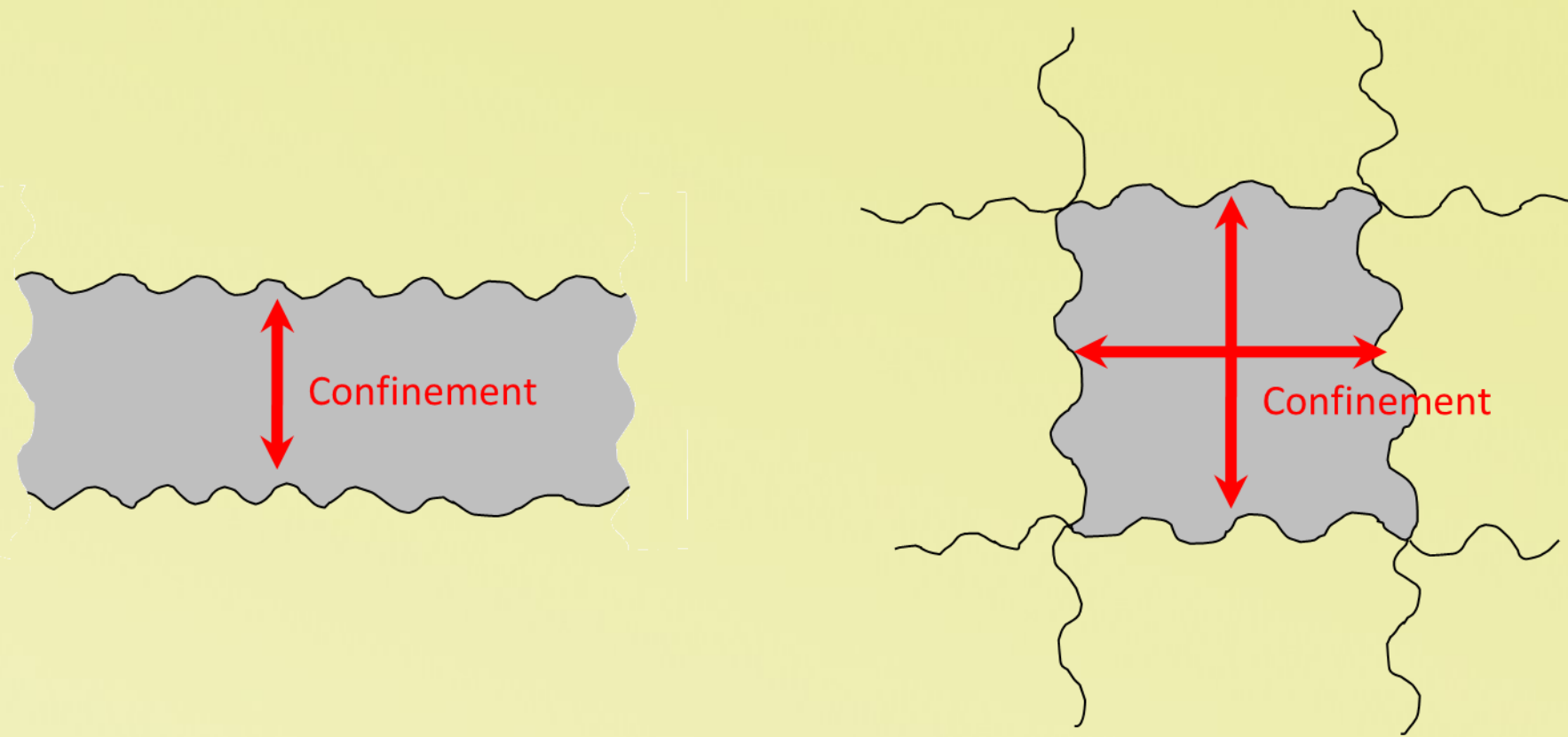


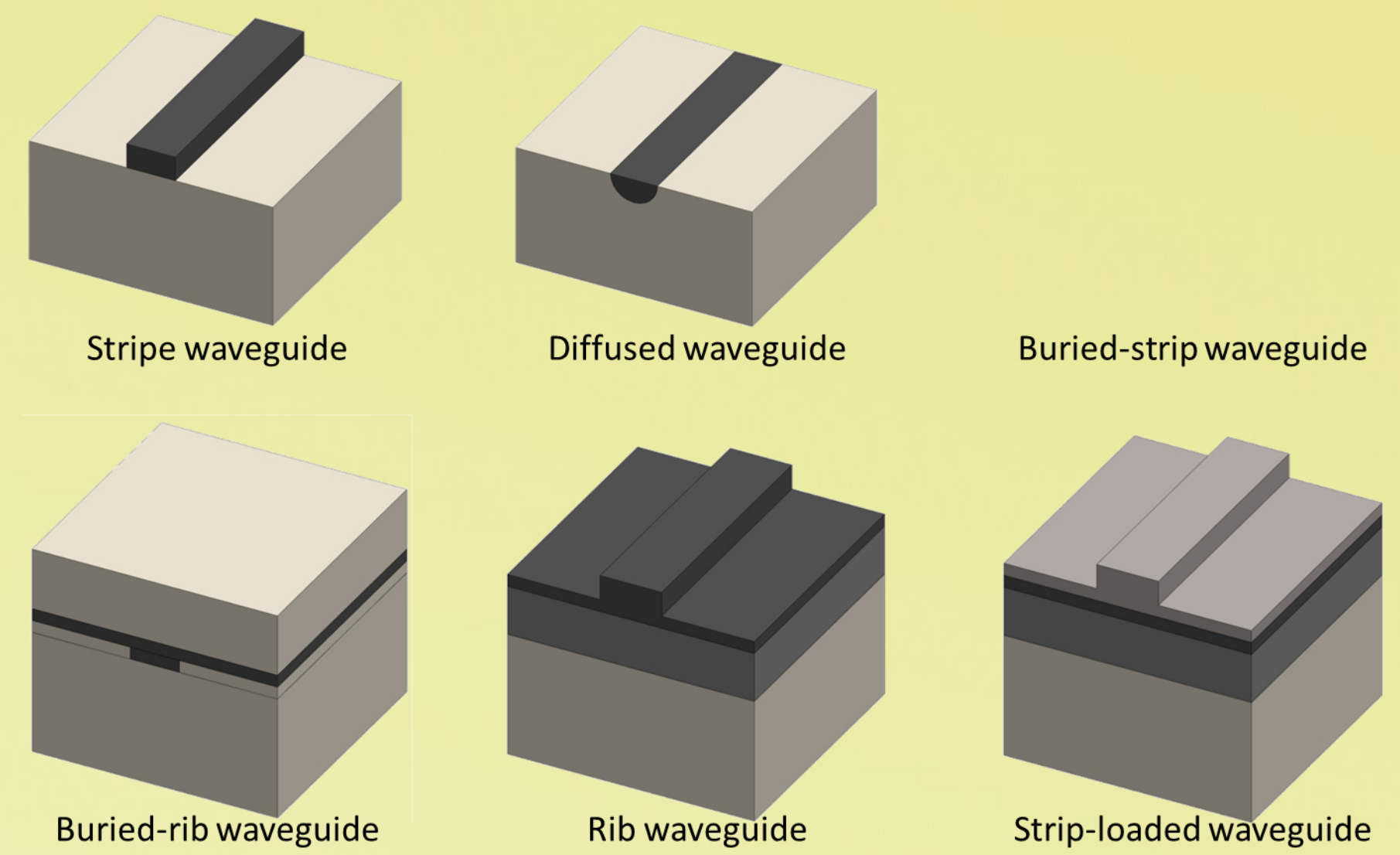
Finite-Difference Analysis of Waveguides

The finite-difference method (FDM) provides a simple way analyze all forms of waveguides. It is simple to formulate and easy to implement. The basis code uses a uniform grid so it is not as efficient as finite element methods that use more efficient unstructured grids.

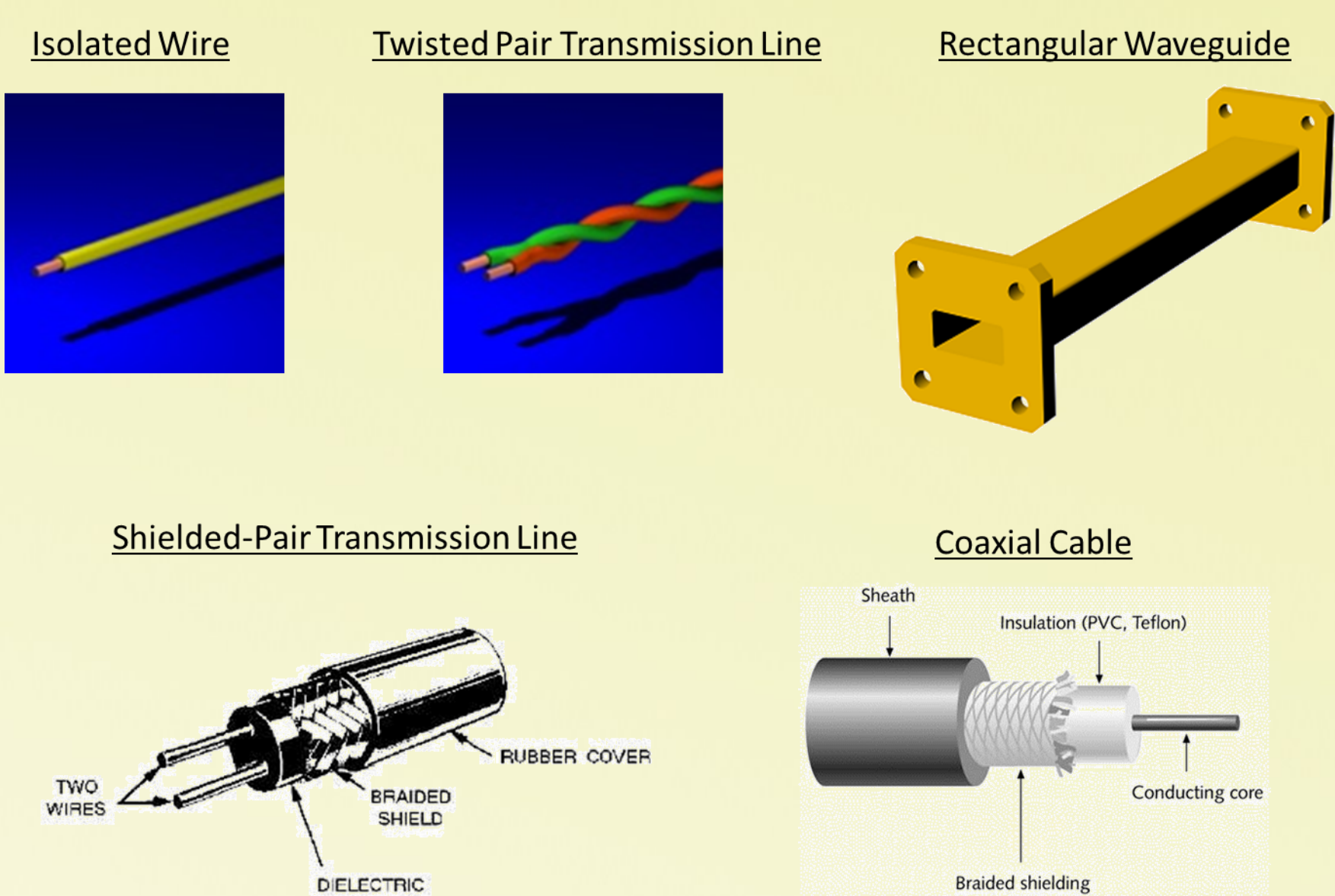
SLAB VS. CHANNEL WAVEGUIDES



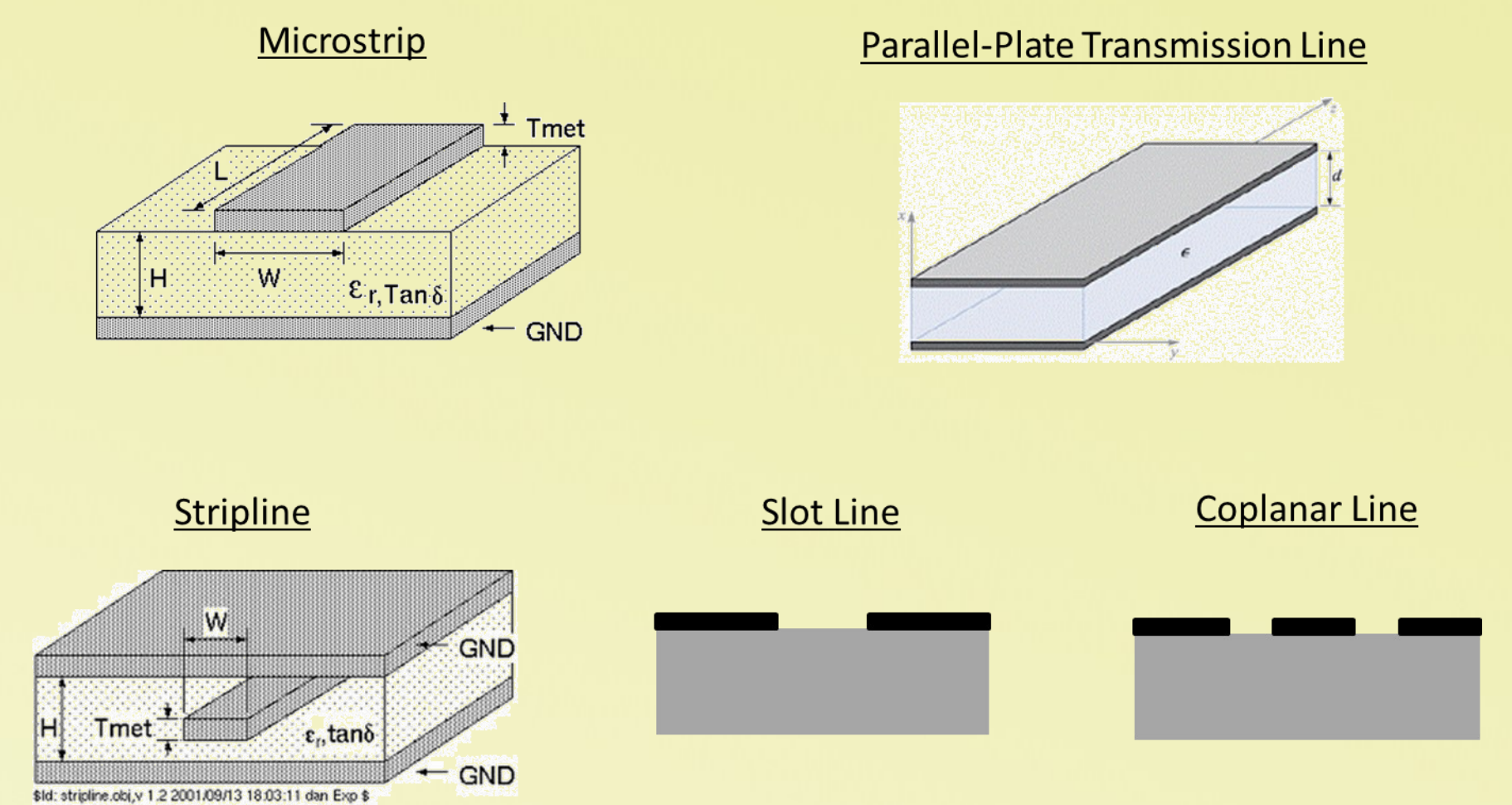
CHANNEL WAVEGUIDES FOR INTEGRATED



CHANNEL WAVEGUIDES FOR RADIO FREQUENCIES



CHANNEL WAVEGUIDES FOR PRINTED CIRCUITS



RIGOROUS HYBRID MODE ANALYSIS

Standard eigen-value equation

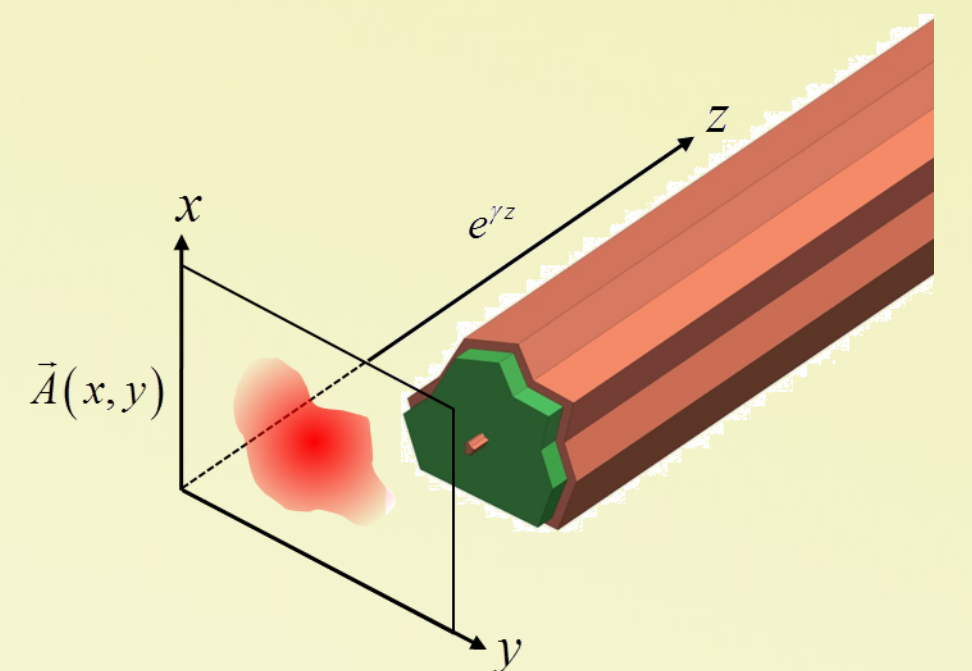
$$\Omega^2 \begin{bmatrix} \mathbf{a}_x \\ \mathbf{a}_y \end{bmatrix} = \tilde{\gamma}^2 \begin{bmatrix} \mathbf{a}_x \\ \mathbf{a}_y \end{bmatrix} \quad \mathbf{A}\mathbf{x} = \lambda\mathbf{x} \quad \mathbf{P} = \begin{bmatrix} -\mathbf{D}_x^e \boldsymbol{\epsilon}_{zz}^{-1} \mathbf{D}_y^h & \mathbf{D}_x^e \boldsymbol{\epsilon}_{zz}^{-1} \mathbf{D}_y^h + \boldsymbol{\mu}_{yy} \\ -(\mathbf{D}_y^e \boldsymbol{\epsilon}_{zz}^{-1} \mathbf{D}_x^h + \boldsymbol{\mu}_{xx}) & \mathbf{D}_y^e \boldsymbol{\epsilon}_{zz}^{-1} \mathbf{D}_x^h \end{bmatrix} \quad \mathbf{Q} = \begin{bmatrix} -\mathbf{D}_x^h \boldsymbol{\mu}_{zz}^{-1} \mathbf{D}_y^e & \mathbf{D}_x^h \boldsymbol{\mu}_{zz}^{-1} \mathbf{D}_y^e + \boldsymbol{\epsilon}_{yy} \\ -(\mathbf{D}_y^h \boldsymbol{\mu}_{zz}^{-1} \mathbf{D}_x^e + \boldsymbol{\epsilon}_{xx}) & \mathbf{D}_y^h \boldsymbol{\mu}_{zz}^{-1} \mathbf{D}_x^e \end{bmatrix}$$

$$\Omega^2 = \mathbf{P}\mathbf{Q} \quad \lambda = \tilde{\gamma}^2$$

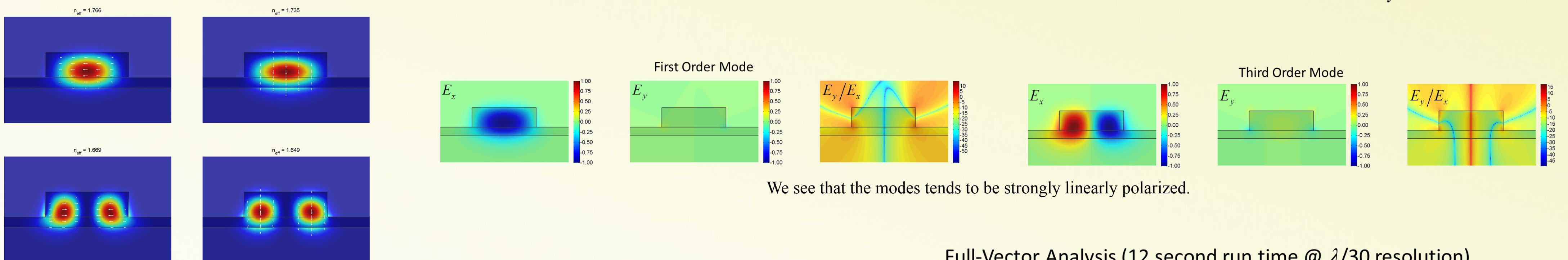
$$\vec{E}(x, y, z) = \vec{A}(x, y) e^{\gamma z}$$

complex amplitude, mode shape accumulation of phase in z direction

$$\gamma = \alpha + j\beta \equiv \text{propagation constant}$$

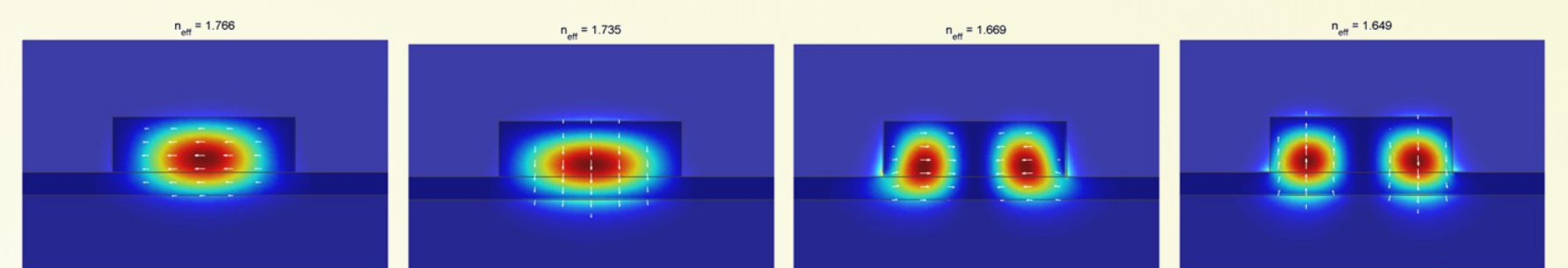


Rib waveguide analysis

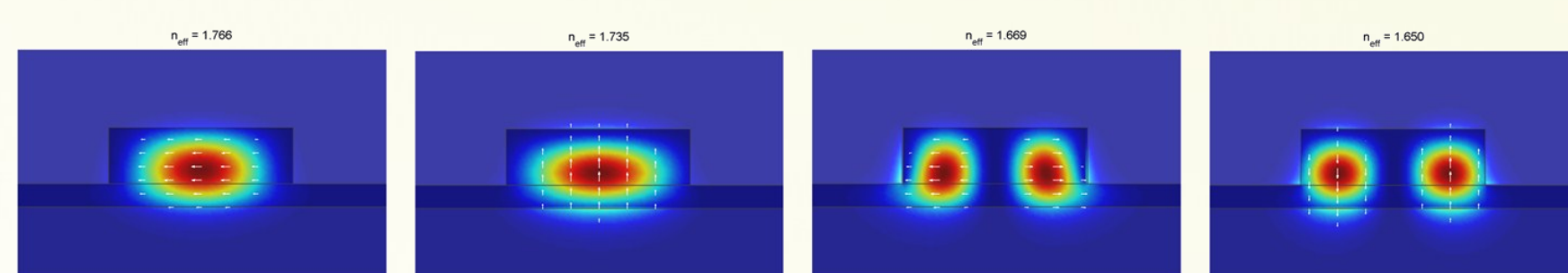


We see that the modes tends to be strongly linearly polarized.

Full-Vector Analysis (12 second run time @ λ/30 resolution)



Quasi-Vectorial Analysis (7 second run time @ λ/30 resolution)



QUASI-TE/TM MODE ANALYSIS

Observing how strongly linearly polarized the hybrid modes are, we can approximate the smaller components to be zero. This simplifies the analysis and makes it more efficient.

$$E_x \text{ Mode: } \Omega_{xx}^2 \mathbf{a}_x = \tilde{\gamma}^2 \mathbf{a}_x \quad \Omega_{xx}^2 = \mathbf{D}_x^e \boldsymbol{\epsilon}_{zz}^{-1} \mathbf{D}_y^h \mathbf{D}_x^h \boldsymbol{\mu}_{zz}^{-1} \mathbf{D}_y^e - (\mathbf{D}_x^e \boldsymbol{\epsilon}_{zz}^{-1} \mathbf{D}_x^h + \boldsymbol{\mu}_{yy}) (\mathbf{D}_y^h \boldsymbol{\mu}_{zz}^{-1} \mathbf{D}_y^e + \boldsymbol{\epsilon}_{xx})$$

$$E_y \text{ Mode: } \Omega_{yy}^2 \mathbf{a}_y = \tilde{\gamma}^2 \mathbf{a}_y \quad \Omega_{yy}^2 = \mathbf{D}_y^e \boldsymbol{\epsilon}_{zz}^{-1} \mathbf{D}_x^h \mathbf{D}_y^h \boldsymbol{\mu}_{zz}^{-1} \mathbf{D}_x^e - (\mathbf{D}_y^e \boldsymbol{\epsilon}_{zz}^{-1} \mathbf{D}_y^h + \boldsymbol{\mu}_{xx}) (\mathbf{D}_x^h \boldsymbol{\mu}_{zz}^{-1} \mathbf{D}_x^e + \boldsymbol{\epsilon}_{yy})$$

