



ELECTROMAGNETIC WAVES

EE 3321 Electromagnetic Field Theory

Pioneering 21st Century
Electromagnetics and Photonics



<http://emlab.utep.edu>

Maxwell's Equations

$$\left. \begin{aligned} \nabla \cdot \vec{D} &= 0 \\ \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{E} &= -j\omega\mu\vec{H} \\ \nabla \times \vec{H} &= j\omega\epsilon\vec{E} \end{aligned} \right\} \begin{aligned} \vec{D}, \vec{E} &\perp \vec{k} \\ \vec{B}, \vec{H} &\perp \vec{k} \end{aligned}$$

Predicts waves

Solution to Wave Equation

$$\nabla^2 E_x + k^2 E_x = 0$$

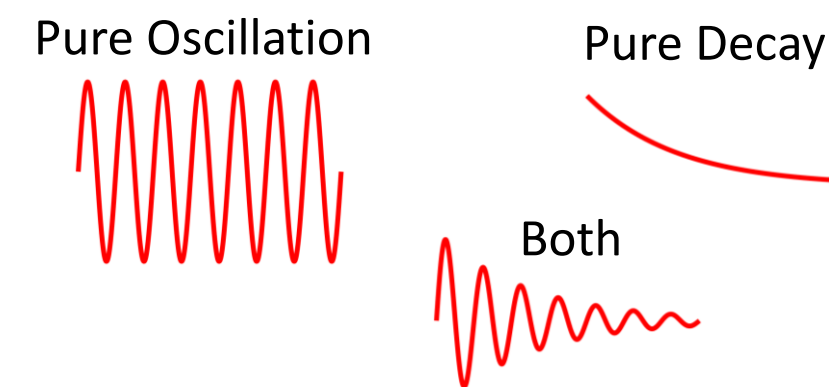
$$\nabla^2 \vec{E} + k^2 \vec{E} = 0 \rightarrow \nabla^2 E_y + k^2 E_y = 0 \rightarrow E_i(z) = A e^{-jkz} + B e^{+jkz}$$

$$\nabla^2 E_z + k^2 E_z = 0$$

Solution is complex exponentials.

Forward Wave Backward Wave

Waves can only do 2.5 things:



Wave Equation

Helmholtz Wave Equation

$$\nabla^2 u + \left(\frac{\omega}{v}\right)^2 u = 0$$

$u \equiv$ disturbance
 $\omega \equiv$ frequency
 $v \equiv$ velocity

Inhomogeneous Media

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \vec{E} \right) = \omega^2 \epsilon \vec{E}$$

Used mostly in numerical analysis.

Homogeneous Media

$$\nabla^2 \vec{E} + \omega^2 \mu \epsilon \vec{E} = 0$$

Used mostly in closed-form analysis.

$$k^2 = \left(\frac{\omega}{v}\right)^2 = \omega^2 \mu \epsilon$$

$k \equiv$ wave number

EM Wave Velocity

$$v = 1/\sqrt{\mu\epsilon}$$

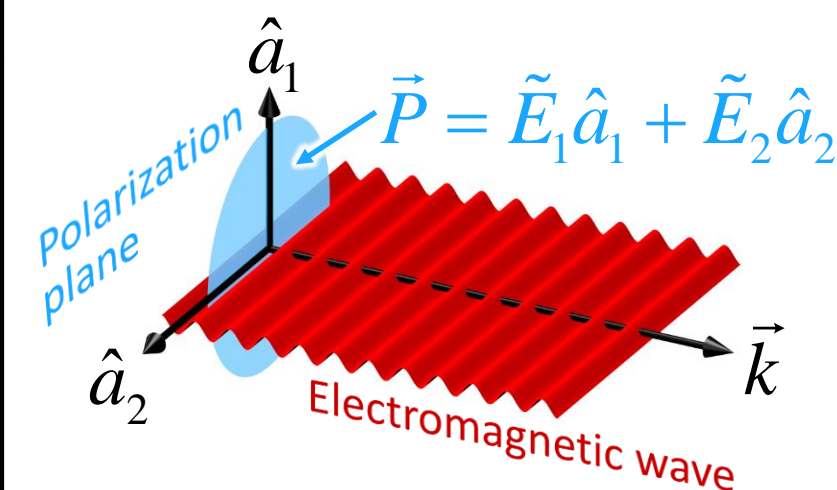
$$c_0 = 1/\sqrt{\mu_r \epsilon_r}$$

$$v = c_0/n$$

$$n = \sqrt{\mu_r \epsilon_r}$$

$c_0 = 299,792,458$ m/s
 $n \equiv$ refractive index

Plane Waves



Time-Domain

$$\vec{E}(t) = \vec{P} \cos(\omega t - \vec{k} \cdot \vec{r})$$

Frequency-Domain

$$\vec{E}(\omega) = \vec{P} \exp(-j\vec{k} \cdot \vec{r})$$

Wave Vector

$$|\vec{k}| = 2\pi/\lambda$$

Conveys wavelength λ inside medium

$$|\vec{k}| = k_0 = 2\pi/\lambda_0$$

In vacuum.

$$|\vec{k}| = k_0 n$$

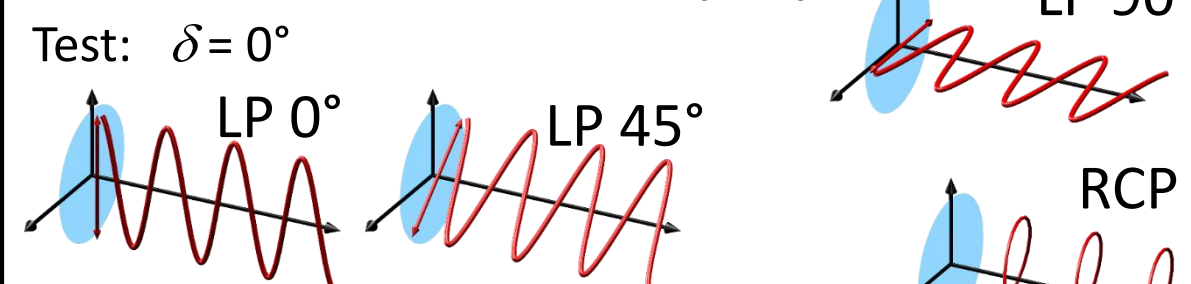
Also conveys refractive index n when frequency is known.

Polarization

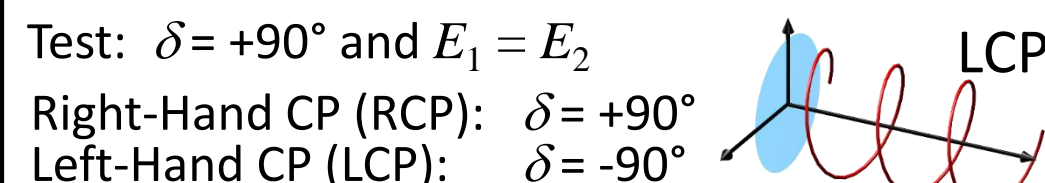
$$\vec{P} = (E_1 \hat{a}_1 + E_2 e^{j\delta} \hat{a}_2) e^{j\theta}$$

Expanded Polarization Vector

Linear Polarization (LP)



Circular Polarization (CP)



Elliptical Polarization (EP)



Properties

Loss Tangent

$$\tan \delta = \epsilon''/\epsilon' \quad P(z) = P_0 e^{-k\delta z}$$

Propagation Constant

$$\gamma = \alpha + j\beta \quad E(z) = E_0 e^{-\gamma z}$$

Attenuation Coefficient

$$\alpha = \sqrt{(k/2) \left[\sqrt{1 + (\sigma/\omega\epsilon)^2} - 1 \right]}$$

Phase Constant

$$\beta = \sqrt{(k/2) \left[\sqrt{1 + (\sigma/\omega\epsilon)^2} + 1 \right]}$$

Absorption Coefficient

$$\alpha_{\text{abs}} = k\delta = 2\alpha \quad P(z) = P_0 e^{-\alpha_{\text{abs}} z}$$

Relation Between E&H

Directionality: $\vec{E} \perp \vec{k} \perp \vec{H}$

Magnetic Field

$$\vec{H}(\omega) = \frac{\vec{k} \times \vec{P}}{\omega\mu} \exp(-j\vec{k} \cdot \vec{r})$$

Impedance

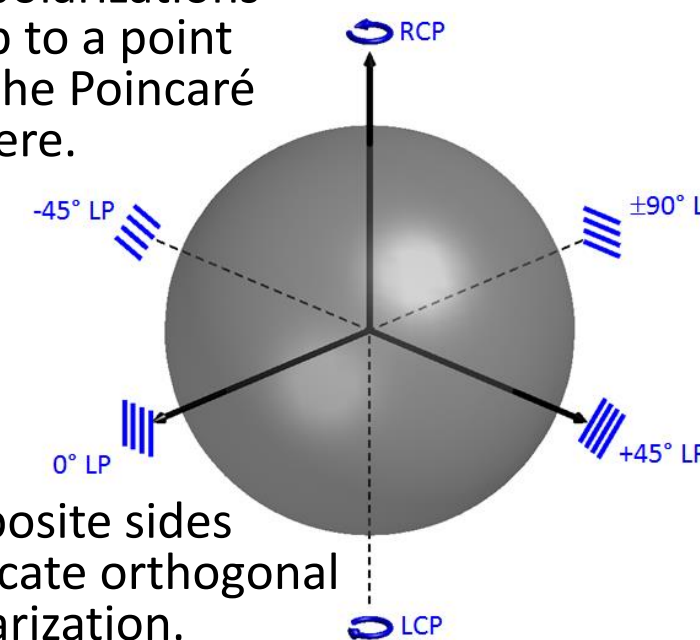
$$\eta = \frac{E_0}{H_0} = \sqrt{\frac{\mu/\epsilon}{1 + \sigma/j\omega\epsilon}}$$

$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\left[1 + (\sigma/\omega\epsilon)^2 \right]^{1/4}}$$

$$\angle \eta = 0.5 \tan^{-1}(\sigma/\omega\epsilon)$$

Poincaré Sphere

All polarizations map to a point on the Poincaré sphere.



Opposite sides indicate orthogonal polarization.