

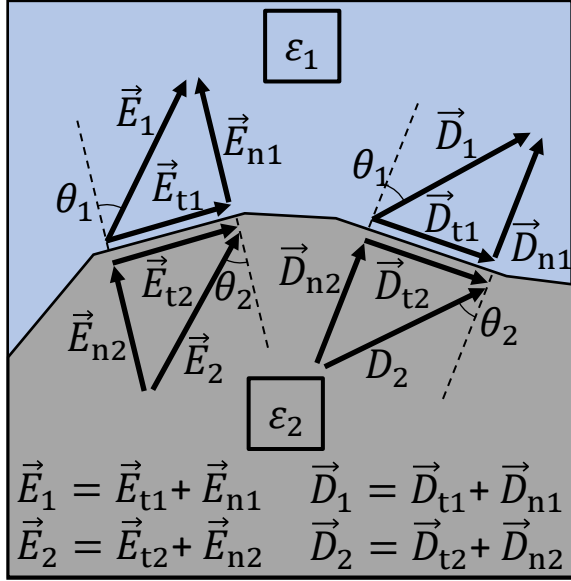
Classes of Materials

- Metals ($\sigma \gg 1$) -- Good conductors
- Semiconductors – Moderate or tunable conduction
- Insulators ($\sigma \ll 1$) -- Poor conductors

Perfect Electric Conductors

- Perfect Electric Conductors (PEC):
- $\vec{E} = 0$ No electric field may exist in a PEC
 - $\nabla V = 0$ Voltage is constant throughout a PEC
- Electric Field at Boundary
- $\vec{D}_t = \vec{E}_t = 0$ No tangential component
 - $\vec{D}_n = \epsilon \vec{E}_n = \rho_s$ Normal component is not zero.

Boundary Conditions



- Tangential Field Components:
- $$\vec{E}_{t1} = \vec{E}_{t2} \quad \frac{\vec{D}_{t1}}{\epsilon_1} = \frac{\vec{D}_{t2}}{\epsilon_2}$$
- Normal Field Components:
- $$\epsilon_1 \vec{E}_{n1} = \epsilon_2 \vec{E}_{n2} \quad \vec{D}_{n1} = \vec{D}_{n2}$$
- Law of Refraction
- $$\frac{\tan \theta_1}{\epsilon_1} = \frac{\tan \theta_2}{\epsilon_2}$$
- Not Snell's Law

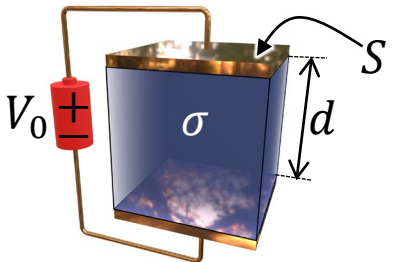
Laplace's Equation

- | | |
|---|--|
| Inhomo. Poisson | Homo. Poisson |
| $\nabla \cdot [\epsilon(\nabla V)] = -\rho_v$ | $\nabla \cdot (\nabla V) = -\rho_v/\epsilon$ |
| Inhomo. Laplace | Homo. Laplace |
| $\nabla \cdot [\epsilon(\nabla V)] = 0$ | $\nabla^2 V = 0$ |

- Solving Laplace's Equation
- Solve $\nabla^2 V = 0$ in each homogeneous region.
 - For 1D, using direct integration.
 - Otherwise, use separation of variables.
 - Apply BCs at edges of regions.
 - Calculate \vec{E} from V using $\vec{E} = -\nabla V$.
 - Calculate \vec{D} from \vec{E} using $\vec{D} = \epsilon \vec{E}$.

Analysis of Resistors

- Choose coordinate system.
- Assume V_0 across terminals.
- Calculate potential by solving $\nabla^2 V = 0$.
- Calculate field using $\vec{E} = -\nabla V$.
- Calculate current using $I = \iint_S \sigma \vec{E} \cdot d\vec{s}$.
- Calculate resistance using $R = V_0/I$.

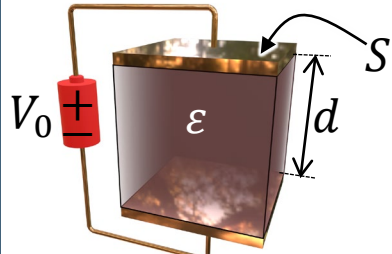


Uniform σ

$$R = \frac{d}{\sigma S}$$

Analysis of Capacitors

- Choose coordinate system.
- Assume plates carry charge $+Q$ and $-Q$.
- Calculate \vec{E} using Gauss' law.
- Calculate V_0 by integrating from plate 1 to plate 2 using $V_0 = -\int_L \vec{E} \cdot d\vec{\ell}$.
- Calculate C using $C = Q/V_0$.



Uniform ϵ : $C = \epsilon S/d$

