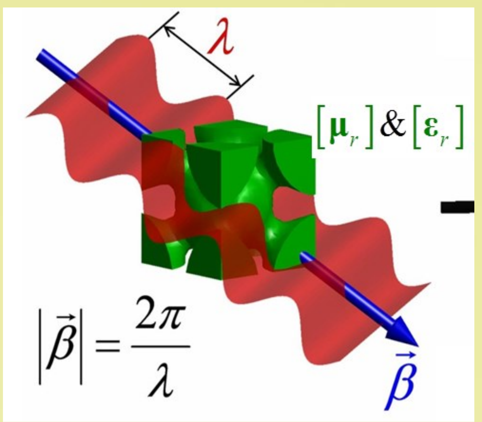
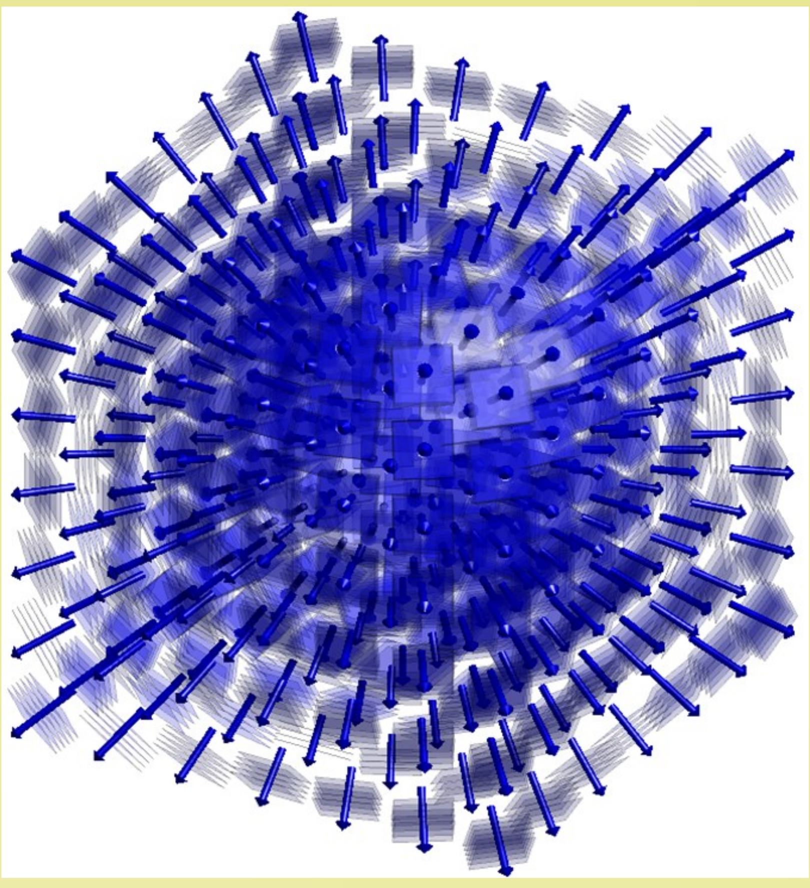
THE UNIVERSITY OF TEXAS AT EL PASO

Plane Wave Expansion Method

and Reduced Bloch Mode Expansion Technique

The plane wave expansion method (PWEM) provides a highly efficient numerical solution to Maxwell's equations for devices with low to moderate dielectric contrast. It expands the field into a set of plane waves and converts Maxwell's equations to matrix form by assigning each plane wave a complex amplitude. The final matrix equation can be solved using any number of standard eigen-value solvers.





MAXWELL'S EQUATIONS IN FOURIER SPACE

Real-Space

$$\frac{\partial \tilde{H}_z}{\partial y}-\frac{\partial \tilde{H}_y}{\partial z}=k_0\varepsilon_r E_x \qquad \frac{\partial \tilde{H}_x}{\partial z}-\frac{\partial \tilde{H}_z}{\partial x}=k_0\varepsilon_r E_y \qquad \frac{\partial \tilde{H}_y}{\partial x}-\frac{\partial \tilde{H}_x}{\partial y}=k_0\varepsilon_r E_z$$

$$\frac{\partial E_z}{\partial y}-\frac{\partial E_y}{\partial z}=k_0\mu_r \tilde{H}_x \qquad \frac{\partial E_x}{\partial z}-\frac{\partial E_z}{\partial x}=k_0\mu_r \tilde{H}_y \qquad \frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}=k_0\mu_r \tilde{H}_z$$

Fourier Space

$$-jk_{y,pqr}U_{z,pqr}+jk_{z,pqr}U_{y,pqr}=k_0\sum_{p'}\sum_{q'}\sum_{r'}a_{p-p',q-q',r-r'}S_{x,p'q'r'}$$

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Matrix Form

$$\mathbf{K}_y\mathbf{u}_z-\mathbf{K}_z\mathbf{u}_y=jk_0\boldsymbol{\varepsilon}_r\mathbf{s}_x \qquad \mathbf{K}_z\mathbf{u}_x-\mathbf{K}_x\mathbf{u}_z=jk_0\boldsymbol{\varepsilon}_r\mathbf{s}_y \qquad \mathbf{K}_x\mathbf{u}_y-\mathbf{K}_y\mathbf{u}_x=jk_0\boldsymbol{\varepsilon}_r\mathbf{s}_z$$

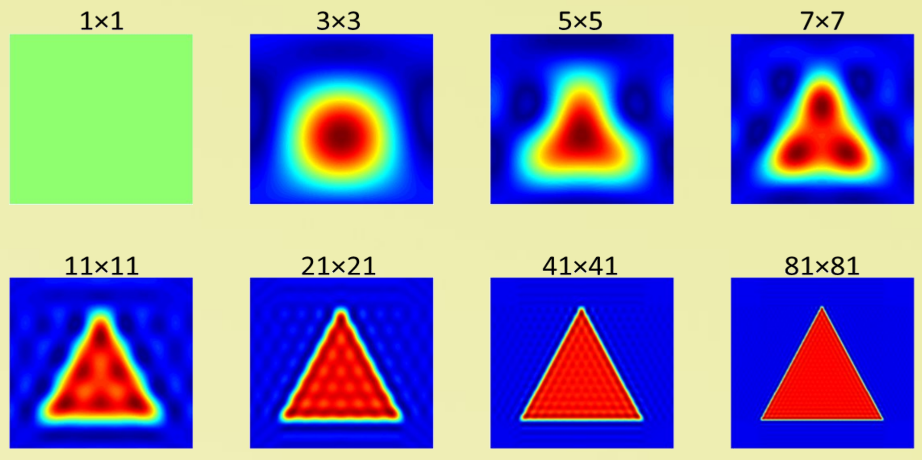
$$\mathbf{K}_y\mathbf{s}_z-\mathbf{K}_z\mathbf{s}_y=jk_0\boldsymbol{\mu}_r\mathbf{u}_x \qquad \mathbf{K}_z\mathbf{s}_x-\mathbf{K}_x\mathbf{s}_z=jk_0\boldsymbol{\mu}_r\mathbf{u}_y \qquad \mathbf{K}_x\mathbf{s}_y-\mathbf{K}_y\mathbf{s}_x=jk_0\boldsymbol{\mu}_r\mathbf{u}_z$$

K Matrices

$$\mathbf{K}_i=\begin{bmatrix}k_{i,1,1,1} & & & \\ & k_{i,1,1,2} & & \\ & & \ddots & \\ & & & k_{i,P,Q,R}\end{bmatrix}$$

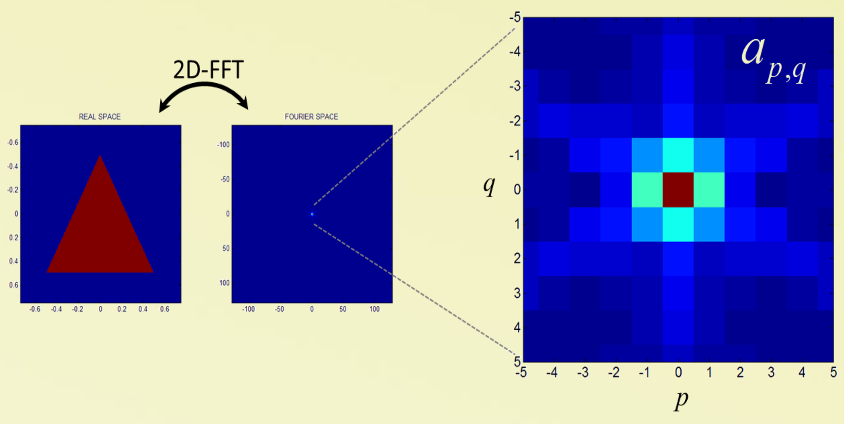
Plane Wave

$$\mathbf{u}_i=\begin{bmatrix}U_{i,1,1,1}\\ U_{i,1,1,2}\\ \vdots \\ U_{i,P,Q,R}\end{bmatrix} \qquad \mathbf{s}_i=\begin{bmatrix}S_{i,1,1,1}\\ S_{i,1,1,2}\\ \vdots \\ S_{i,P,Q,R}\end{bmatrix}$$

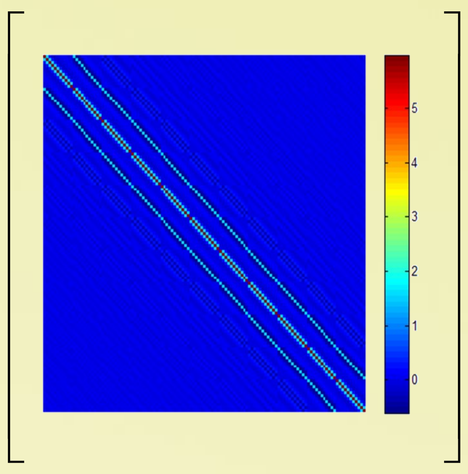


Effect of the number of spatial harmonics.

THE CONVOLUTION MATRICES



$$\boldsymbol{\varepsilon}_r=$$



3D PWEM FORMULATION

$$\begin{bmatrix}|\vec{\mathbf{K}}|\left[\hat{P}_2\bullet\right]\boldsymbol{\varepsilon}_r^{-1}\hat{\mathbf{P}}_2|\vec{\mathbf{K}}| & -|\vec{\mathbf{K}}|\left[\hat{P}_2\bullet\right]\boldsymbol{\varepsilon}_r^{-1}\hat{\mathbf{P}}_1|\vec{\mathbf{K}}| \\ -|\vec{\mathbf{K}}|\left[\hat{P}_1\bullet\right]\boldsymbol{\varepsilon}_r^{-1}\hat{\mathbf{P}}_2|\vec{\mathbf{K}}| & |\vec{\mathbf{K}}|\left[\hat{P}_1\bullet\right]\boldsymbol{\varepsilon}_r^{-1}\hat{\mathbf{P}}_1|\vec{\mathbf{K}}|\end{bmatrix}\begin{bmatrix}\mathbf{u}_1\\ \mathbf{u}_2\end{bmatrix}=k_0^2\begin{bmatrix}\left[\hat{P}_1\bullet\right]\boldsymbol{\mu}_r\hat{\mathbf{P}}_1 & \left[\hat{P}_1\bullet\right]\boldsymbol{\mu}_r\hat{\mathbf{P}}_2 \\ \left[\hat{P}_2\bullet\right]\boldsymbol{\mu}_r\hat{\mathbf{P}}_1 & \left[\hat{P}_2\bullet\right]\boldsymbol{\mu}_r\hat{\mathbf{P}}_2\end{bmatrix}\begin{bmatrix}\mathbf{u}_1\\ \mathbf{u}_2\end{bmatrix}$$

$\hat{\mathbf{P}}_i \equiv$ polarization vectors orthogonal to K

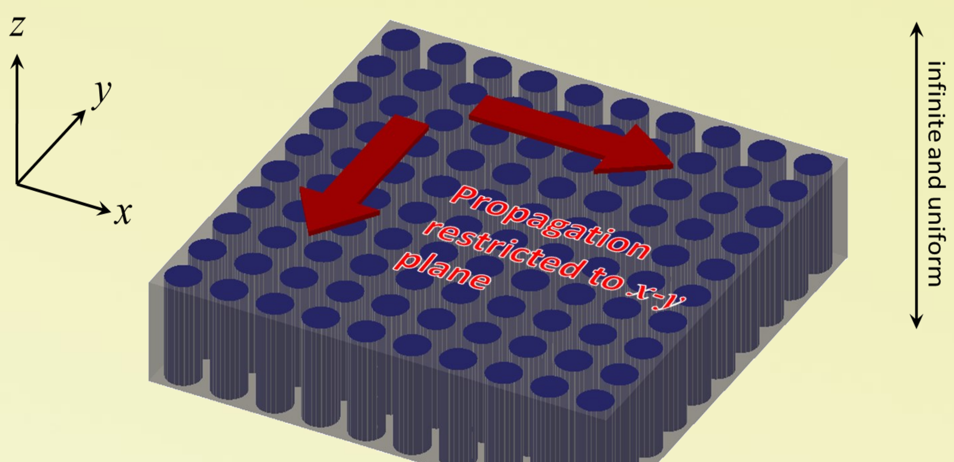
2D PWEM FORMULATION

E Mode

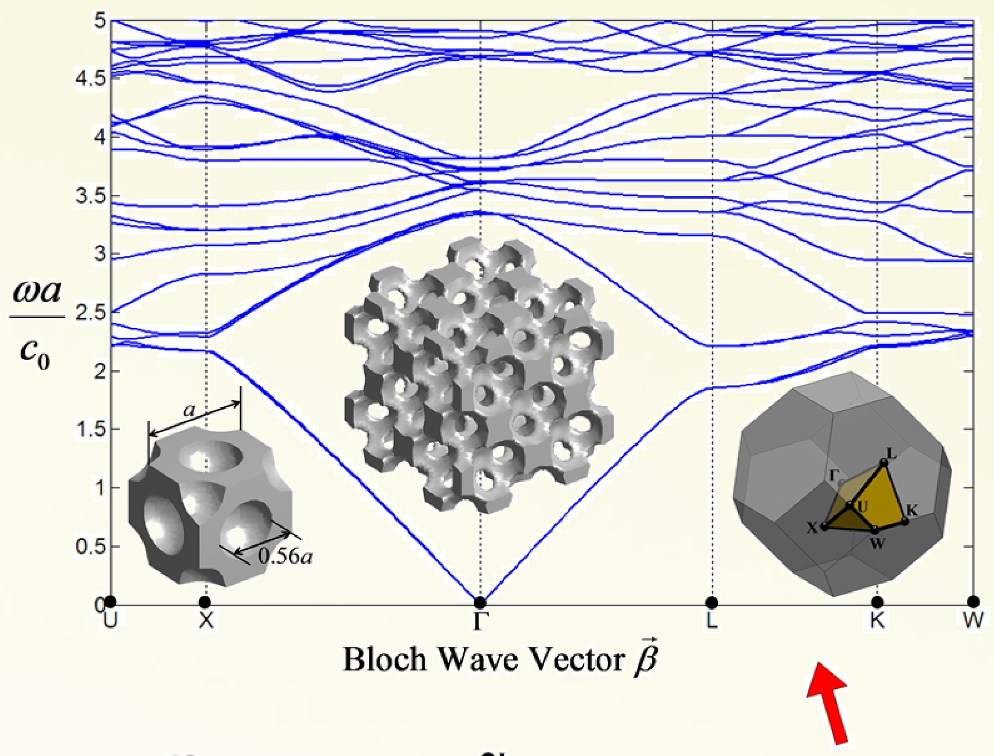
$$\left(\mathbf{K}_x\boldsymbol{\mu}_r^{-1}\mathbf{K}_x+\mathbf{K}_y\boldsymbol{\mu}_r^{-1}\mathbf{K}_y\right)\mathbf{s}_z=k_0^2\boldsymbol{\varepsilon}_r\mathbf{s}_z$$

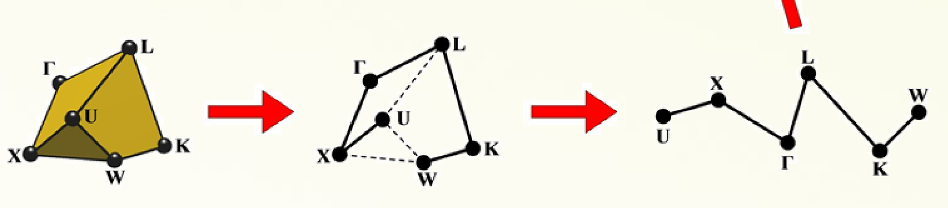
H Mode

$$\left(\mathbf{K}_x\boldsymbol{\varepsilon}_r^{-1}\mathbf{K}_x+\mathbf{K}_y\boldsymbol{\varepsilon}_r^{-1}\mathbf{K}_y\right)\mathbf{u}_z=k_0^2\boldsymbol{\mu}_r\mathbf{u}_z$$

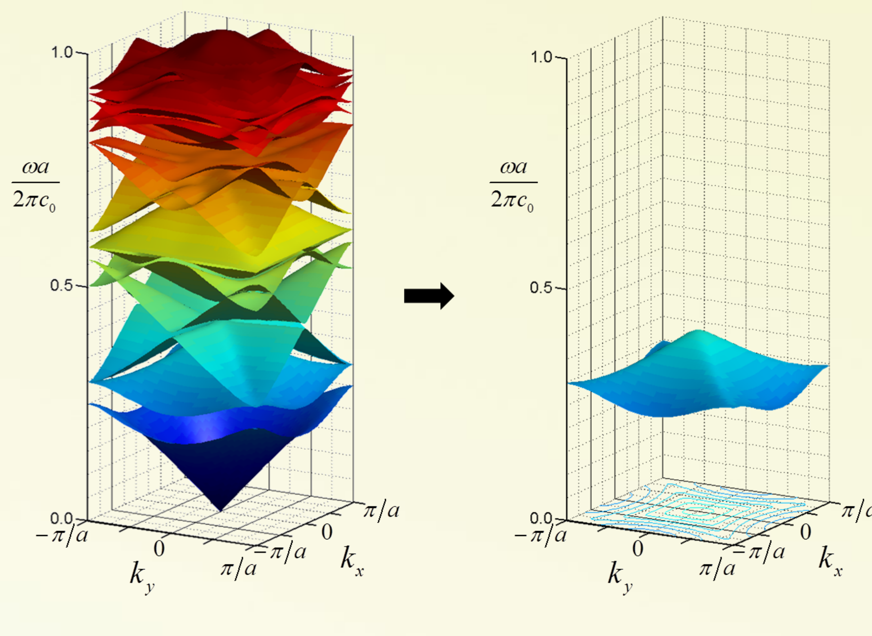


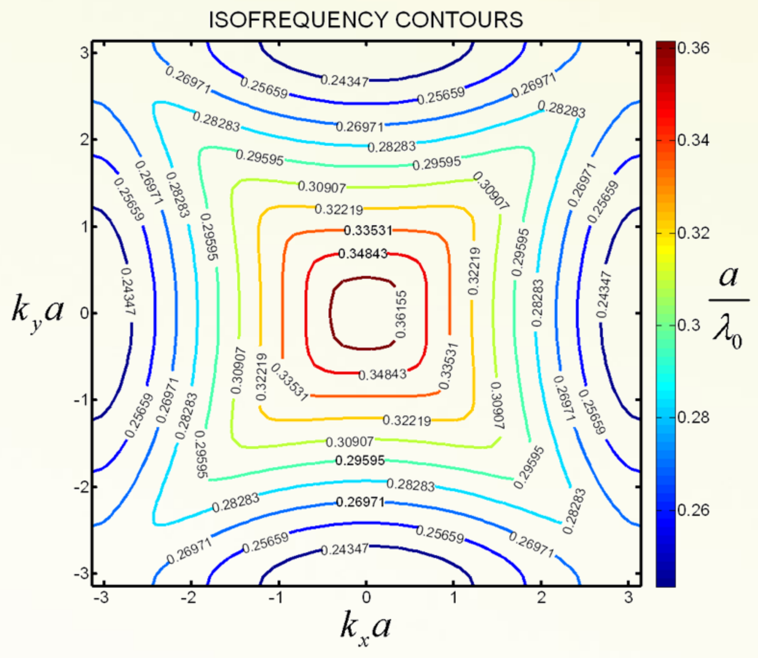
BAND DIAGRAMS





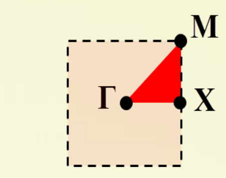
ISOFREQUENCY CONTOURS





REDUCED BLOCH MODE EXPANSION

Step 1: Calculate the eigen-vector matrices at the key points of symmetry



$$\mathbf{V}_\Gamma=\left[\mathbf{v}_\Gamma^{(1)}\ \mathbf{v}_\Gamma^{(2)}\ \mathbf{v}_\Gamma^{(3)}\ \cdots\ \mathbf{v}_\Gamma^{(N)}\right] \quad \mathbf{V}_X=\left[\mathbf{v}_X^{(1)}\ \mathbf{v}_X^{(2)}\ \mathbf{v}_X^{(3)}\ \cdots\ \mathbf{v}_X^{(N)}\right] \quad \mathbf{V}_M=\left[\mathbf{v}_M^{(1)}\ \mathbf{v}_M^{(2)}\ \mathbf{v}_M^{(3)}\ \cdots\ \mathbf{v}_M^{(N)}\right]$$

Step 2: Construct Bloch mode

$$\mathbf{U}'=\left[\mathbf{v}_\Gamma^{(1)}\ \cdots\ \mathbf{v}_\Gamma^{(M)}\ \mathbf{v}_X^{(1)}\ \cdots\ \mathbf{v}_X^{(M)}\ \mathbf{v}_M^{(1)}\ \cdots\ \mathbf{v}_M^{(M)}\right] \qquad \mathbf{U}=\text{GramSchmidt}(\mathbf{U}')$$

Step 3: Calculate eigen-value problem using standard PWEM

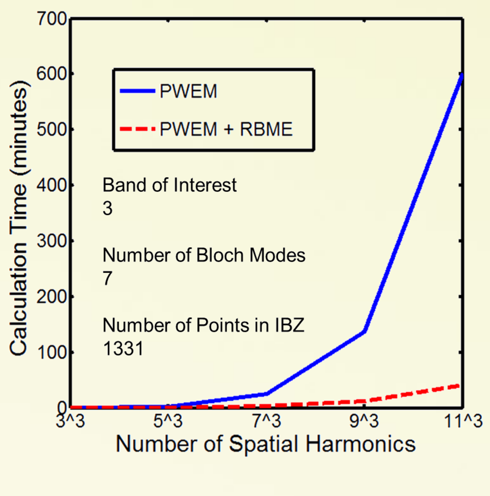
$$\mathbf{A}\mathbf{x}=\lambda\mathbf{B}\mathbf{x}$$

Step 4: Calculate and solve a reduced eigen-value problem.

$$\mathbf{A}'=\mathbf{U}^H\mathbf{A}\mathbf{U} \qquad \mathbf{B}'=\mathbf{U}^H\mathbf{B}\mathbf{U} \qquad \tilde{\mathbf{A}}\tilde{\mathbf{x}}=\lambda\tilde{\mathbf{B}}\tilde{\mathbf{x}} \rightarrow \tilde{\mathbf{V}},\lambda$$

Step 5: If needed, the eigen-vectors can be transformed back to the plane wave basis.

$$\mathbf{V}=\mathbf{U}\tilde{\mathbf{V}}\mathbf{U}^H$$



M. I. Hussein, "Reduced Bloch mode expansion for periodic media band structure calculations," Proc. Roy. Soc. Lond. Ser. A465, 2825–2848 (2009).

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