Magnetostatic Materials & Devices

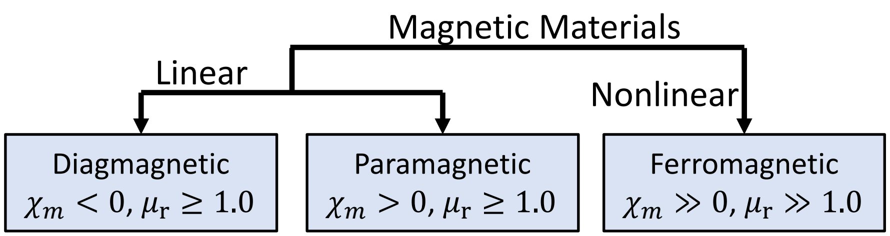
EMPossible
https://empossible.net/

Classes of Materials

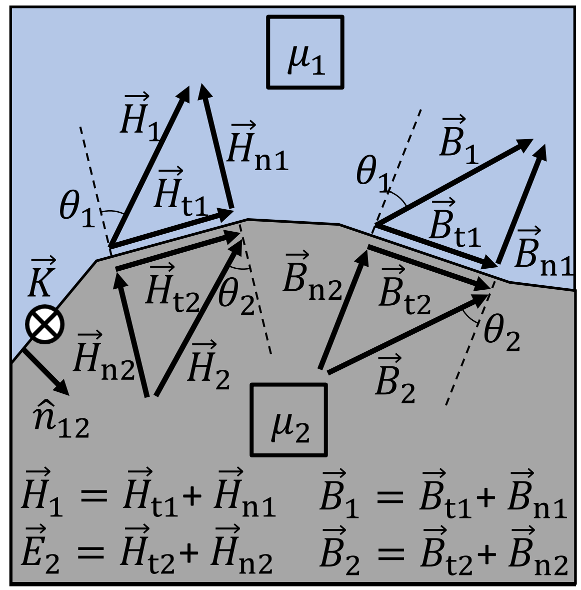
Diamagnetic – No permanent magnetic moment. Very weak response to a magnetic field. Dipoles align to oppose applied field.

Paramagnetic – Weak magnetic response to applied field, but not permanent.

Ferromagnetic – Large permanent magnetic moments. Strongly magnetized by an applied field.



Boundary Conditions



Tangential Field Components:

$$\vec{H}_{t1} = \vec{H}_{t2} \qquad \frac{\vec{B}_{t1}}{\mu_1} = \frac{\vec{B}_{t2}}{\mu_2}$$

Normal Field Components:

$$\left(\vec{H}_1 - \vec{H}_2\right) \times \hat{n}_{12} = \vec{K}$$

$$\mu_1 \vec{H}_{n1} = \mu_2 \vec{H}_{n2} \qquad \vec{B}_{n1} = \vec{B}_{n2}$$

Law of Refraction

$$\frac{\tan\theta_1}{\mu_1} = \frac{\tan\theta_2}{\mu_2}$$

Not Snell's Law

Total Magnetic Energy

$$W_m = \frac{1}{2}\iiint_V \left(\vec{B} \times \vec{H}\right) dv \qquad \text{General Case}$$

$$W_m = \frac{1}{2}\iiint_V \mu \left|\vec{H}\right|^2 dv \qquad \text{LHI Media}$$

Inductance L

An inductor is a device that can store and discharge magnetic energy. It generates potential so as to oppose a change in current. They can generate very high voltages to do this!

$$L = \frac{\lambda}{I} = \frac{N\psi}{I} = \frac{2W_m}{I^2}$$

Flux Linkage λ

Like flux, but flux linkage accounts for multiple loops.

$$\lambda = N\psi = LI = 2W_m/I$$

Stored Magnetic Energy W_m

$$W_m = \frac{1}{2}LI^2$$

Analysis of Inductors

1. Choose coordinate system.
2. Assume inductor carries current I_0.
3. Calculate magnetic field intensity $\vec{H}$.
   a) If symmetry exists, use Ampere's circuit law: $I = \int_L \vec{H} \cdot d\vec{\ell}$
   b) Otherwise, use Biot-Savart law
4. Calculate $\vec{B}$ from $\vec{H}$ using $\vec{B} = \mu\vec{H}$.
5. Calculate flux using $\psi = \iint_S \vec{B} \cdot d\vec{s}$
6. Calculate inductance L as

$$L = \frac{N\psi}{I}$$

Line Current

$$\vec{H} = \int_L \frac{Id\vec{\ell} \times \hat{a}_R}{4\pi R^2}$$

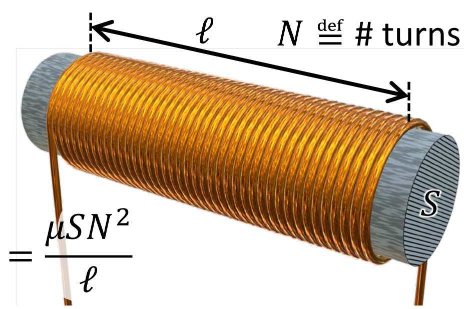
Surface Current

$$\vec{H} = \iint_S \frac{\vec{K}ds \times \hat{a}_R}{4\pi R^2}$$

Volume Current

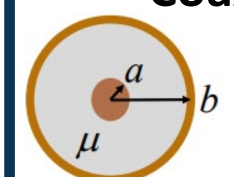
$$\vec{H} = \iint_S \frac{\vec{K}ds \times \hat{a}_R}{4\pi R^2}$$

The Solenoid



$$L = \frac{\mu S N^2}{\ell}$$

Coaxial Line



$$L\ (\text{H}) = \frac{\mu\ell}{2\pi}\left[\frac{1}{4} + \ln\left(\frac{b}{a}\right)\right]$$