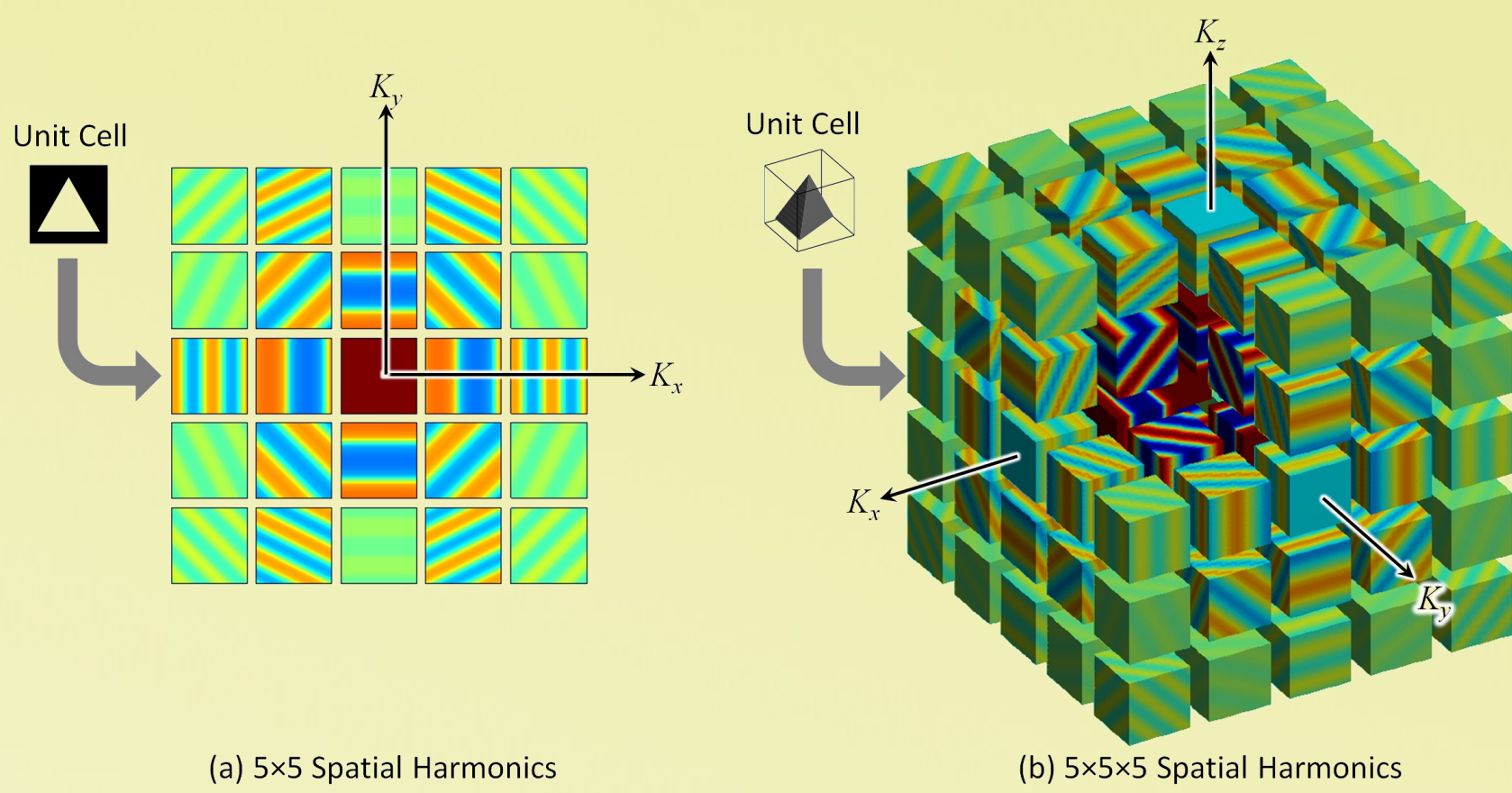


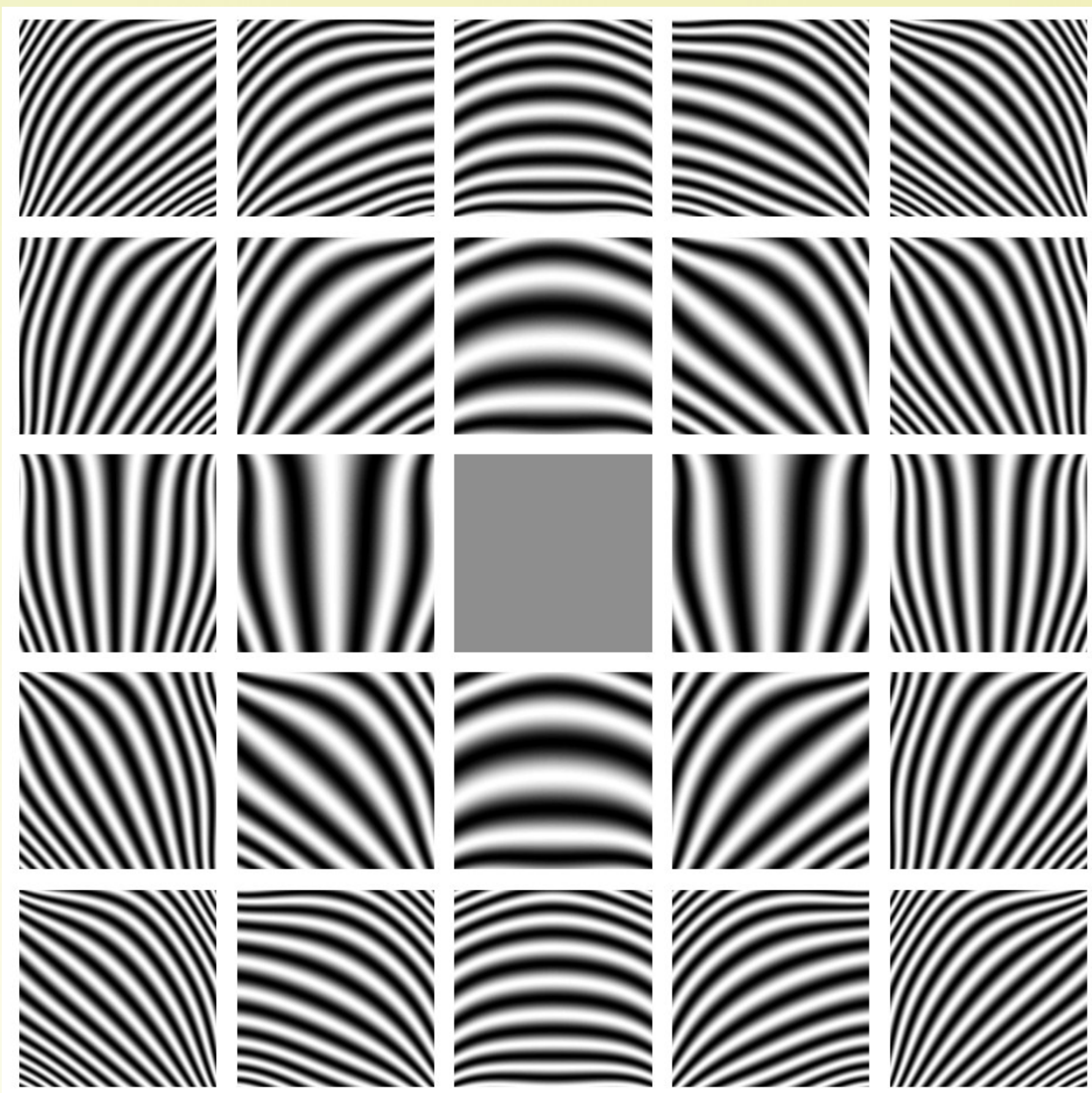
# Synthesis of Spatially Variant Lattices

The EM Lab has developed a unique and powerful tool to spatially vary any attribute of a periodic structure while leaving the overall lattice smooth and continuous. Attributes include unit cell orientation, lattice spacing, fill fraction, material composition, geometry, and more. The tool can be used to functionally grade metamaterials, exploit directional phenomena, develop multi-functional structures, and more.

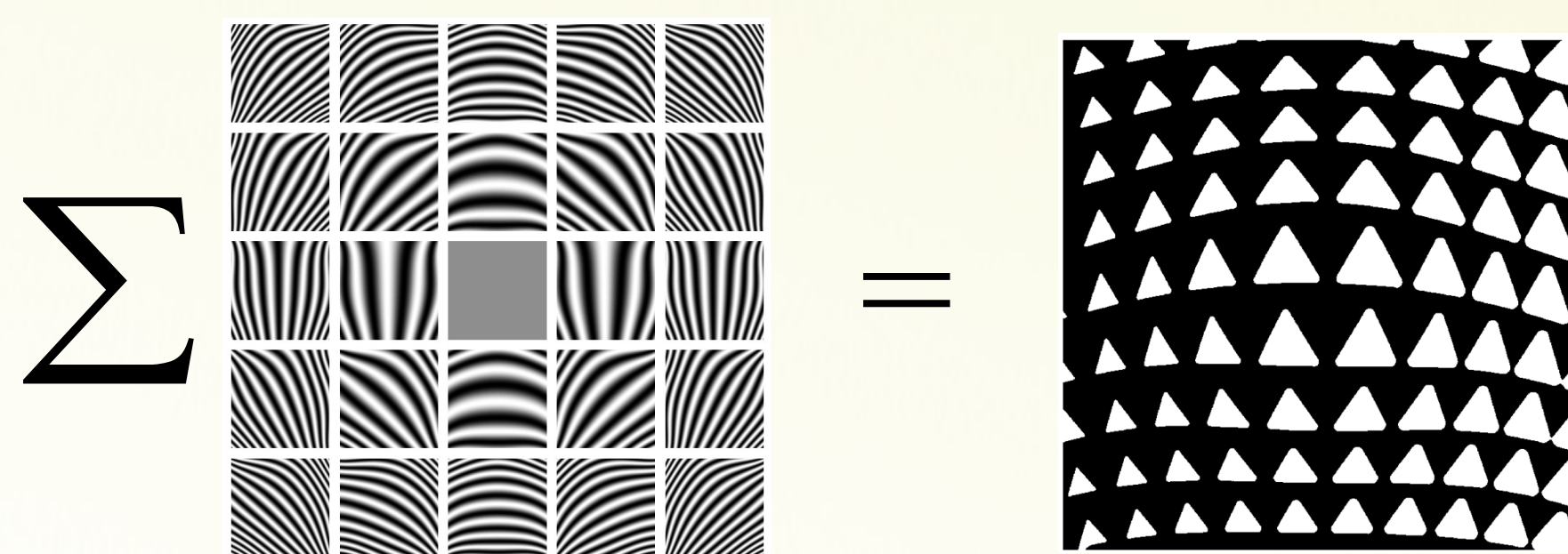
## 1. Decompose Unit Cell into Spatial Harmonics



## 2. Spatially Vary Each Harmonic Separately



## 3. Add Above Results



## INPUT TO SYNTHESIS ALGORITHM

**Orientation Function**

- Map of how unit cells should be oriented throughout the lattice.
- Can be arbitrary.
- Function should be as smooth and continuous as possible.
- Lattice spacing is held remarkably constant despite unit cell being spatially varied.

**Lattice Spacing Function**

- Map of the lattice spacing as a function of position in the lattice.
- Can be arbitrary.
- Function should be as smooth and continuous as possible.

**Fill Fraction Function**

- Map of the fill fraction as a function of position in the lattice.
- Can be arbitrary and abrupt.

**Any Combination**

- All of the above, and more, can be spatially varied at the same time.
- Each attribute of the lattice can be spatially varied in a pattern unique from the rest.
- Multiple functions can be incorporated into a lattice.
- Can be used to compensate for lattice distortions.

## PROCEDURE FOR $m^{\text{th}}$ HARMONIC

$$\nabla \phi_m = \vec{K}_m$$

$$\tilde{\epsilon}_m = a_m e^{j\phi_m}$$

## CONSTRUCTING THE OVERALL LATTICE

Harmonic 1 + Harmonic 2 = Harmonic 1 + 2

$$\tilde{\epsilon}_T = \text{Re} \left[ \sum \tilde{\epsilon}_m \right]$$

The overall lattice is constructed by:

- (1) summing the individual spatially variant 1D lattices, and
- (2) applying a threshold function.

Analog Lattice  $\Rightarrow$   $\epsilon_1$   $\epsilon_2$

$\epsilon_T = \begin{cases} \epsilon_{r1} & \tilde{\epsilon}_T \leq \gamma \\ \epsilon_{r2} & \tilde{\epsilon}_T > \gamma \end{cases}$

