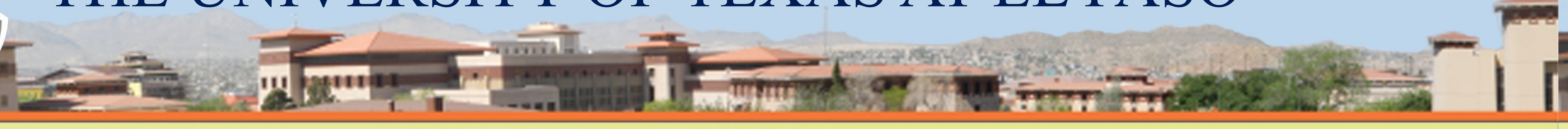
THE UNIVERSITY OF TEXAS AT EL PASO



Slice Absorption Method

for Rigorous Electromagnetic Analysis of Volumetrically Complex Periodic Structures

The slice absorption method (SAM) is specifically tailored to model periodic structures that are volumetrically complex. It was inspired by semi-analytical methods in its ability to model structures one layer at a time, but is fully numerical and avoids the computationally expensive eigen-system computation.

Real-Space Formulation

Block Matrix Form

$$\mathbf{C}^e\vec{\mathbf{e}} = [\boldsymbol{\mu}]\vec{\tilde{\mathbf{h}}}$$
$$\mathbf{C}^h\vec{\tilde{\mathbf{h}}} = [\boldsymbol{\varepsilon}]\vec{\mathbf{e}}$$

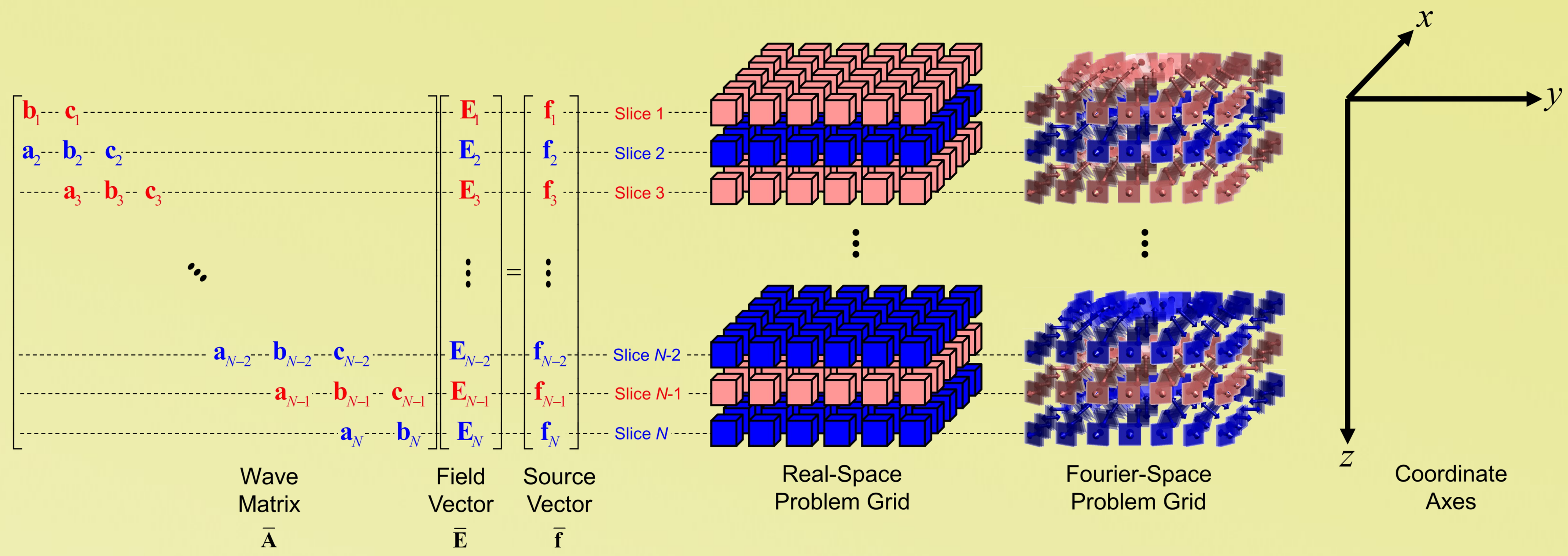
$$\vec{\mathbf{e}} = \begin{bmatrix}\mathbf{e}_x\\ \mathbf{e}_y\\ \mathbf{e}_z\end{bmatrix} \qquad \vec{\tilde{\mathbf{h}}} = \begin{bmatrix}\tilde{\mathbf{h}}_x\\ \tilde{\mathbf{h}}_y\\ \tilde{\mathbf{h}}_z\end{bmatrix}$$

$$[\boldsymbol{\varepsilon}] = \begin{bmatrix}\boldsymbol{\varepsilon}_{xx} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\varepsilon}_{yy} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \boldsymbol{\varepsilon}_{zz}\end{bmatrix} \qquad [\boldsymbol{\mu}] = \begin{bmatrix}\boldsymbol{\mu}_{xx} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{\mu}_{yy} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \boldsymbol{\mu}_{zz}\end{bmatrix}$$

$$\mathbf{C}^e = \begin{bmatrix}\mathbf{0} & -\mathbf{D}^e_{z'} & \mathbf{D}^e_{y'}\\ \mathbf{D}^e_{z'} & \mathbf{0} & -\mathbf{D}^e_{x'}\\ -\mathbf{D}^e_{y'} & \mathbf{D}^e_{x'} & \mathbf{0}\end{bmatrix} \qquad \mathbf{C}^h = \begin{bmatrix}\mathbf{0} & -\mathbf{D}^h_{z'} & \mathbf{D}^h_{y'}\\ \mathbf{D}^h_{z'} & \mathbf{0} & -\mathbf{D}^h_{x'}\\ -\mathbf{D}^h_{y'} & \mathbf{D}^h_{x'} & \mathbf{0}\end{bmatrix}$$

Matrix Wave Equations

$$\mathbf{A}_e\vec{\tilde{\mathbf{h}}} = \mathbf{0} \qquad \mathbf{A}_e = \mathbf{C}^e[\boldsymbol{\varepsilon}]^{-1}\mathbf{C}^h - [\boldsymbol{\mu}]$$
$$\mathbf{A}_h\vec{\mathbf{e}} = \mathbf{0} \qquad \mathbf{A}_h = \mathbf{C}^h[\boldsymbol{\mu}]^{-1}\mathbf{C}^e - [\boldsymbol{\varepsilon}]$$



Fourier-Space Formulation

Block Matrix Form

$$\mathbf{C}^s\vec{\mathbf{s}} = \boldsymbol{\mu}*\ \vec{\tilde{\mathbf{u}}}$$
$$\mathbf{C}^u\vec{\tilde{\mathbf{u}}} = \boldsymbol{\varepsilon}*\ \vec{\mathbf{s}}$$

$$\vec{\mathbf{s}} = \begin{bmatrix}\mathbf{s}_x\\ \mathbf{s}_y\\ \mathbf{s}_z\end{bmatrix} \qquad \vec{\tilde{\mathbf{u}}} = \begin{bmatrix}\tilde{\mathbf{u}}_x\\ \tilde{\mathbf{u}}_y\\ \tilde{\mathbf{u}}_z\end{bmatrix}$$

$$[\boldsymbol{\varepsilon}] = \begin{bmatrix}\boldsymbol{\varepsilon}_{xx}* & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & [\![\boldsymbol{\varepsilon}_{yy}*]\!] & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \boldsymbol{\varepsilon}_{zz}*\end{bmatrix} \qquad [\boldsymbol{\mu}] = \begin{bmatrix}\boldsymbol{\mu}_{xx}* & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & [\![\boldsymbol{\mu}_{yy}*]\!] & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \boldsymbol{\mu}_{zz}*\end{bmatrix}$$

$$\mathbf{C}^s = \begin{bmatrix}\mathbf{0} & -\mathbf{D}^s_{z'} & \tilde{\mathbf{K}}^s_y\\ \mathbf{D}^s_{z'} & \mathbf{0} & -\tilde{\mathbf{K}}^s_x\\ -\tilde{\mathbf{K}}^s_y & \tilde{\mathbf{K}}^s_x & \mathbf{0}\end{bmatrix} \qquad \mathbf{C}^u = \begin{bmatrix}\mathbf{0} & -\mathbf{D}^u_{z'} & \tilde{\mathbf{K}}^u_y\\ \mathbf{D}^u_{z'} & \mathbf{0} & -\tilde{\mathbf{K}}^u_x\\ -\tilde{\mathbf{K}}^u_y & \tilde{\mathbf{K}}^u_x & \mathbf{0}\end{bmatrix}$$

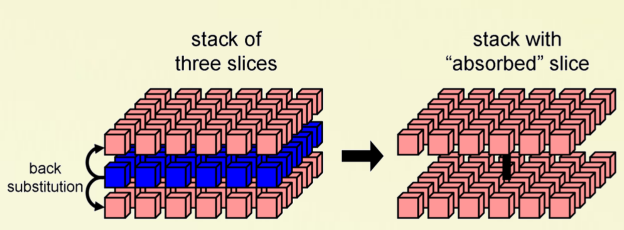
Matrix Wave Equations

$$\mathbf{A}_s\vec{\tilde{\mathbf{u}}} = \mathbf{0} \qquad \mathbf{A}_s = \mathbf{C}^s\ \boldsymbol{\varepsilon}*\ ^{-1}\mathbf{C}^u - \ \boldsymbol{\mu}*$$
$$\mathbf{A}_u\vec{\mathbf{s}} = \mathbf{0} \qquad \mathbf{A}_u = \mathbf{C}^u\ \boldsymbol{\mu}*\ ^{-1}\mathbf{C}^s - \ \boldsymbol{\varepsilon}*$$

K Matrices

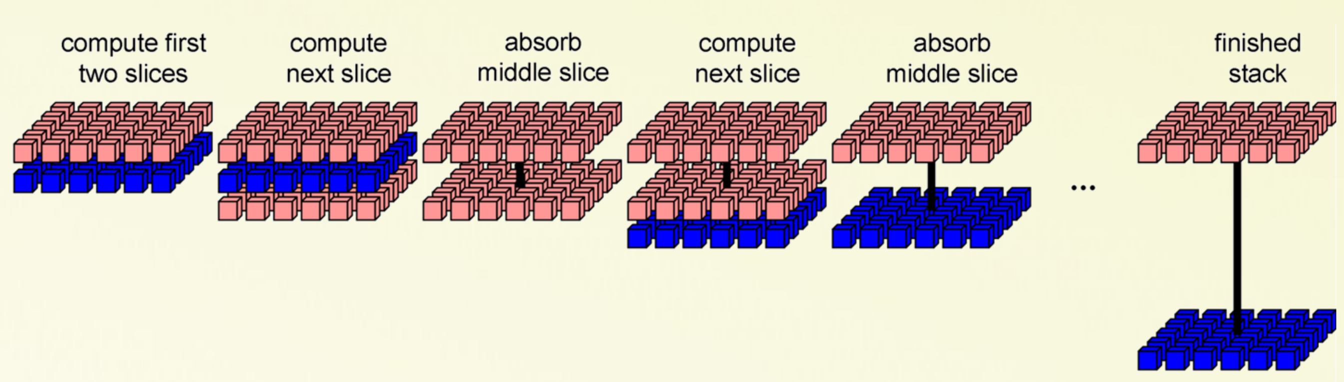
Absorbing the i^{th} Slice

$$\begin{aligned}\mathbf{a}_{i-1}\mathbf{E}_{i-2} + \mathbf{b}_{i-1}\mathbf{E}_{i-1} + \mathbf{c}_{i-1}\mathbf{E}_i &= \mathbf{f}_{i-1}\\ \mathbf{a}_i\mathbf{E}_{i-1} + \mathbf{b}_i\mathbf{E}_i + \mathbf{c}_i\mathbf{E}_{i+1} &= \mathbf{f}_i\\ \mathbf{a}_{i+1}\mathbf{E}_i + \mathbf{b}_{i+1}\mathbf{E}_{i+1} + \mathbf{c}_{i+1}\mathbf{E}_{i+2} &= \mathbf{f}_{i+1}\end{aligned} \quad\rightarrow\quad \begin{aligned}\mathbf{a}'_{i-1}\mathbf{E}_{i-2} + \mathbf{b}'_{i-1}\mathbf{E}_{i-1} - \mathbf{c}'_{i-1}\mathbf{E}_{i+1} &= \mathbf{f}'_{i-1}\\ \mathbf{a}'_{i+1}\mathbf{E}_{i-1} + \mathbf{b}'_{i+1}\mathbf{E}_{i+1} - \mathbf{c}'_{i+1}\mathbf{E}_{i+2} &= \mathbf{f}'_{i+1}\end{aligned}$$

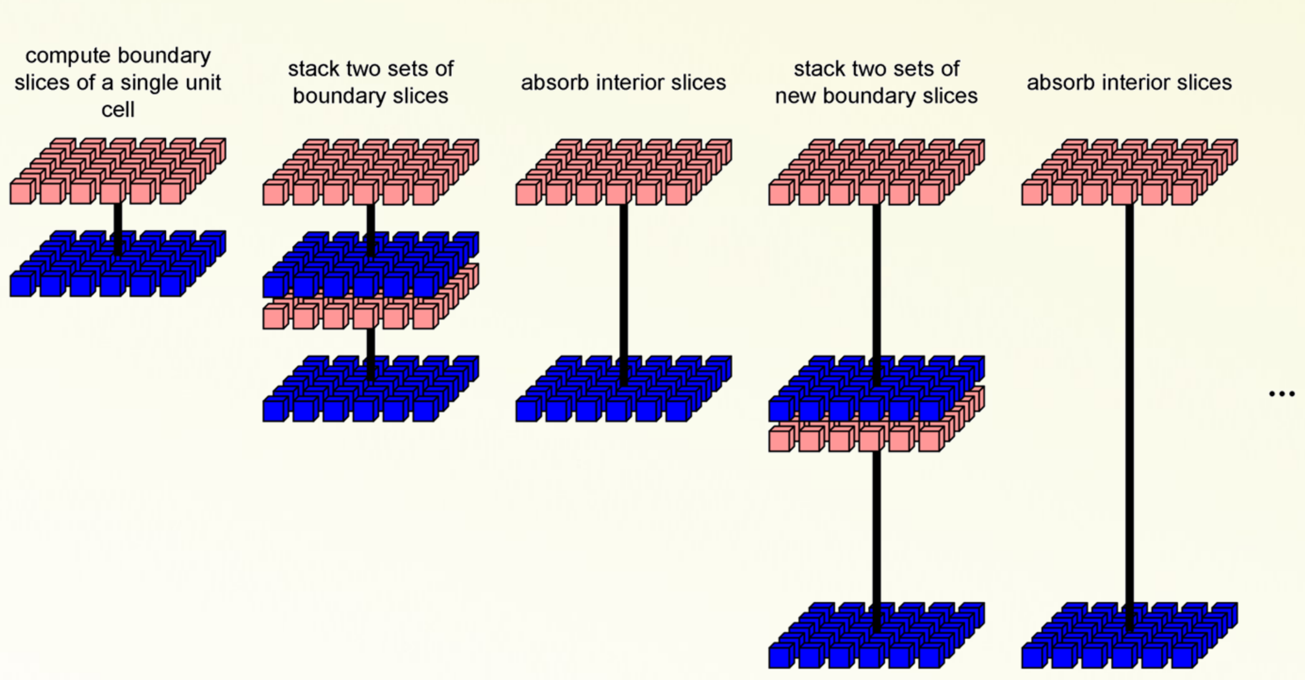


$$\begin{aligned}\mathbf{a}'_{i-1} &= \mathbf{a}_{i-1}\\ \mathbf{b}'_{i-1} &= \mathbf{b}_{i-1} - \mathbf{c}_{i-1}\mathbf{b}_i^{-1}\mathbf{a}_i\\ \mathbf{c}'_{i-1} &= -\mathbf{c}_{i-1}\mathbf{b}_i^{-1}\mathbf{c}_i\\ \mathbf{f}'_{i-1} &= \mathbf{f}_{i-1} - \mathbf{c}_{i-1}\mathbf{b}_i^{-1}\mathbf{f}_i\end{aligned} \qquad \begin{aligned}\mathbf{a}'_{i+1} &= -\mathbf{a}_{i+1}\mathbf{b}_i^{-1}\mathbf{a}_i\\ \mathbf{b}'_{i+1} &= \mathbf{b}_{i+1} - \mathbf{a}_{i+1}\mathbf{b}_i^{-1}\mathbf{c}_i\\ \mathbf{c}'_{i+1} &= \mathbf{c}_{i+1}\\ \mathbf{f}'_{i+1} &= \mathbf{f}_{i+1} - \mathbf{a}_{i+1}\mathbf{b}_i^{-1}\mathbf{f}_i\end{aligned}$$

Absorbing a Stack of Slices



Cascading and Doubling Algorithm



Calculating Transmitted and Reflected Power

Step 1: Calculate amplitudes of spatial harmonics

$$\vec{\mathbf{s}} = (\mathbf{FT})\vec{\mathbf{e}}$$

Step 2: Calculate diffraction efficiencies

$$P_{m,n} = \left|\vec{S}_{m,n}\right|^2 \cdot \mathrm{Re}\left[\frac{k_{z,m,n}}{k_z^{\text{inc}}}\frac{\mu_r^{\text{inc}}}{\mu_r}\right]$$

Calculating the Slice Data

Step 1: Calculate the A matrix for three consecutive slices.

$$\mathbf{A}\vec{\mathbf{e}} = \mathbf{0}$$

$$\vec{\mathbf{e}} = \begin{bmatrix}E_x^{1,1,1}\\ \vdots\\ E_x^{N_x,N_y,N_z}\\ E_y^{1,1,1}\\ \vdots\\ E_y^{N_x,N_y,N_z}\\ E_z^{1,1,1}\\ \vdots\\ E_z^{N_x,N_y,N_z}\end{bmatrix}$$

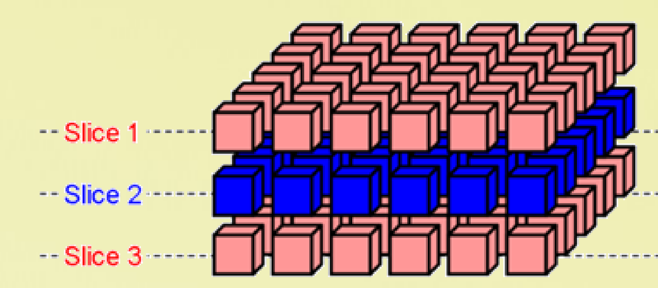
Step 2: Reorder the **A** matrix.

$$\tilde{\mathbf{A}} = \text{reorder}(\mathbf{A})$$
$$\tilde{\mathbf{e}} = \text{reorder}(\vec{\mathbf{e}})$$
$$\langle\tilde{\mathbf{A}}\rangle_{\tilde{p}\tilde{q}} = \langle\mathbf{A}\rangle_{pq}$$
$$p = 1 + N_xN_yN_z\cdot\text{mod}_3(\tilde{p}-1) + \lfloor(\tilde{p}-1)\div 3\rfloor$$
$$q = 1 + N_xN_yN_z\cdot\text{mod}_3(\tilde{q}-1) + \lfloor(\tilde{q}-1)\div 3\rfloor$$

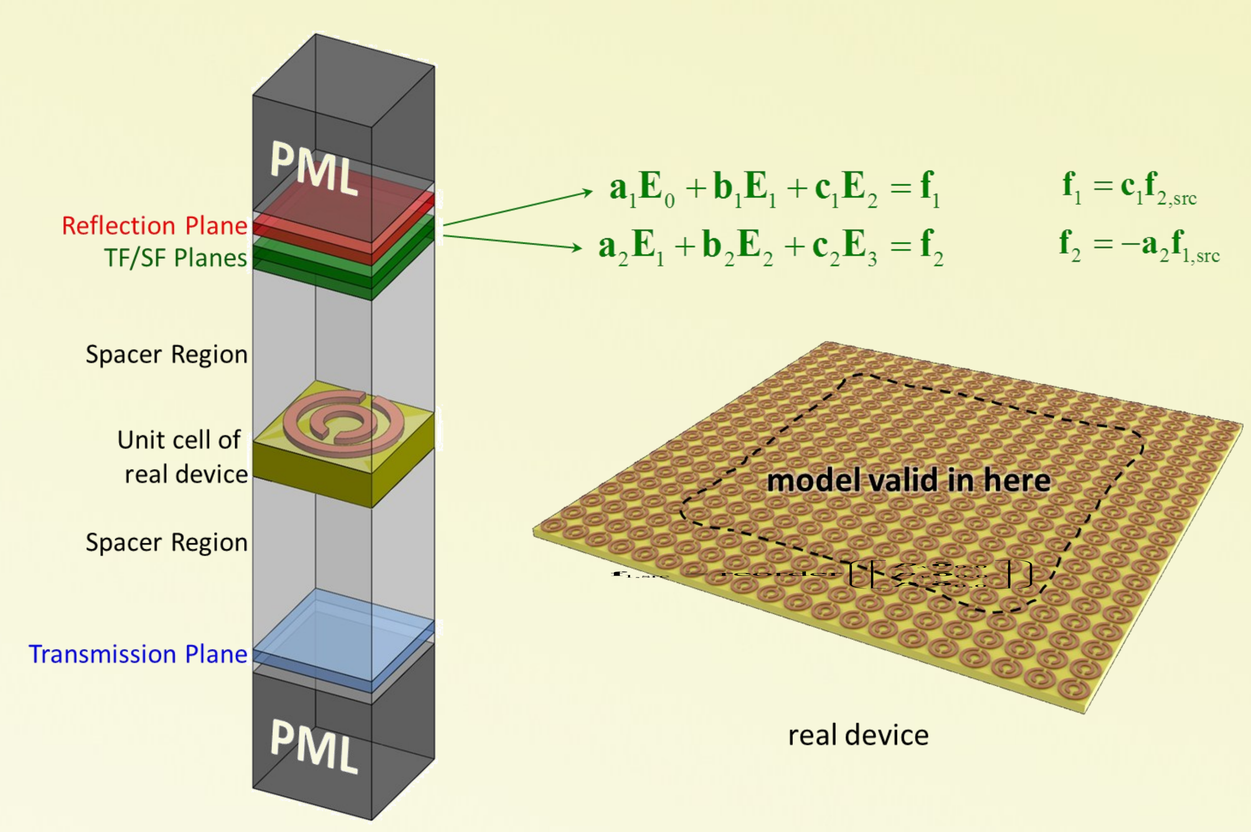
$$\tilde{\mathbf{e}} = \begin{bmatrix}E_x^{1,1,1}\\ E_y^{1,1,1}\\ E_z^{1,1,1}\\ E_x^{2,1,1}\\ E_y^{2,1,1}\\ E_z^{2,1,1}\\ \vdots\\ E_x^{N_x,1,1}\\ E_y^{N_x,1,1}\\ E_z^{N_x,1,1}\\ \vdots\\ E_x^{N_x,N_y,N_z}\\ E_y^{N_x,N_y,N_z}\\ E_z^{N_x,N_y,N_z}\end{bmatrix}$$

Step 3: Extract the slice data $\mathbf{a}_2$, $\mathbf{b}_2$, $\mathbf{c}_2$, and $\mathbf{f}_2$.

$$\tilde{\mathbf{A}}\tilde{\mathbf{e}} = \mathbf{f} \;\rightarrow\; \begin{bmatrix}\mathbf{b}_1 & \mathbf{c}_1 & \mathbf{0}\\ \mathbf{a}_2 & \mathbf{b}_2 & \mathbf{c}_2\\ \mathbf{0} & \mathbf{a}_3 & \mathbf{b}_3\end{bmatrix}\begin{bmatrix}\mathbf{e}_1\\ \mathbf{e}_2\\ \mathbf{e}_3\end{bmatrix} = \begin{bmatrix}\mathbf{f}_1\\ \mathbf{f}_2\\ \mathbf{f}_3\end{bmatrix}$$



Incorporating a Plane Wave Source



$$\mathbf{a}_1\mathbf{E}_0 + \mathbf{b}_1\mathbf{E}_1 + \mathbf{c}_1\mathbf{E}_2 = \mathbf{f}_1 \qquad \mathbf{f}_1 = \mathbf{c}_1\mathbf{f}_{2,\text{src}}$$
$$\mathbf{a}_2\mathbf{E}_1 + \mathbf{b}_2\mathbf{E}_2 + \mathbf{c}_2\mathbf{E}_3 = \mathbf{f}_2 \qquad \mathbf{f}_2 = -\mathbf{a}_2\mathbf{f}_{1,\text{src}}$$

Real-Space Source

$$\mathbf{f}_{1,\text{src}} = \text{reorder}\left(\begin{bmatrix}p_x e^{-j(k_{x,\text{inc}}\mathbf{x} + k_{x,\text{inc}}\mathbf{y})}\\ p_y e^{-j(k_{x,\text{inc}}\mathbf{x} + k_{x,\text{inc}}\mathbf{y})}\\ p_z e^{-j(k_{x,\text{inc}}\mathbf{x} + k_{x,\text{inc}}\mathbf{y})}\end{bmatrix}\right)$$

Fourier-Space Source

$$\mathbf{f}_{1,\text{src}} = \text{reorder}\left(\begin{bmatrix}p_x\boldsymbol{\delta}_{0,0}\\ p_y\boldsymbol{\delta}_{0,0}\\ p_z\boldsymbol{\delta}_{0,0}\end{bmatrix}\right)$$

$$\mathbf{f}_{2,\text{src}} = e^{-jk_{z,\text{inc}}\Delta z}\mathbf{f}_{1,\text{src}}$$

Incorporating Transparent Boundary Conditions

Step 1: Calculate tilt operator **T**

$$\mathbf{T}' = \exp\left[j\left(k_{x,\text{inc}}\mathbf{X} + k_{y,\text{inc}}\mathbf{Y}\right)\right]$$
$$\mathbf{T} = \text{reorder}\left(\begin{bmatrix}\mathbf{T}' & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{T}' & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{T}'\end{bmatrix}\right)$$

Step 2: Calculate Fourier transform operator **F**

$$H(m,n) = \sum_p\sum_q h(p,q)e^{-j2\pi\left(\frac{mp}{M}+\frac{nq}{N}\right)} \;\rightarrow\; \mathbf{H} = \mathbf{F}'\mathbf{h}$$
$$\mathbf{F} = \text{reorder}\left(\begin{bmatrix}\mathbf{F}' & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{F}' & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{F}'\end{bmatrix}\right)$$

Step 3: Calculator phase propagators **Z**

$$\mathbf{Z}'_{\text{ref}} = \begin{bmatrix}e^{-jk^{\text{ref}}_{z,1,1}\Delta z} & & 0\\ & \ddots & \\ 0 & & e^{-jk^{\text{ref}}_{z,M,N}\Delta z}\end{bmatrix} \qquad \mathbf{Z}_{\text{ref}} = \text{reorder}\left(\begin{bmatrix}\mathbf{Z}'_{\text{ref}} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{Z}'_{\text{ref}} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{Z}'_{\text{ref}}\end{bmatrix}\right)$$

$$\mathbf{Z}'_{\text{trn}} = \begin{bmatrix}e^{-jk^{\text{trn}}_{z,1,1}\Delta z} & & 0\\ & \ddots & \\ 0 & & e^{-jk^{\text{trn}}_{z,M,N}\Delta z}\end{bmatrix} \qquad \mathbf{Z}_{\text{trn}} = \text{reorder}\left(\begin{bmatrix}\mathbf{Z}'_{\text{trn}} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{Z}'_{\text{trn}} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{Z}'_{\text{trn}}\end{bmatrix}\right)$$

Step 4: Calculate field propagator **P**

Real-Space Propagators

$$\mathbf{P}^e_{\text{ref}} = \mathbf{T}^{-1}\mathbf{F}^{-1}\mathbf{Z}_{\text{ref}}\mathbf{FT} = (\mathbf{FT})^{-1}\mathbf{Z}_{\text{ref}}(\mathbf{FT})$$
$$\mathbf{P}^e_{\text{trn}} = \mathbf{T}^{-1}\mathbf{F}^{-1}\mathbf{Z}_{\text{trn}}\mathbf{FT} = (\mathbf{FT})^{-1}\mathbf{Z}_{\text{trn}}(\mathbf{FT})$$

Fourier-Space Propagators

$$\mathbf{P}^s_{\text{ref}} = \mathbf{Z}_{\text{ref}}$$
$$\mathbf{P}^s_{\text{trn}} = \mathbf{Z}_{\text{trn}}$$

Step 5: Adjust slice equations at top and bottom of grid.

$$\mathbf{b}'_1\mathbf{E}_1 + \mathbf{c}_1\mathbf{E}_2 = \mathbf{f}_1 \qquad \mathbf{b}'_1 = \mathbf{b}_1 + \mathbf{a}_1\mathbf{P}_{\text{ref}}$$
$$\mathbf{a}_N\mathbf{E}_{N-1} + \mathbf{b}'_N\mathbf{E}_N = \mathbf{f}_N \qquad \mathbf{b}'_N = \mathbf{b}_N + \mathbf{c}_N\mathbf{P}_{\text{trn}}$$

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