THE UNIVERSITY OF TEXAS AT EL PASO

Finite-Difference Time-Domain

The finite-difference time-domain (FDTD) method simulates electromagnetic devices by evolving the fields over time. It can model a device over an enormous band of frequencies in a single simulation making it well suited to broadband and transient analysis. The model scales near linearly making it the dominant method for modeling electrically large structures with complex geometries. It accommodates parallel processing very well.

FLOW OF MAXWELL'S EQUATIONS

$$\nabla \times \vec{E}(t) = -\frac{\partial \vec{B}(t)}{\partial t}$$

A circulating ***E*** field induces a change in the ***B*** field at the center of circulation.

$$\vec{B}(t) = [\mu(t)] * \vec{H}(t)$$

A ***B*** field induces an ***H*** field in proportion to the permeability.

$$\nabla \times \vec{H}(t) = \frac{\partial \vec{D}(t)}{\partial t}$$

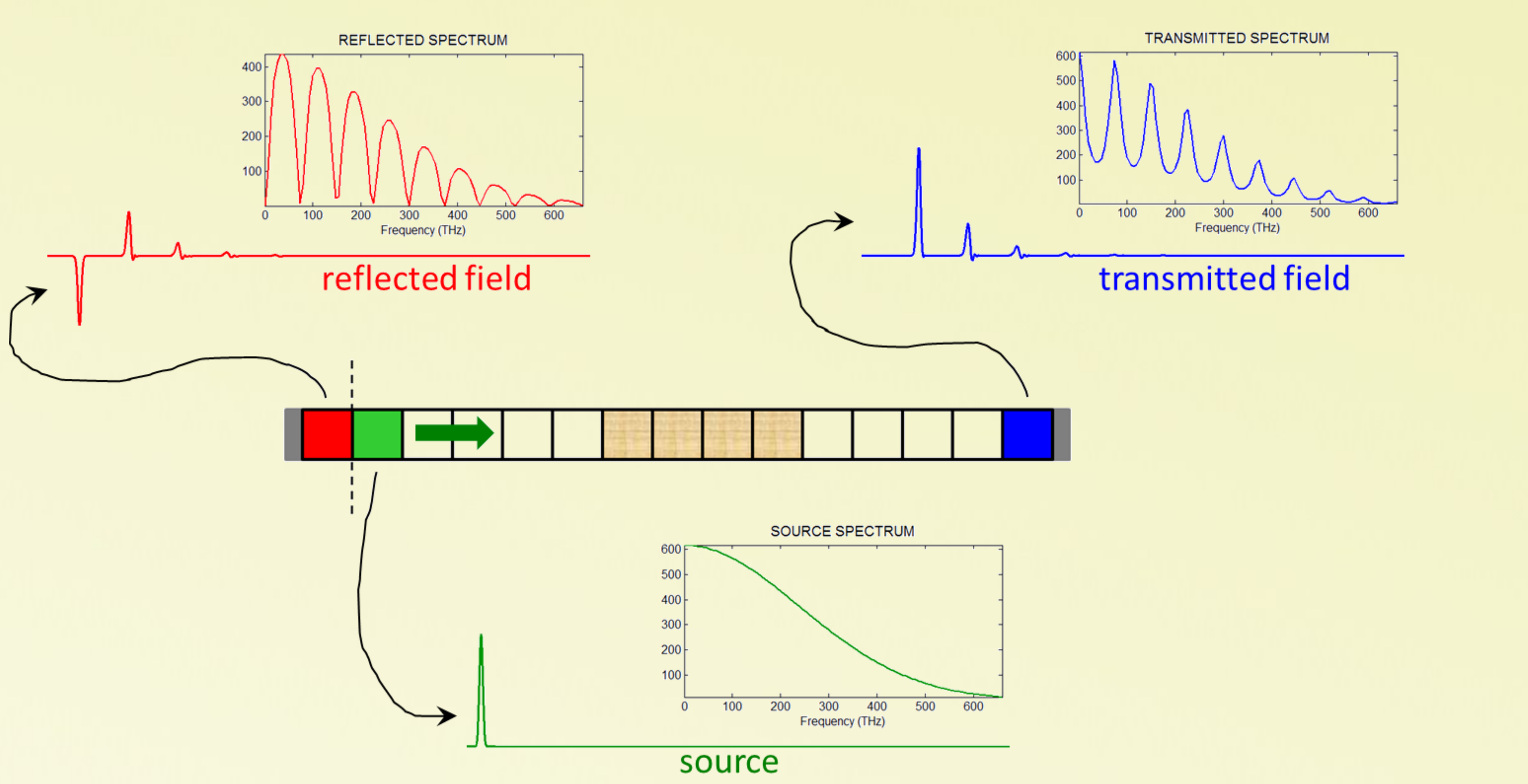
A circulating ***H*** field induces a change in the ***D*** field at the center of circulation.

$$\vec{D}(t) = [\varepsilon(t)] * \vec{E}(t)$$

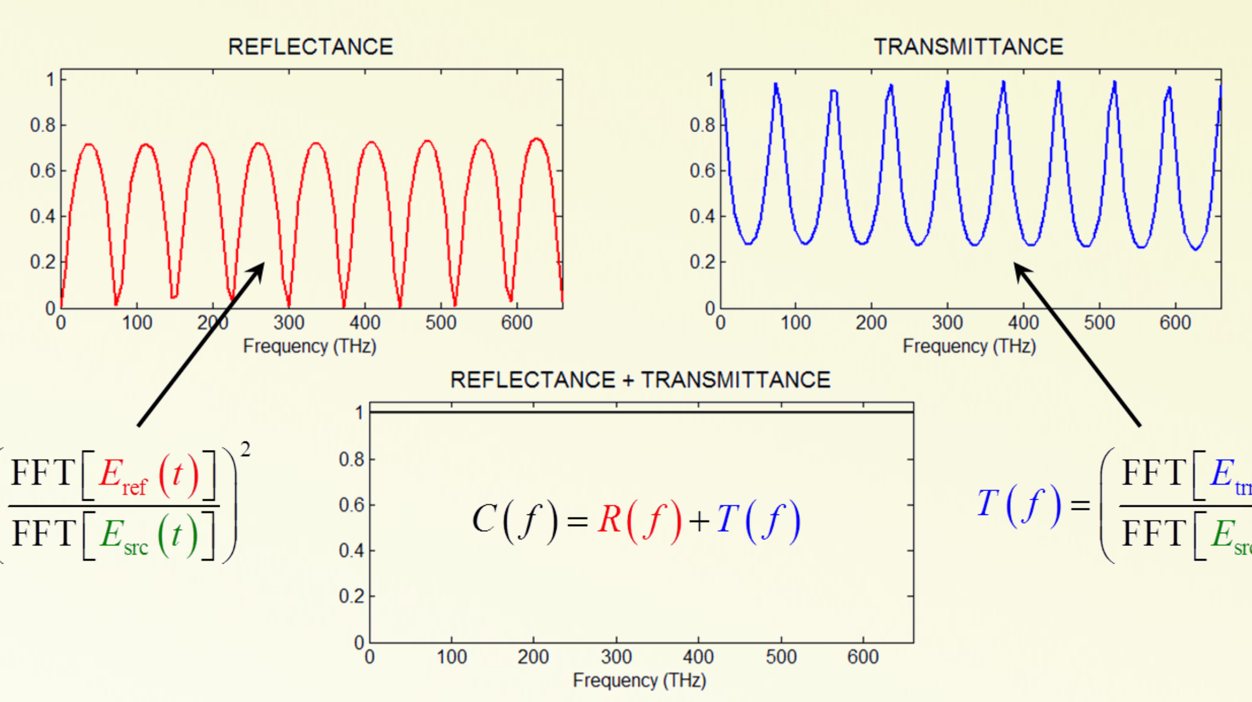
A ***D*** field induces an ***E*** field in proportion to the permittivity.

TRANSMITTANCE AND REFLECTANCE

We excite the simulation with an impulse and record the impulse response wherever we are interested.



We must normalize our spectra according to the source spectrum.



$$R(f) = \left(\frac{\mathrm{FFT}[E_{\mathrm{ref}}(t)]}{\mathrm{FFT}[E_{\mathrm{src}}(t)]}\right)^2 \qquad C(f) = R(f) + T(f) \qquad T(f) = \left(\frac{\mathrm{FFT}[E_{\mathrm{trn}}(t)]}{\mathrm{FFT}[E_{\mathrm{src}}(t)]}\right)^2$$

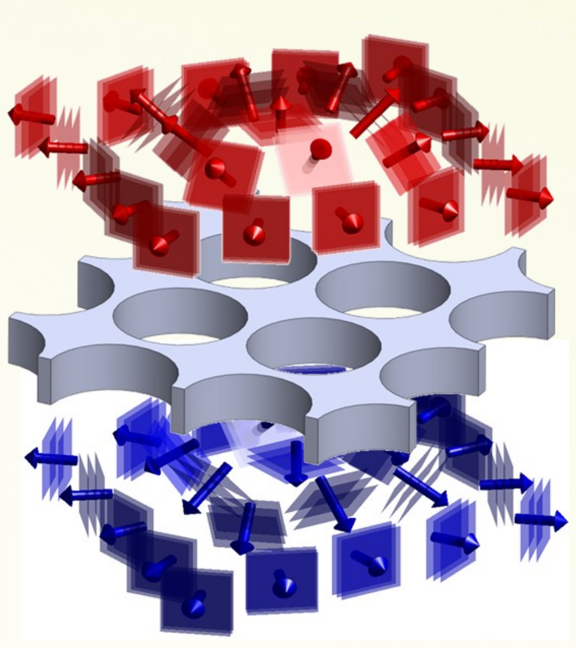
We calculate the diffraction efficiency of all the spatial harmonics separately.

$$S_{x,m,n} = \mathrm{FFT2}\{E_x^{i,j}\} \qquad S_{y,m,n} = \mathrm{FFT2}\{E_x^{i,j}\}$$

$$S_{z,m,n} = -\frac{k_{x,m}S_{x,m,n} + k_{y,n}S_{y,m,n}}{k_{z,m,n}}$$

$$\mathrm{DE}_{m,n}^{\mathrm{ref}}(f) = \left|\vec{S}_{m,n}^{\mathrm{ref}}(f)\right|^2 \cdot \mathrm{Re}\left[\frac{k_{z,m,n}^{\mathrm{ref}}(f)}{k_{z,\mathrm{inc}}(f)}\right]$$

$$\mathrm{DE}_{m,n}^{\mathrm{trn}}(f) = \left|\vec{S}_{m,n}^{\mathrm{trn}}(f)\right|^2 \cdot \mathrm{Re}\left[\frac{k_{z,m,n}^{\mathrm{trn}}(f)}{k_{z,\mathrm{inc}}(f)}\frac{\mu_{r,\mathrm{ref}}}{\mu_{r,\mathrm{trn}}}\right]$$



APPROXIMATING THE TIME-DERIVATIVES

To approximate the time derivatives using central finite-differences, the E and H fields are staggered in time.

$$\nabla \times \vec{E}(t) = -\mu\frac{\partial \vec{H}(t)}{\partial t} \;\Rightarrow\; \nabla \times \vec{E}\Big|_t = -\mu\frac{\vec{H}\big|_{t+\Delta t/2} - \vec{H}\big|_{t-\Delta t/2}}{\Delta t}$$

$$\nabla \times \vec{H}(t) = \varepsilon\frac{\partial \vec{E}(t)}{\partial t} \;\Rightarrow\; \nabla \times \vec{H}\Big|_{t+\Delta t/2} = \varepsilon\frac{\vec{E}\big|_{t+\Delta t} - \vec{E}\big|_t}{\Delta t}$$

The update equations are derived by solving the above equations for the fields at the future time value.

$$\vec{H}\Big|_{t+\Delta t/2} = \vec{H}\Big|_{t-\Delta t/2} - \frac{\Delta t}{\mu}\left(\nabla \times \vec{E}\Big|_t\right) \qquad \vec{E}\Big|_{t+\Delta t} = \vec{E}\Big|_t + \frac{\Delta t}{\varepsilon}\left(\nabla \times \vec{H}\Big|_{t+\Delta t/2}\right)$$

UNIAXIAL PERFECTLY MATCHED LAYER

We wish to incorporate loss at the outer boundaries of the grid to absorb outgoing waves, but this must be done in a manner with perfectly matched impedance to prevent reflections from the lossy regions themselves.

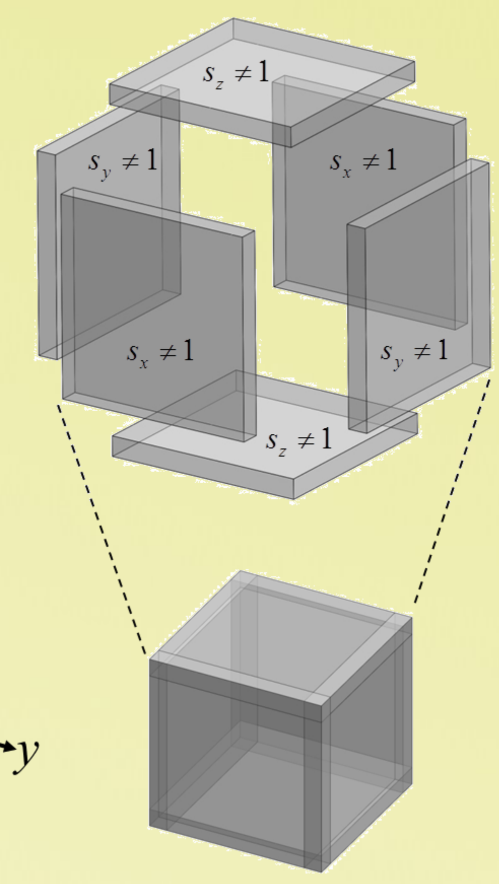
$$[s] = \begin{bmatrix} \frac{s_y s_z}{s_x} & 0 & 0 \\ 0 & \frac{s_x s_z}{s_y} & 0 \\ 0 & 0 & \frac{s_x s_y}{s_z} \end{bmatrix}$$

$$s_x(x) = 1 + \frac{\sigma_x'(x)}{j\omega\varepsilon_0} \qquad \sigma_x'(x) = \frac{\varepsilon_0}{2\Delta t}\left(\frac{x}{L_x}\right)^3$$

$$s_y(y) = 1 + \frac{\sigma_y'(y)}{j\omega\varepsilon_0} \qquad \sigma_y'(y) = \frac{\varepsilon_0}{2\Delta t}\left(\frac{y}{L_y}\right)^3$$

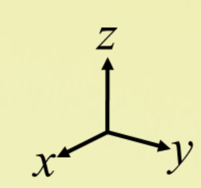
$$s_z(z) = 1 + \frac{\sigma_z'(z)}{j\omega\varepsilon_0} \qquad \sigma_z'(z) = \frac{\varepsilon_0}{2\Delta t}\left(\frac{z}{L_z}\right)^3$$

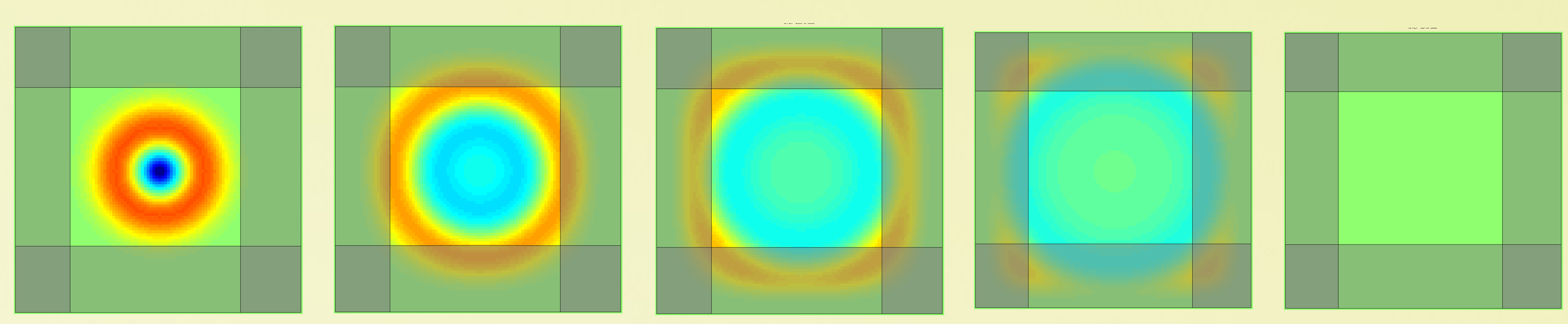
$L_?$ ≡ length of the PML in the ? direction



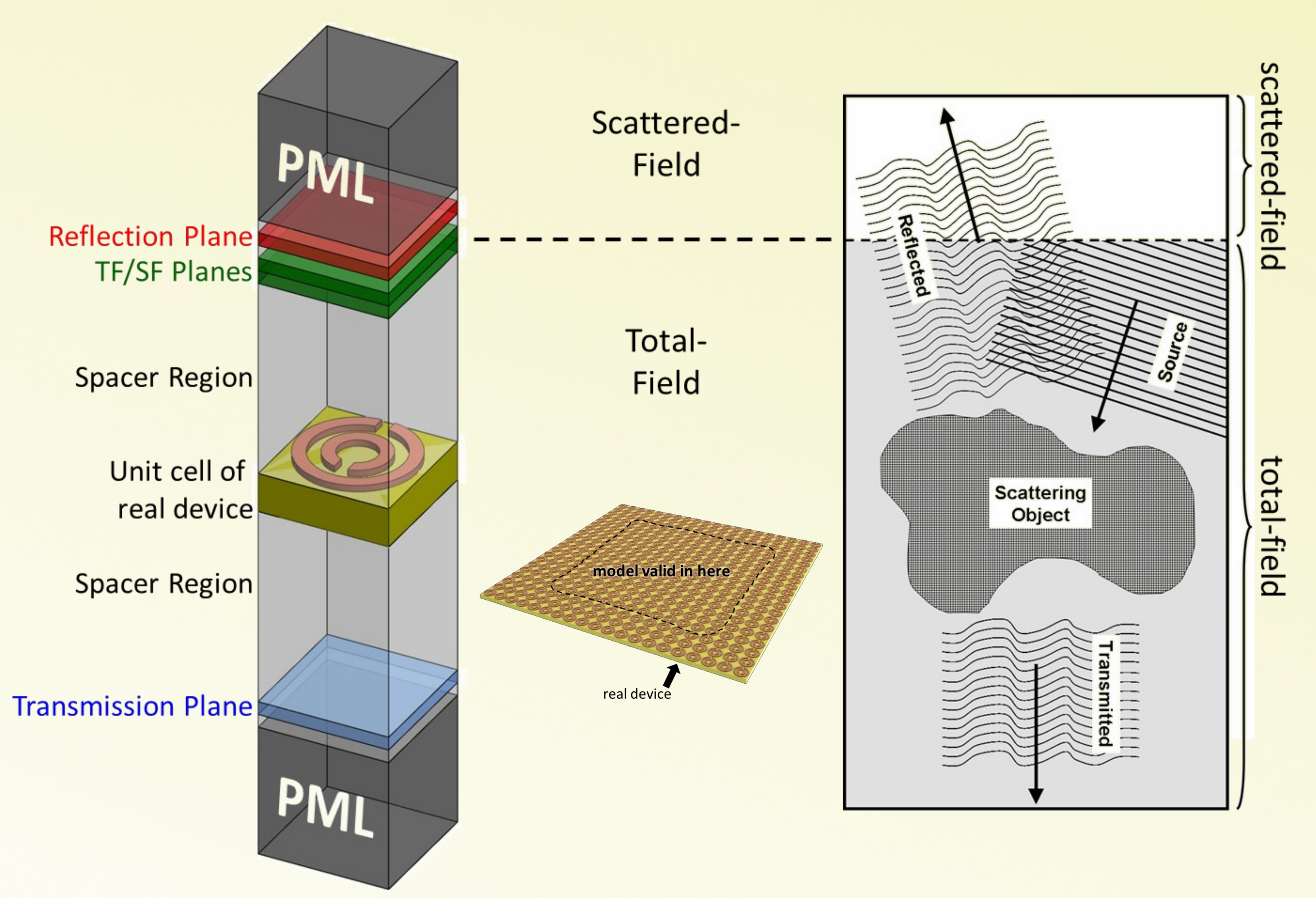
The PML is a frequency-domain concept and is incorporated into Maxwell's equations much like it was a constitutive parameter.

$$\nabla \times \vec{E}(\omega) = -j\omega\mu_0[\mu_r][s]\vec{H}(\omega)$$
$$\nabla \times \vec{H}(\omega) = \sigma\vec{E}(\omega) + j\omega[s]\vec{D}(\omega)$$
$$\vec{D}(\omega) = \varepsilon_0[\varepsilon_r]\vec{E}(\omega)$$





TOTAL-FIELD/SCATTERED-FIELD FRAMEWORK



W.M. KECK CENTER
FOR 3D INNOVATION

Pioneering 21st Century Electromagnetics and Photonics

LAB