Electromagnetic Material Parameters & Relations

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Fundamental Parameters

These parameters are fundamental to Maxwell's equations, but it is difficult to conclude their effect on fields and waves.

Permittivity ε (F/m)

Ability of a medium to store electric energy.

$\varepsilon \overset{\text{def}}{=}$ permittivity (F/m)
$\varepsilon_0 \overset{\text{def}}{=}$ free space permittivity (F/m)
$\varepsilon_\mathrm{r} \overset{\text{def}}{=}$ relative permittivity (no units)
$\varepsilon_0 = 8.8541878176 \times 10^{-12}$ F/m

$$\varepsilon = \varepsilon_0 \varepsilon_\mathrm{r}$$
$$\varepsilon_\mathrm{r} \geq 1$$

Permeability μ (H/m)

Ability of a medium to store magnetic energy.

$\mu \overset{\text{def}}{=}$ permeability (H/m)
$\mu_0 \overset{\text{def}}{=}$ free space permeability (H/m)
$\mu_\mathrm{r} \overset{\text{def}}{=}$ relative permeability (no units)

$$\mu = \mu_0 \mu_\mathrm{r}$$
$$\mu_\mathrm{r} \geq 1$$

$\mu_0 = 4\pi \times 10^{-7}$ H/m $= 1.2566370614 \times 10^{-6}$ H/m

Electrical Conductivity σ $(1/\Omega \cdot \mathrm{m})$

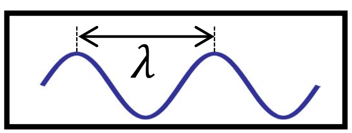
Ability of a medium to conduct electricity.

Two Popular Models for Materials

Model 1	Model 2
Real permittivity ε	Complex permittivity
Real conductivity σ	$\tilde{\varepsilon} = \varepsilon' - j\varepsilon''$

$$\tilde{\varepsilon} = \varepsilon' - j\varepsilon'' = \varepsilon + \sigma/j\omega$$
$$\varepsilon' = \varepsilon \qquad \varepsilon'' = \sigma/\omega$$

Frequency, Velocity, and Wavelength



$f \overset{\text{def}}{=}$ ordinary frequency (Hz)
$\omega \overset{\text{def}}{=}$ angular frequency (rad/sec)
$\lambda_0 \overset{\text{def}}{=}$ free space wavelength (m)
$\lambda \overset{\text{def}}{=}$ wavelength inside medium (m)
$c_0 \overset{\text{def}}{=}$ speed of light in vacuum (m/s)
$v \overset{\text{def}}{=}$ wave velocity (m)
$c_0 = f\lambda_0 = 299{,}792{,}458$ m/s

$$\omega = 2\pi f$$
$$\lambda = \lambda/n_\mathrm{o}$$
$$v = f\lambda$$
$$c_0 = v/n_\mathrm{o}$$
$$v = 1/\sqrt{\mu_\mathrm{r}\varepsilon_\mathrm{r}}$$
$$c_0 = 1/\sqrt{\mu_0\varepsilon_0}$$

Meaningful Parameters

These parameters isolate specific information about fields and waves into single quantities. They are more intuitive and meaningful than the fundamental parameters.

Refractive Index n (no units)

$n \overset{\text{def}}{=}$ refractive index (no units)
$n_\mathrm{o} \overset{\text{def}}{=}$ ordinary refractive index (no units)
$\kappa \overset{\text{def}}{=}$ extinction coefficient (no units)

$$n = n_\mathrm{o} - j\kappa = \sqrt{\mu_\mathrm{r}\varepsilon_\mathrm{r}}$$
$$E(z) = \exp(-jk_0 n z) = \underbrace{\exp(-jk_0 n_\mathrm{o} z)}_{\text{speed/oscillation}} \underbrace{\exp(-j\kappa z)}_{\text{growth/decay}}$$

Best Way to Calculate to Resolve Sign
$n = \sqrt{\mu_\mathrm{r}}\sqrt{\varepsilon_\mathrm{r}}$

Nonmagnetic Materials
$n = \sqrt{\varepsilon_\mathrm{r}} \qquad \eta = \eta_0/n$

$v = c_0/n_\mathrm{o}$

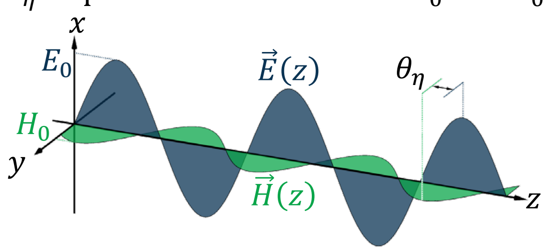
Impedance η (Ω)

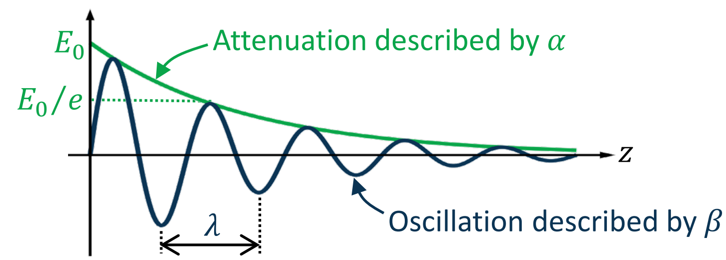
Impedance quantifies the amplitude and phase relationship between $\vec{E}$ and $\vec{H}$ of a wave.

$$\eta = \frac{E_0}{H_0}$$

$\eta \overset{\text{def}}{=}$ impedance (Ω)
$\eta_0 \overset{\text{def}}{=}$ vacuum impedance (Ω)
$\eta' \overset{\text{def}}{=}$ resistive component (Ω)
$\eta'' \overset{\text{def}}{=}$ reactive component (Ω)
$|\eta| \overset{\text{def}}{=}$ amplitude relation between E_0 and H_0
$\theta_\eta \overset{\text{def}}{=}$ phase relation between E_0 and H_0

$$\eta = \eta' + j\eta'' = |\eta|\angle\theta_\eta = \sqrt{\mu/\varepsilon}$$
$$\eta_0 = \sqrt{\mu_0/\varepsilon_0} = 376.73031346177\ \Omega$$





Propagation Constant γ (m^{-1})

$\gamma \overset{\text{def}}{=}$ propagation constant (m^{-1})
$\beta \overset{\text{def}}{=}$ phase constant (m^{-1})
$\alpha \overset{\text{def}}{=}$ attenuation coefficient (m^{-1})

$$\gamma = \alpha + j\beta = jk_0 n$$
$$E(z) = \exp(-\gamma z) = \underbrace{\exp(-j\beta z)}_{\text{speed/oscillation}} \underbrace{\exp(-\alpha z)}_{\text{growth/decay}}$$
$$\beta = k_0 n_\mathrm{o} = 2\pi/\lambda$$
$$\alpha = k_0 \kappa$$

Power Loss Parameters

$\kappa \overset{\text{def}}{=}$ exctinction coefficient (no units)
$\alpha \overset{\text{def}}{=}$ attenuation coefficient (m^{-1})
$\alpha_\mathrm{P} \overset{\text{def}}{=}$ absorption coefficient (m^{-1})
$\tan\delta \overset{\text{def}}{=}$ loss tangent (no units)

$$P(z) = P_0 \exp(-\alpha_\mathrm{P} z) = P_0 \exp(-\delta k_0 n_\mathrm{o} z)$$
$$\alpha_\mathrm{P} = 2\alpha \qquad \delta = \frac{2\kappa}{n_\mathrm{o}} = \frac{2\alpha}{k_0 n_\mathrm{o}}$$