ELECTROSTATIC MATERIALS & DEVICES

EE 3321 Electromagnetic Field Theory

Pioneering 21[st] Century Electromagnetics and Photonics
http://emlab.utep.edu

Classes of Materials

Metals – good conductors ($\sigma >> 1$).
Semiconductors – moderate or tunable conduction.
Insulators – poor conductors ($\sigma << 1$).

Inside Perfect Conductors:

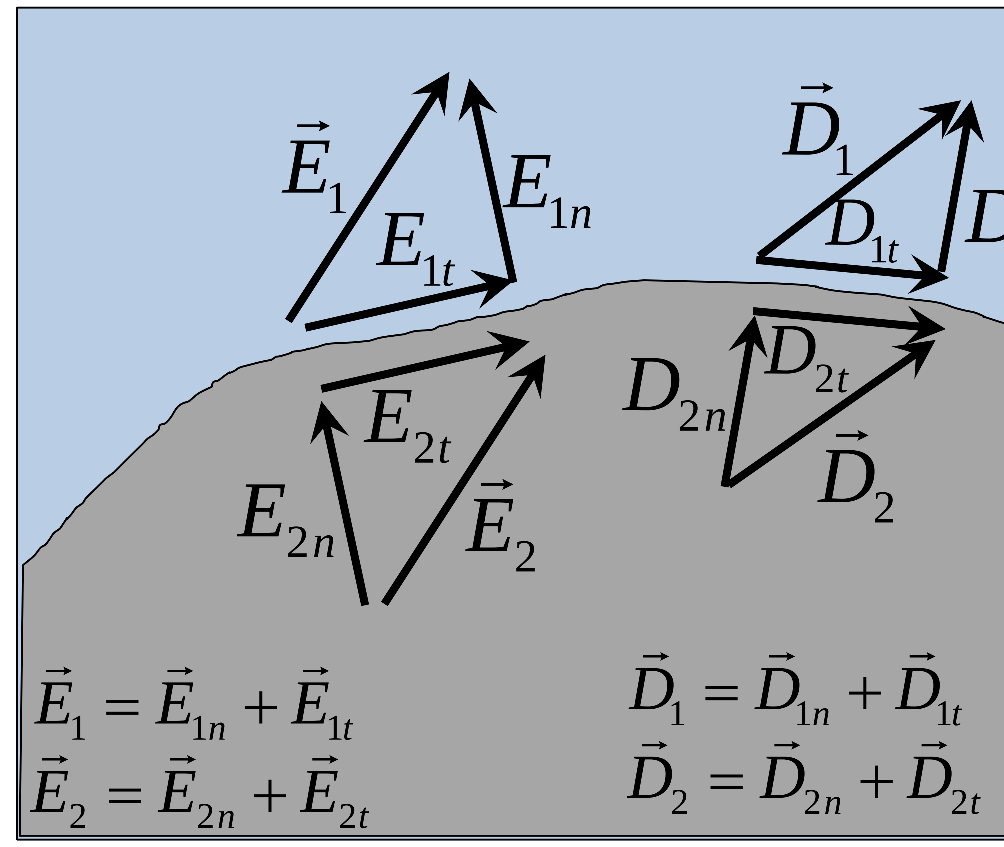
$\vec{E} = 0$ No electric field may exist within a perfect conductor.

$\nabla V = 0$ Voltage is constant throughout a conductor.

Electric field at the boundary of a metal has no tangential component.

$$D_t = E_t = 0 \qquad D_n = \varepsilon E_n = \rho_s$$

Boundary Conditions



Tangential Components:

$$E_{1t} = E_{2t} \qquad \frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}$$

Normal Components:

$$\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n} \qquad D_{1n} = D_{2n}$$

Refraction:

$$\frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}$$

Not Snell's Law

Solving Laplace

1. Solve $\nabla^2 V=0$ in each homo. region.
   a) For 1D problems, use direct integration.
   b) Otherwise, use separation of variables.
2. Apply BC's at edges of homo. regions.
3. Calculate E from V using $V = -\nabla V$.
4. Calculate D from E using $D=\varepsilon E$.

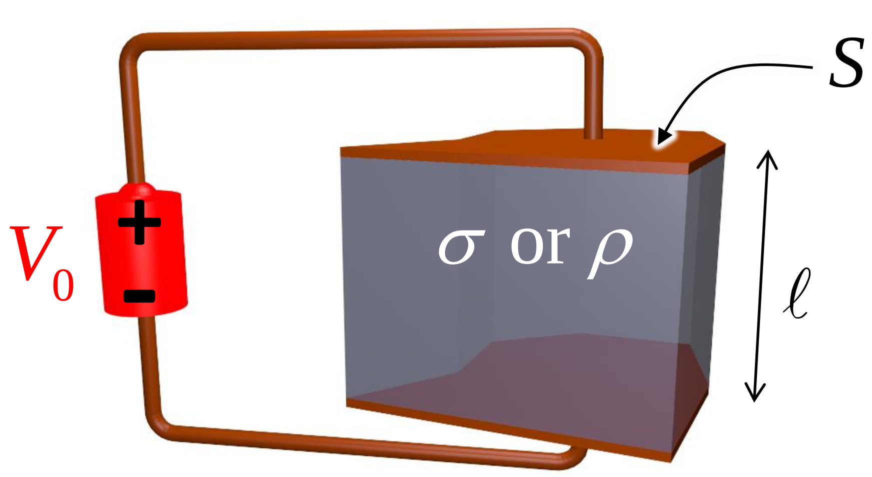
Governing Equations

Uniqueness Thm.: There is only one solution.

Inhomog. Poisson	Homog. Poisson	Inhomog. Laplace	Homog. Laplace
$\nabla \bullet \left[\varepsilon\left(\nabla V\right)\right] = -\rho_v$	$\nabla \bullet \left(\nabla V\right) = -\rho_v/\varepsilon$	$\nabla \bullet \left[\varepsilon\left(\nabla V\right)\right] = 0$	$\nabla^2 V = 0$

Analysis of Resistors

1. Choose suitable coordinate system.
2. Assume V_0 is potential across terminals.
3. Calculate V by solving $\nabla^2 V = 0$.
4. Calculate E using $E = -\nabla V$.
5. Calculate I using $I = \iint_S \sigma \vec{E} \bullet d\vec{s}$
6. Calculate R using $R = V_0/I$.



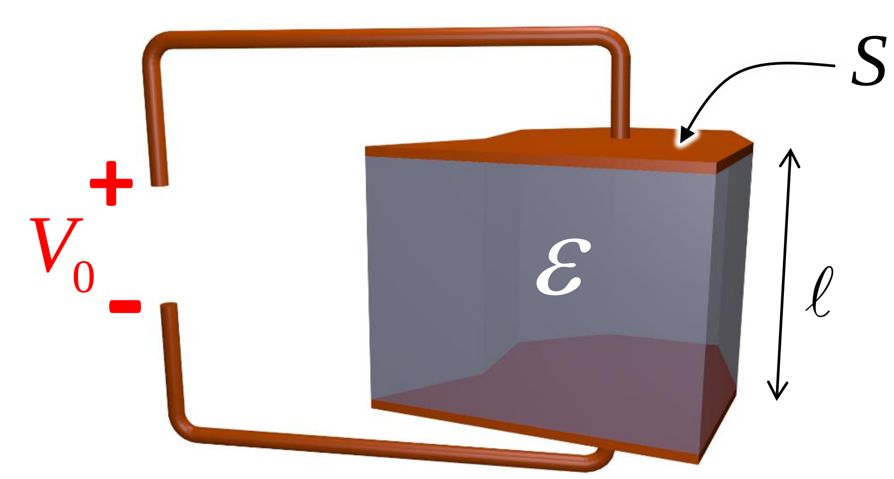
$$R = \frac{\ell}{\sigma S} = \frac{\rho \ell}{S}$$

Analysis of Capacitors

Two Methods for Analysis

$$C = \frac{Q}{V_0}$$

1. Assume Q and calculate V_0 using Coulomb's law or Gauss' law.
2. Assume V_0 and calculate Q using Laplace's equation.



$$C = \frac{\varepsilon S}{\ell}$$

Method 1

1. Choose suitable coordinate system.
2. Assume the plates carry charges $+Q$ and $-Q$.
3. Calculate E using Coulomb's law or Gauss' law.
4. Calculate V_0 by integrating from plate 1 to plate 2 using $V_0 = -\int_L \vec{E} \bullet d\vec{\ell}$
5. Calculate C using $C = Q/V_0$.

Series/Parallel Combinations