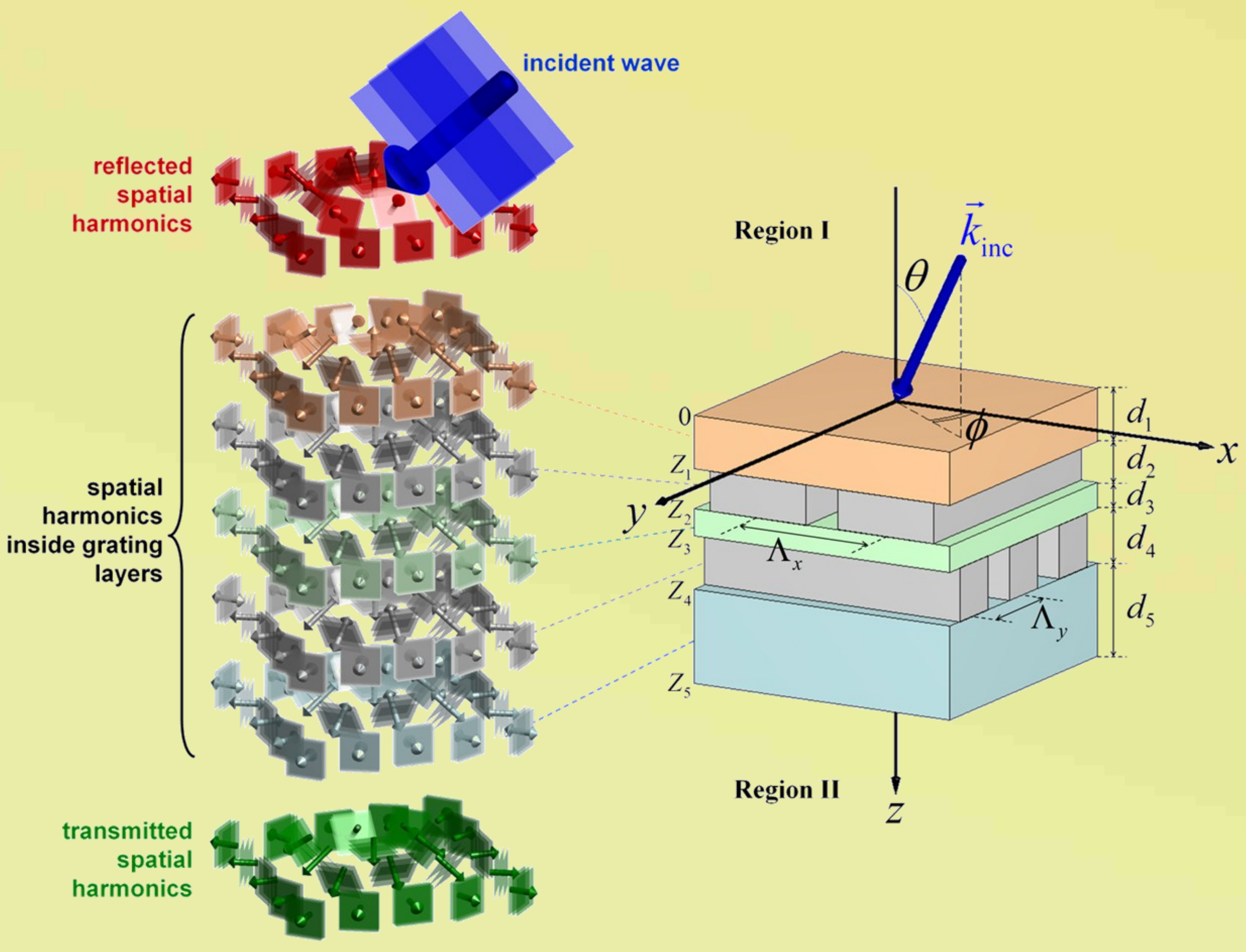
THE UNIVERSITY OF TEXAS AT EL PASO

Fast Analysis of All-Dielectric Structures Using

Rigorous Coupled-Wave Analysis

Rigorous coupled-wave analysis (RCWA) is an extremely efficient algorithm for modeling scattering through all-dielectric structures. It is most efficient for devices with low to moderate dielectric contrast.



SEMI-ANALYTICAL FORMULATION OF MAXWELL'S EQUATIONS

Wave Equation for i^{th} Layer

$$\frac{d^2}{d\tilde{z}^2}\begin{bmatrix}\mathbf{s}_x^{(i)}\\ \mathbf{s}_y^{(i)}\end{bmatrix}-\mathbf{\Omega}_i^2\begin{bmatrix}\mathbf{s}_x^{(i)}\\ \mathbf{s}_y^{(i)}\end{bmatrix}=\mathbf{0}$$

$$\mathbf{\Omega}_i^2=\mathbf{P}_i\mathbf{Q}_i$$

$$\mathbf{Q}_i=\begin{bmatrix}\mathbf{K}_x\boldsymbol{\mu}_{r,i}^{-1}\mathbf{K}_y & \boldsymbol{\varepsilon}_{r,i}-\mathbf{K}_x\boldsymbol{\mu}_{r,i}^{-1}\mathbf{K}_x\\ \mathbf{K}_y\boldsymbol{\mu}_{r,i}^{-1}\mathbf{K}_y-\boldsymbol{\varepsilon}_{r,i} & -\mathbf{K}_y\boldsymbol{\mu}_{r,i}^{-1}\mathbf{K}_x\end{bmatrix}$$

$$\mathbf{P}_i=\begin{bmatrix}\mathbf{K}_x\boldsymbol{\varepsilon}_{r,i}^{-1}\mathbf{K}_y & \boldsymbol{\mu}_{r,i}-\mathbf{K}_x\boldsymbol{\varepsilon}_{r,i}^{-1}\mathbf{K}_x\\ \mathbf{K}_y\boldsymbol{\varepsilon}_{r,i}^{-1}\mathbf{K}_y-\boldsymbol{\mu}_{r,i} & -\mathbf{K}_y\boldsymbol{\varepsilon}_{r,i}^{-1}\mathbf{K}_x\end{bmatrix}$$

Solution

$$\mathbf{\Omega}_i^2 \rightarrow \begin{matrix}\mathbf{W}_i \equiv \text{eigen-vector matrix}\\ \boldsymbol{\lambda}_i^2 \equiv \text{eigen-value matrix}\end{matrix}$$

$$\mathbf{V}=\mathbf{QW}\boldsymbol{\lambda}^{-1}$$

$$\boldsymbol{\psi}(\tilde{z})=\begin{bmatrix}\mathbf{s}_x(\tilde{z})\\ \mathbf{s}_y(\tilde{z})\\ \mathbf{u}_x(\tilde{z})\\ \mathbf{u}_y(\tilde{z})\end{bmatrix}=\begin{bmatrix}\mathbf{W} & \mathbf{W}\\ -\mathbf{V} & \mathbf{V}\end{bmatrix}\begin{bmatrix}e^{-\lambda\tilde{z}} & \mathbf{0}\\ \mathbf{0} & e^{\lambda\tilde{z}}\end{bmatrix}\begin{bmatrix}\mathbf{c}^+\\ \mathbf{c}^-\end{bmatrix}$$

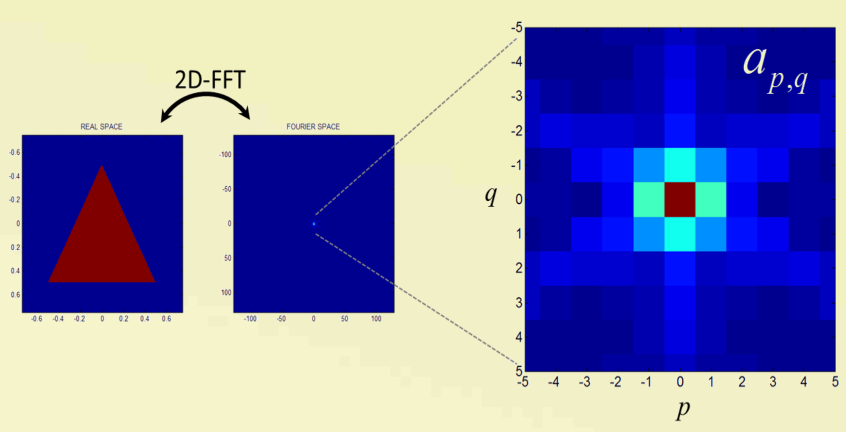
K Matrices

$$\mathbf{K}_i=\begin{bmatrix}k_{i,1,1,1} & & & \\ & k_{i,1,1,2} & & \\ & & \ddots & \\ & & & k_{i,P,Q,R}\end{bmatrix}$$

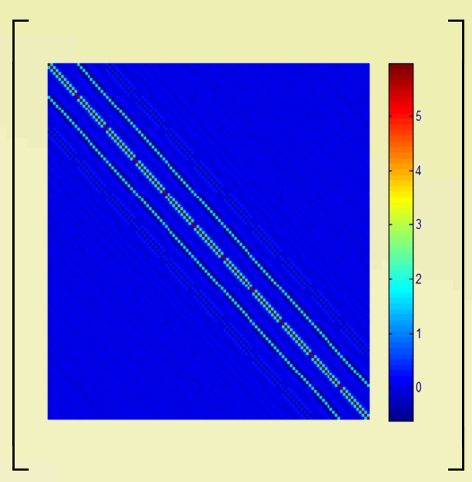
Plane Wave Amplitudes

$$\mathbf{u}_i=\begin{bmatrix}U_{i,1,1,1}\\ U_{i,1,1,2}\\ \vdots\\ U_{i,P,Q,R}\end{bmatrix}\qquad \mathbf{s}_i=\begin{bmatrix}S_{i,1,1,1}\\ S_{i,1,1,2}\\ \vdots\\ S_{i,P,Q,R}\end{bmatrix}$$

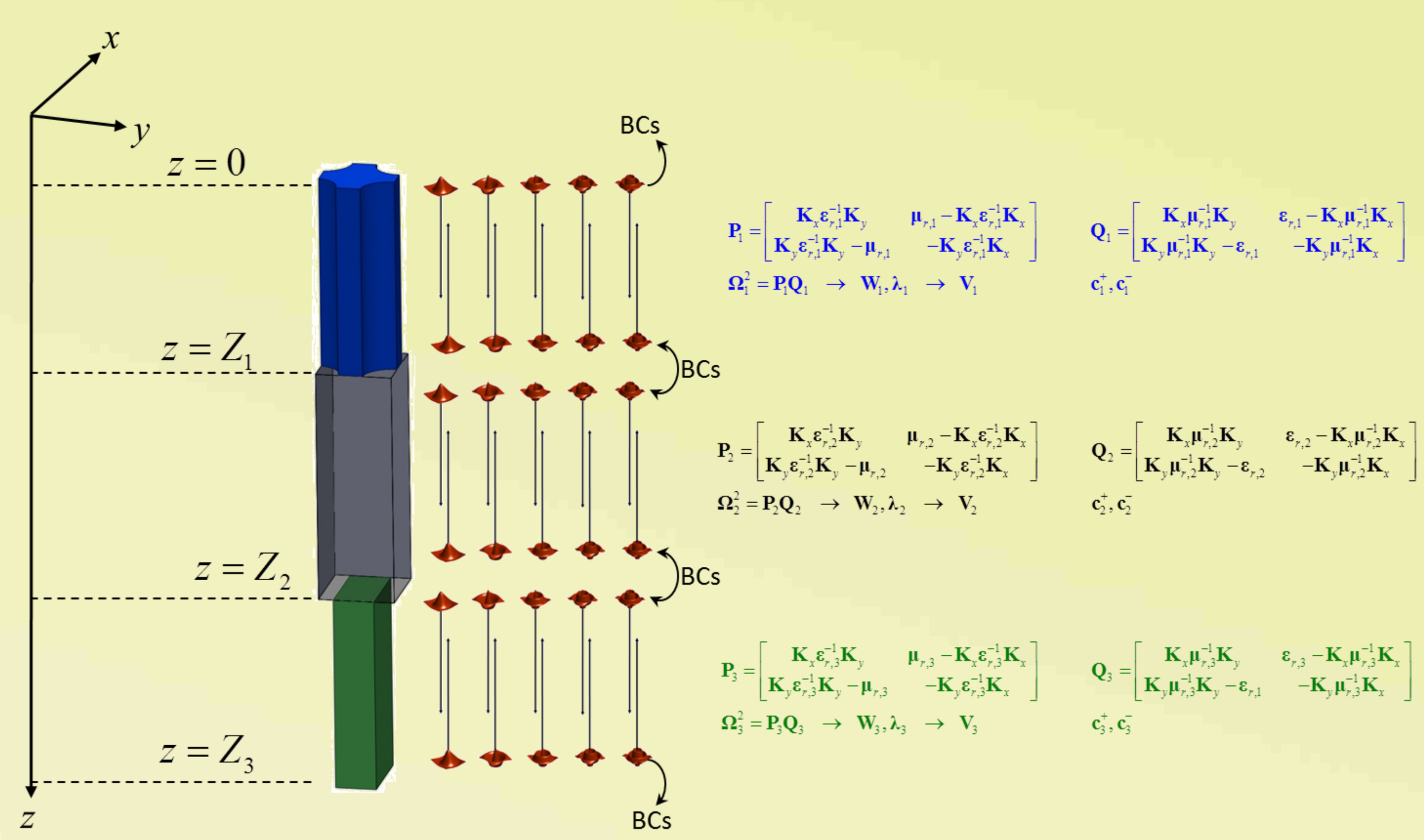
THE CONVOLUTION MATRICES



$\boldsymbol{\varepsilon}_r =$

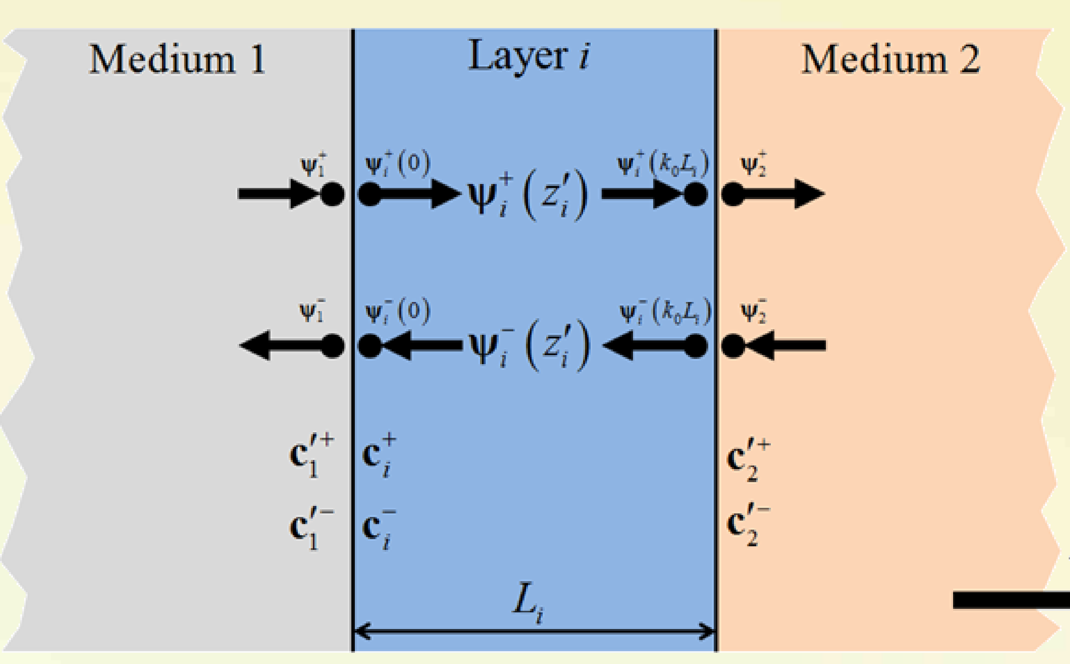


THE SEMI-ANALYTICAL SOLUTION



THE SCATTERING MATRIX

The formulation for the general asymmetric case is



$$\begin{bmatrix}\mathbf{c}_1'^-\\ \mathbf{c}_2'^+\end{bmatrix}=\begin{bmatrix}\mathbf{S}_{11}^{(i)} & \mathbf{S}_{12}^{(i)}\\ \mathbf{S}_{21}^{(i)} & \mathbf{S}_{22}^{(i)}\end{bmatrix}\begin{bmatrix}\mathbf{c}_1'^+\\ \mathbf{c}_2'^-\end{bmatrix}$$

$$\mathbf{A}_{ij}=\mathbf{W}_i^{-1}\mathbf{W}_j+\mathbf{V}_i^{-1}\mathbf{V}_j$$

$$\mathbf{B}_{ij}=\mathbf{W}_i^{-1}\mathbf{W}_j-\mathbf{V}_i^{-1}\mathbf{V}_j$$

$$\mathbf{X}_i=e^{-\lambda_i k_0 L_i}$$

$$\mathbf{S}_{11}^{(i)}=\left(\mathbf{A}_{i1}-\mathbf{X}_i\mathbf{B}_{i2}\mathbf{A}_{i2}^{-1}\mathbf{X}_i\mathbf{B}_{i1}\right)^{-1}\left(\mathbf{X}_i\mathbf{B}_{i2}\mathbf{A}_{i2}^{-1}\mathbf{X}_i\mathbf{A}_{i1}-\mathbf{B}_{i1}\right)$$

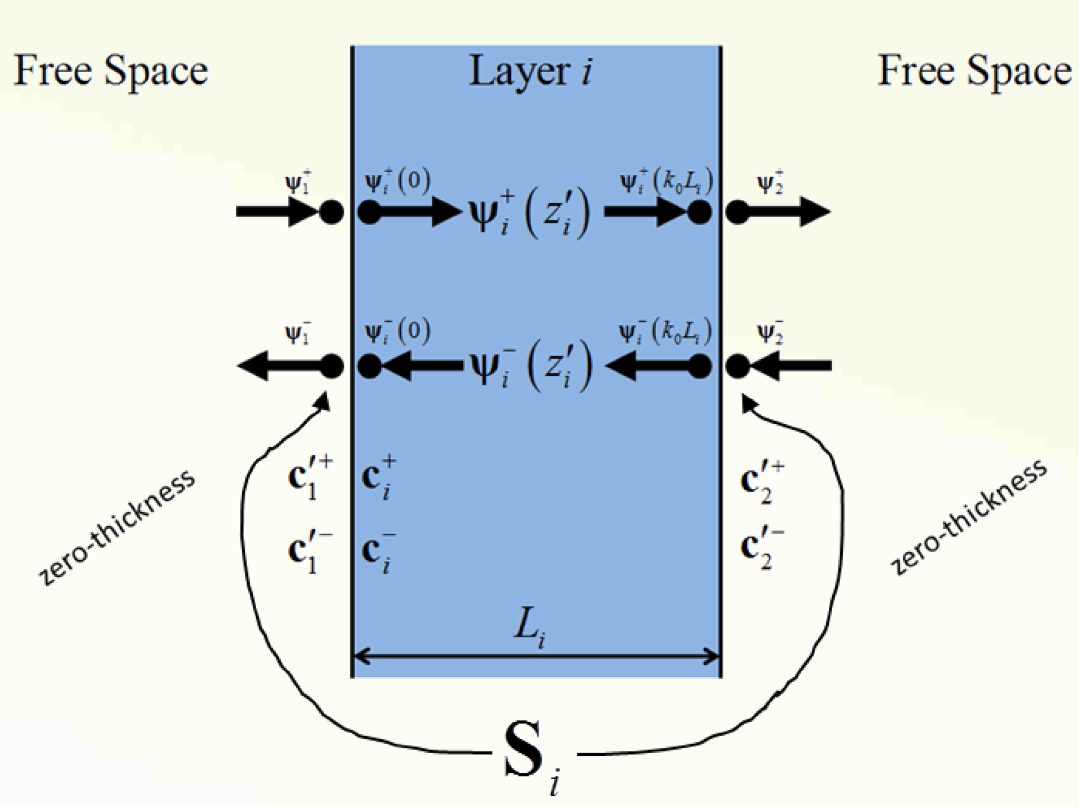
$$\mathbf{S}_{12}^{(i)}=\left(\mathbf{A}_{i1}-\mathbf{X}_i\mathbf{B}_{i2}\mathbf{A}_{i2}^{-1}\mathbf{X}_i\mathbf{B}_{i1}\right)^{-1}\mathbf{X}_i\left(\mathbf{A}_{i2}-\mathbf{B}_{i2}\mathbf{A}_{i2}^{-1}\mathbf{B}_{i2}\right)$$

$$\mathbf{S}_{21}^{(i)}=\left(\mathbf{A}_{i2}-\mathbf{X}_i\mathbf{B}_{i1}\mathbf{A}_{i1}^{-1}\mathbf{X}_i\mathbf{B}_{i2}\right)^{-1}\mathbf{X}_i\left(\mathbf{A}_{i1}-\mathbf{B}_{i1}\mathbf{A}_{i1}^{-1}\mathbf{B}_{i1}\right)$$

$$\mathbf{S}_{22}^{(i)}=\left(\mathbf{A}_{i2}-\mathbf{X}_i\mathbf{B}_{i1}\mathbf{A}_{i1}^{-1}\mathbf{X}_i\mathbf{B}_{i2}\right)^{-1}\left(\mathbf{X}_i\mathbf{B}_{i1}\mathbf{A}_{i1}^{-1}\mathbf{X}_i\mathbf{A}_{i2}-\mathbf{B}_{i2}\right)$$

An improved formulation offers the following benefits:

1. Symmetric scattering matrices
2. Faster the calculate.
3. More memory efficient.
4. Interchangeable scattering matrices.



$$\mathbf{A}_i=\mathbf{W}_i^{-1}\mathbf{W}_0+\mathbf{V}_i^{-1}\mathbf{V}_0$$

$$\mathbf{B}_i=\mathbf{W}_i^{-1}\mathbf{W}_0-\mathbf{V}_i^{-1}\mathbf{V}_0$$

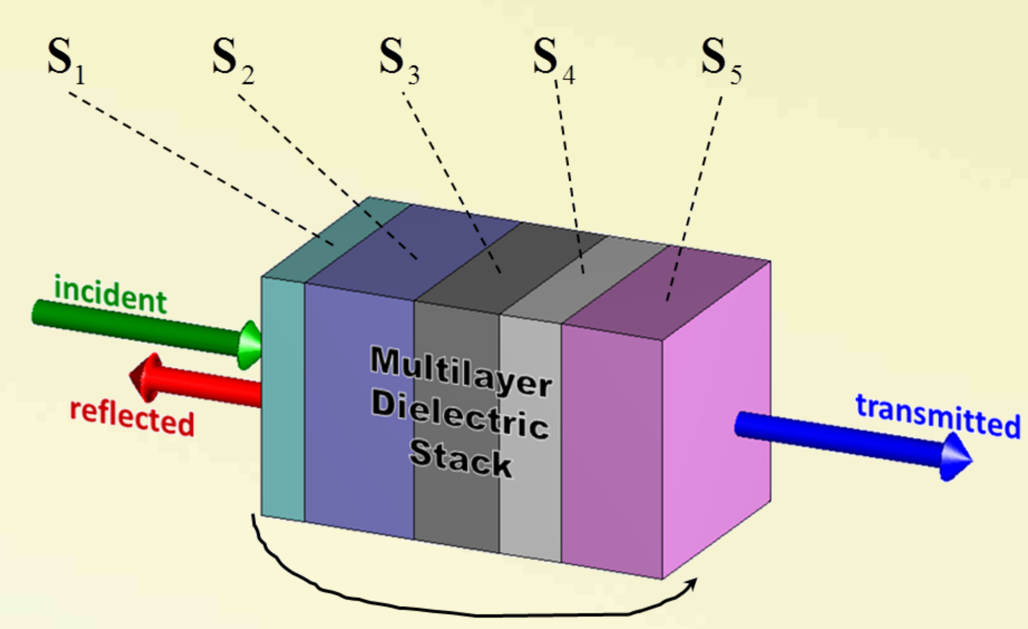
$$\mathbf{S}_{11}^{(i)}=\left(\mathbf{A}_i-\mathbf{X}_i\mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{X}_i\mathbf{B}_i\right)^{-1}\left(\mathbf{X}_i\mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{X}_i\mathbf{A}_i-\mathbf{B}_i\right)$$

$$\mathbf{S}_{12}^{(i)}=\left(\mathbf{A}_i-\mathbf{X}_i\mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{X}_i\mathbf{B}_i\right)^{-1}\mathbf{X}_i\left(\mathbf{A}_i-\mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{B}_i\right)$$

$$\mathbf{S}_{21}^{(i)}=\mathbf{S}_{12}^{(i)}$$

$$\mathbf{S}_{22}^{(i)}=\mathbf{S}_{11}^{(i)}$$

THE SCATTERING MATRIX ALGORITHM



$$\mathbf{S}_{\text{global}}=\mathbf{S}_1\otimes\mathbf{S}_2\otimes\mathbf{S}_3\otimes\mathbf{S}_4\otimes\mathbf{S}_5$$

Redheffer Star Product:

$$\mathbf{S}=\mathbf{S}_a\otimes\mathbf{S}_b$$

$$\mathbf{S}_{11}=\mathbf{a}_{11}+\mathbf{a}_{12}\left(\mathbf{I}-\mathbf{b}_{11}\mathbf{a}_{22}\right)^{-1}\mathbf{b}_{11}\mathbf{a}_{21}$$

$$\mathbf{S}_{12}=\mathbf{a}_{12}\left(\mathbf{I}-\mathbf{b}_{11}\mathbf{a}_{22}\right)^{-1}\mathbf{b}_{12}$$

$$\mathbf{S}_{21}=\mathbf{b}_{21}\left(\mathbf{I}-\mathbf{a}_{22}\mathbf{b}_{11}\right)^{-1}\mathbf{a}_{21}$$

$$\mathbf{S}_{22}=\mathbf{b}_{22}+\mathbf{b}_{21}\left(\mathbf{I}-\mathbf{a}_{22}\mathbf{b}_{11}\right)^{-1}\mathbf{a}_{22}\mathbf{b}_{12}$$

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