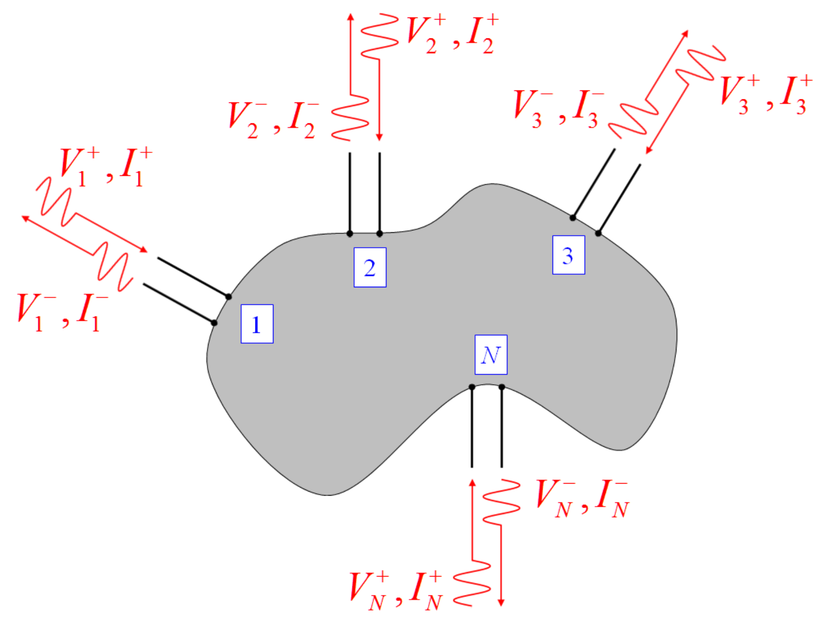
NETWORK MATRICES

EE 4380/5390 Microwave Engineering

Pioneering 21st Century Electromagnetics and Photonics

http://emlab.utep.edu/

N-Port Networks



Total Voltage and Current

$V_n = V_n^+ + V_n^-$ $I_n = I_n^+ - I_n^-$

V_n^+ & $I_n^+ \equiv$ forward waves

V_n^- & $I_n^- \equiv$ backward waves

Terms

Impedance

$Z = R + jX$ $R \equiv$ resistance, $X \equiv$ reactance

$Z(-\omega) = Z^*(\omega)$ Hermitian

$R(-\omega) = R(-\omega)$ even

$X(-\omega) = -X(\omega)$ odd

$|\Gamma(-\omega)| = |\Gamma(-\omega)|$ even

Admittance

$Y = G + jB = Z^{-1}$ $G \equiv$ conductance, $B \equiv$ susceptance

Foster's Reactance Thm.

$\frac{\partial X}{\partial \omega} > 0$ and $\frac{\partial B}{\partial \omega} > 0$

ABCD Matrix $[T]$

Definition

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$V_1 = AV_2 + BI_2$

$I_1 = CV_2 + DI_2$

$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ open circuit voltage ratio

$B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$ short circuit transfer impedance

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$ open circuit transfer impedance

$D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$ short circuit current ratio

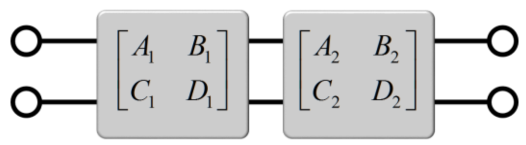
Reciprocal

$\det[T] = 0$

Lossless

$\text{Im}(A) = \text{Re}(B) = \text{Re}(C) = \text{Im}(D) = 0$

Cascading



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

Scattering Matrix $[S]$

Definition

$$\begin{bmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ \vdots & \vdots & \ddots & \\ S_{N1} & S_{N2} & & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{bmatrix}$$

$S_{ij} = \left.\frac{V_i^-}{V_j^+}\right|_{\text{no other applied voltages}}$

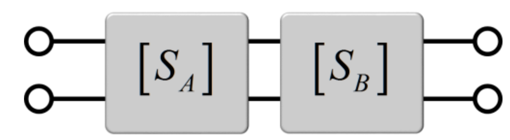
Reciprocal

$[S] = [S]^T$

Lossless

$[S]$ is unitary

Cascading



$[S_{AB}] = [S_A] \otimes [S_B]$

$$[S_{AB}] = \frac{1}{1 - S_{22}^A S_{11}^B} \begin{bmatrix} S_{11}^A - S_{11}^B \det[S_A] & S_{12}^A S_{12}^B \\ S_{21}^A S_{21}^B & S_{22}^B - S_{22}^A \det[S_B] \end{bmatrix}$$

Impedance Matrix $[Z]$

Definition

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \ddots & \\ Z_{N1} & Z_{N2} & & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

$Z_{ij} = \left.\frac{V_i}{I_j}\right|_{\text{all other currents zero}}$

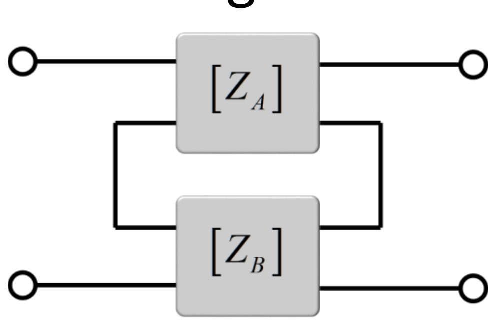
Reciprocal

$[Z] = [Z]^T$

Lossless

$\text{Re}\{Z_{ij}\} = 0$

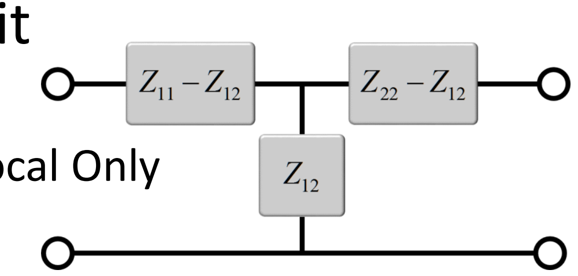
Cascading



$[Z_{AB}] = [Z_A] + [Z_B]$

Matrix → Circuit

$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$ Reciprocal Only



Admittance Matrix $[Y]$

Definition

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & \cdots & Y_{2N} \\ \vdots & \vdots & \ddots & \\ Y_{N1} & Y_{N2} & & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix}$$

$Y_{ij} = \left.\frac{I_i}{V_j}\right|_{\text{all other voltages zero}}$

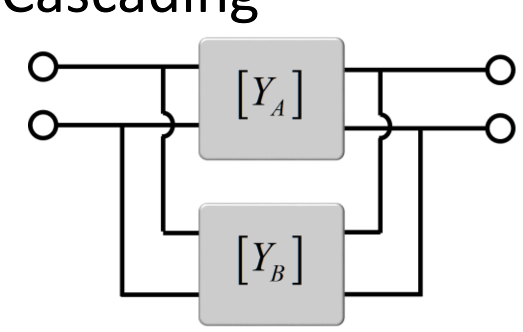
Reciprocal

$[Y] = [Y]^T$

Lossless

$\text{Re}\{Y_{ij}\} = 0$

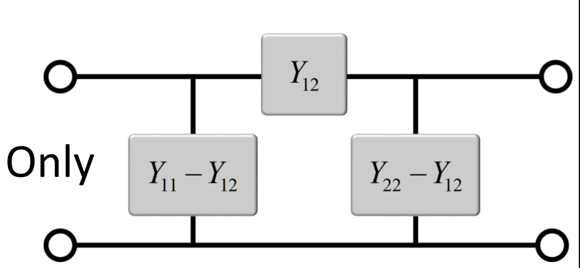
Cascading



$[Y_{AB}] = [Y_A] + [Y_B]$

Matrix → Circuit

$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$ Reciprocal Only



Performance Metrics Through Network

Input Impedance

$$Z_{\text{in}} = \frac{AZ_L + B}{CZ_L + D}$$

Conjugate Matching

$Z_L = Z_s^*$

$R_L = R_s$ $X_L = -X_s$

Max Available Power

$$P_A = \frac{|V_s|^2}{8R_s} \quad \text{when } Z_L = Z_s^*$$

Input Reflection

$$\Gamma_{\text{in}} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

Average Power to Load

$$P_L = \frac{|V_s|^2}{8Z_0} \frac{|S_{21}|^2 \left(1 - |\Gamma_L|^2\right) |1 - \Gamma_s|^2}{|1 - S_{22}\Gamma_L|^2 |1 - \Gamma_s \Gamma_{\text{in}}|^2}$$

Insertion Loss

$$L_I = 10\log_{10} \left| \frac{AZ_L + B + CZ_s Z_L + DZ_s}{Z_s + Z_L} \right|^2$$

Insertion Phase

$$\phi_I = \tan^{-1} \left(\frac{\text{Im}[AZ_L + B + CZ_s Z_L + DZ_s]}{\text{Re}[AZ_L + B + CZ_s Z_L + DZ_s]} \right)$$

Transducer Loss

$$L_T = 10\log_{10} \left(\frac{|AZ_L + B + CZ_s Z_L + DZ_s|^2}{4R_s R_L} \right)$$

Gain through Network

$$G = \frac{P_L}{P_{\text{in}}} = \frac{|S_{21}|^2 \left(1 - |\Gamma_L|^2\right)}{|1 - S_{22}\Gamma_L|^2 \left(1 - |\Gamma_{\text{in}}|^2\right)}$$

Mismatch Loss

$$L_M = 10\log_{10} \left(\frac{1}{1 - |\Gamma_{\text{in}}|^2} \right)$$

Loss Term Relation

$L_T = L_M + L_\sigma$

$L_\sigma \equiv$ dissipative loss