{i,j-1,k}E_z^{i,j-1,k}}{4}$$

$$\frac{E_y^{i+1,j,k} - E_y^{i,j,k}}{\Delta x'} - \frac{E_x^{i,j+1,k} - E_x^{i,j,k}}{\Delta y'} = \frac{\mu_{zx}'^{i,j,k}\tilde{H}_x^{i,j,k} + \mu_{zx}'^{i+1,j,k}\tilde{H}_x^{i+1,j,k} + \mu_{zx}'^{i+1,j,k-1}\tilde{H}_x^{i+1,j,k-1} + \mu_{zx}'^{i,j,k-1}\tilde{H}_x^{i,j,k-1}}{4} + \frac{\mu_{zy}'^{i,j,k}\tilde{H}_y^{i,j,k} + \mu_{zy}'^{i,j+1,k}\tilde{H}_y^{i,j+1,k} + \mu_{zy}'^{i,j+1,k-1}\tilde{H}_y^{i,j+1,k-1} + \mu_{zy}'^{i,j,k-1}\tilde{H}_y^{i,j,k-1}}{4} + \mu_{zz}'^{i,j,k}\tilde{H}_z^{i,j,k}$$

MATRIX FORM

$$\mathbf{D}_y^h\tilde{\mathbf{h}}_z - \mathbf{D}_z^h\tilde{\mathbf{h}}_y = \boldsymbol{\varepsilon}_{xx}'\mathbf{e}_x + \mathbf{R}_x^+\mathbf{R}_y^-\boldsymbol{\varepsilon}_{xy}'\mathbf{e}_y + \mathbf{R}_x^+\mathbf{R}_z^-\boldsymbol{\varepsilon}_{xz}'\mathbf{e}_z$$

$$\mathbf{D}_z^h\tilde{\mathbf{h}}_x - \mathbf{D}_x^h\tilde{\mathbf{h}}_z = \mathbf{R}_x^-\mathbf{R}_y^+\boldsymbol{\varepsilon}_{yx}'\mathbf{e}_x + \boldsymbol{\varepsilon}_{yy}'\mathbf{e}_y + \mathbf{R}_y^+\mathbf{R}_z^-\boldsymbol{\varepsilon}_{yz}'\mathbf{e}_z$$

$$\mathbf{D}_x^h\tilde{\mathbf{h}}_y - \mathbf{D}_y^h\tilde{\mathbf{h}}_x = \mathbf{R}_x^-\mathbf{R}_z^+\boldsymbol{\varepsilon}_{zx}'\mathbf{e}_x + \mathbf{R}_y^-\mathbf{R}_z^+\boldsymbol{\varepsilon}_{zy}'\mathbf{e}_y + \boldsymbol{\varepsilon}_{zz}'\mathbf{e}_z$$

$$\mathbf{D}_y^e\mathbf{e}_z - \mathbf{D}_z^e\mathbf{e}_y = \boldsymbol{\mu}_{xx}'\tilde{\mathbf{h}}_x + \mathbf{R}_x^-\mathbf{R}_y^+\boldsymbol{\mu}_{xy}'\tilde{\mathbf{h}}_y + \mathbf{R}_x^-\mathbf{R}_z^+\boldsymbol{\mu}_{xz}'\tilde{\mathbf{h}}_z$$

$$\mathbf{D}_z^e\mathbf{e}_x - \mathbf{D}_x^e\mathbf{e}_z = \mathbf{R}_x^+\mathbf{R}_y^-\boldsymbol{\mu}_{yx}'\tilde{\mathbf{h}}_x + \boldsymbol{\mu}_{yy}'\tilde{\mathbf{h}}_y + \mathbf{R}_y^-\mathbf{R}_z^+\boldsymbol{\mu}_{yz}'\tilde{\mathbf{h}}_z$$

$$\mathbf{D}_x^e\mathbf{e}_y - \mathbf{D}_y^e\mathbf{e}_x = \mathbf{R}_x^+\mathbf{R}_z^-\boldsymbol{\mu}_{zx}'\tilde{\mathbf{h}}_x + \mathbf{R}_y^+\mathbf{R}_z^-\boldsymbol{\mu}_{zy}'\tilde{\mathbf{h}}_y + \boldsymbol{\mu}_{zz}'\tilde{\mathbf{h}}_z$$

3D ANALYSIS

$$\left(\mathbf{C}^H\left[\boldsymbol{\mu}_r^R\right]^{-1}\mathbf{C}^E - \left[\boldsymbol{\varepsilon}_r^R\right]\right)\vec{\mathbf{e}} = \mathbf{0} \qquad \vec{\tilde{\mathbf{h}}} = \left[\boldsymbol{\mu}_r^R\right]^{-1}\mathbf{C}^E\vec{\mathbf{e}}$$

$$\mathbf{C}^h = \begin{bmatrix} \mathbf{0} & -\mathbf{D}_z^h & \mathbf{D}_y^h \\ \mathbf{D}_z^h & \mathbf{0} & -\mathbf{D}_x^h \\ -\mathbf{D}_y^h & \mathbf{D}_z^h & \mathbf{0} \end{bmatrix} \qquad \mathbf{C}^e = \begin{bmatrix} \mathbf{0} & -\mathbf{D}_z^e & \mathbf{D}_y^e \\ \mathbf{D}_z^e & \mathbf{0} & -\mathbf{D}_x^e \\ -\mathbf{D}_y^e & \mathbf{D}_z^E & \mathbf{0} \end{bmatrix}$$

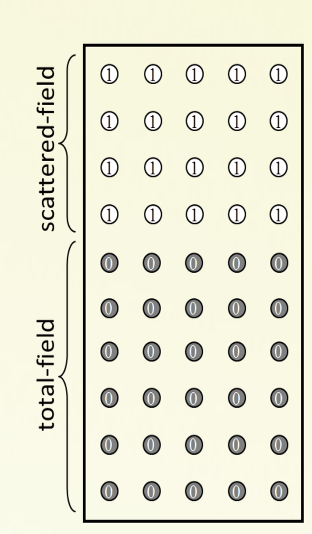
$$\vec{\tilde{\mathbf{h}}} = \begin{bmatrix} \tilde{\mathbf{h}}_x \\ \tilde{\mathbf{h}}_y \\ \tilde{\mathbf{h}}_z \end{bmatrix} \qquad \vec{\mathbf{e}} = \begin{bmatrix} \mathbf{e}_x \\ \mathbf{e}_y \\ \mathbf{e}_z \end{bmatrix}$$

$$\left[\boldsymbol{\varepsilon}_r^R\right] = \begin{bmatrix} \boldsymbol{\varepsilon}_{xx}' & \mathbf{R}_x^+\mathbf{R}_y^-\boldsymbol{\varepsilon}_{xy}' & \mathbf{R}_x^+\mathbf{R}_z^-\boldsymbol{\varepsilon}_{xz}' \\ \mathbf{R}_y^+\mathbf{R}_x^-\boldsymbol{\varepsilon}_{yx}' & \boldsymbol{\varepsilon}_{yy}' & \mathbf{R}_y^+\mathbf{R}_z^-\boldsymbol{\varepsilon}_{yz}' \\ \mathbf{R}_z^+\mathbf{R}_x^-\boldsymbol{\varepsilon}_{zx}' & \mathbf{R}_z^+\mathbf{R}_y^-\boldsymbol{\varepsilon}_{zy}' & \boldsymbol{\varepsilon}_{zz}' \end{bmatrix}$$

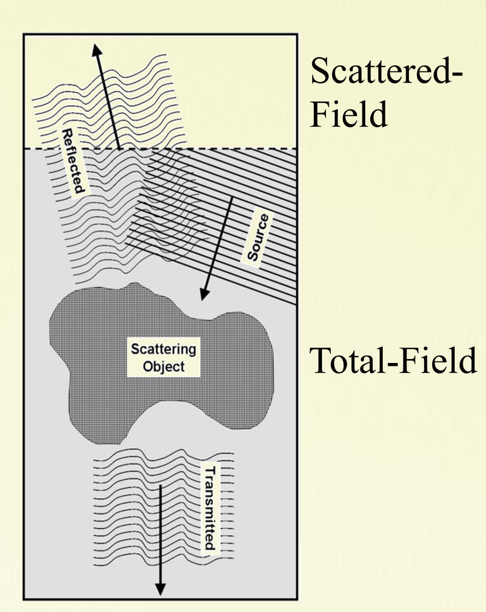
$$\left[\boldsymbol{\mu}_r^R\right] = \begin{bmatrix} \boldsymbol{\mu}_{xx}' & \mathbf{R}_x^-\mathbf{R}_y^+\boldsymbol{\mu}_{xy}' & \mathbf{R}_x^-\mathbf{R}_z^+\boldsymbol{\mu}_{xz}' \\ \mathbf{R}_y^-\mathbf{R}_x^+\boldsymbol{\mu}_{yx}' & \boldsymbol{\mu}_{yy}' & \mathbf{R}_y^-\mathbf{R}_z^+\boldsymbol{\mu}_{yz}' \\ \mathbf{R}_z^-\mathbf{R}_x^+\boldsymbol{\mu}_{zx}' & \mathbf{R}_z^-\mathbf{R}_y^+\boldsymbol{\mu}_{zy}' & \boldsymbol{\mu}_{zz}' \end{bmatrix}$$

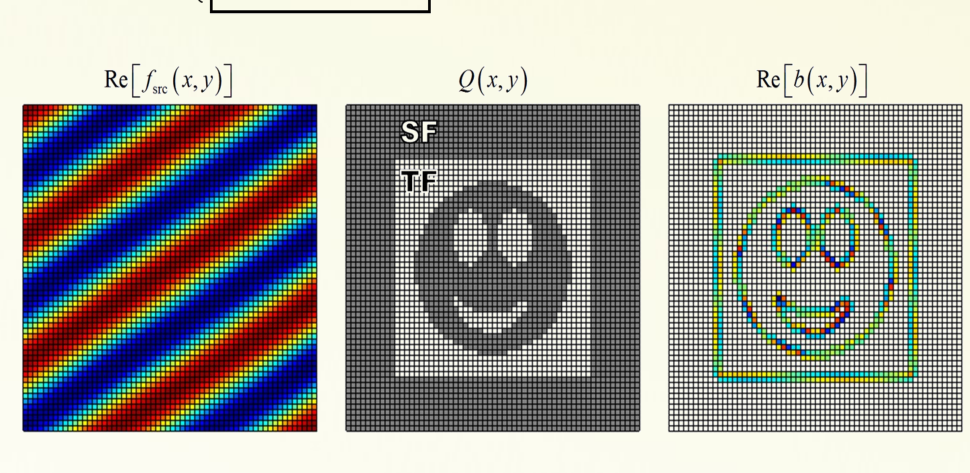
TOTAL-FIELD/SCATTERED-FIELD

$$\mathbf{b} = (\mathbf{QA} - \mathbf{AQ})\mathbf{f}_{\text{src}}$$



$$\mathbf{Q} = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 0 & \\ & & & & 0 \\ & & & & & 0 \end{bmatrix}$$





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