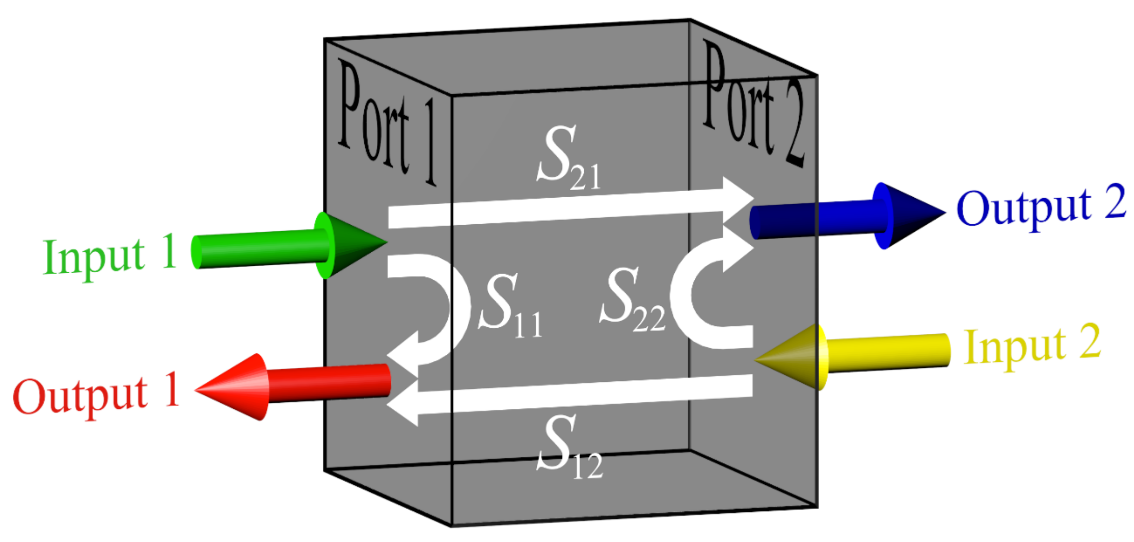
SCATTERING MATRICES FOR CEM

EE 5337 Computational Electromagnetics

Pioneering 21st Century Electromagnetics and Photonics

http://emlab.utep.edu

S-MATRIX DEFINITION



$S_{11} \equiv$ reflection coefficient from Port 1

$S_{12} \equiv$ transmission coefficient from Port 2 to Port 1

$S_{21} \equiv$ transmission coefficient from Port 1 to Port 2

$S_{22} \equiv$ reflection coefficient from Port 2

LAYER PARAMETERS

Eigen-Value Problem

$$\frac{d^2\boldsymbol{\psi}}{dz'^2} - \boldsymbol{\Omega}^2\boldsymbol{\psi} = \mathbf{0} \qquad \boldsymbol{\Omega}^2 = \mathbf{PQ}$$

$\mathbf{W} \equiv$ Eigen-vector matrix of $\boldsymbol{\Omega}^2$

$\boldsymbol{\lambda}^2 \equiv$ Eigen-value matrix of $\boldsymbol{\Omega}^2$

Field Solution $\quad \mathbf{V} = \mathbf{QW}\boldsymbol{\lambda}^{-1}$

$$\boldsymbol{\psi}(z') = \begin{bmatrix} \mathbf{W} & \mathbf{W} \\ \mathbf{V} & -\mathbf{V} \end{bmatrix} \begin{bmatrix} e^{\boldsymbol{\lambda} z'} & \mathbf{0} \\ \mathbf{0} & e^{-\boldsymbol{\lambda} z'} \end{bmatrix} \begin{bmatrix} \mathbf{c}^+ \\ \mathbf{c}^- \end{bmatrix}$$

P and Q Matrices

	$\mathbf{P} =$	$\mathbf{Q} =$
TMM	$\frac{1}{\varepsilon_r}\begin{bmatrix} \tilde{k}_x\tilde{k}_y & \mu_r\varepsilon_r - \tilde{k}_x^2 \\ \tilde{k}_y^2 - \mu_r\varepsilon_r & -\tilde{k}_x\tilde{k}_y \end{bmatrix}$	$\frac{1}{\mu_r}\begin{bmatrix} \tilde{k}_x\tilde{k}_y & \mu_r\varepsilon_r - \tilde{k}_x^2 \\ \tilde{k}_y^2 - \mu_r\varepsilon_r & -\tilde{k}_x\tilde{k}_y \end{bmatrix}$
RCWA	$\begin{bmatrix} \tilde{\mathbf{K}}_x \boldsymbol{\varepsilon}_r^{-1}\tilde{\mathbf{K}}_y & \boldsymbol{\mu}_r - \tilde{\mathbf{K}}_x \boldsymbol{\varepsilon}_r^{-1}\tilde{\mathbf{K}}_x \\ \tilde{\mathbf{K}}_y \boldsymbol{\varepsilon}_r^{-1}\tilde{\mathbf{K}}_y - \boldsymbol{\mu}_r & -\tilde{\mathbf{K}}_y \boldsymbol{\varepsilon}_r^{-1}\tilde{\mathbf{K}}_x \end{bmatrix}$	$\begin{bmatrix} \tilde{\mathbf{K}}_x \boldsymbol{\mu}_r^{-1}\tilde{\mathbf{K}}_y & \boldsymbol{\varepsilon}_r - \tilde{\mathbf{K}}_x \boldsymbol{\mu}_r^{-1}\tilde{\mathbf{K}}_x \\ \tilde{\mathbf{K}}_y \boldsymbol{\mu}_r^{-1}\tilde{\mathbf{K}}_y - \boldsymbol{\varepsilon}_r & -\tilde{\mathbf{K}}_y \boldsymbol{\mu}_r^{-1}\tilde{\mathbf{K}}_x \end{bmatrix}$
MoL	$\begin{bmatrix} -\mathbf{D}^e_{x'}\boldsymbol{\varepsilon}_{zz}^{-1}\mathbf{D}^h_{y'} & \left(\boldsymbol{\mu}_{yy} + \mathbf{D}^e_{x'}\boldsymbol{\varepsilon}_{zz}^{-1}\mathbf{D}^h_{x'}\right) \\ -\left(\boldsymbol{\mu}_{xx} + \mathbf{D}^e_{y'}\boldsymbol{\varepsilon}_{zz}^{-1}\mathbf{D}^h_{y'}\right) & \mathbf{D}^e_{y'}\boldsymbol{\varepsilon}_{zz}^{-1}\mathbf{D}^h_{x'} \end{bmatrix}$	$\mathbf{Q} = \begin{bmatrix} -\mathbf{D}^h_{x'}\boldsymbol{\mu}_{zz}^{-1}\mathbf{D}^e_{y'} & \left(\boldsymbol{\varepsilon}_{yy} + \mathbf{D}^h_{x'}\boldsymbol{\mu}_{zz}^{-1}\mathbf{D}^e_{x'}\right) \\ -\left(\boldsymbol{\varepsilon}_{xx} + \mathbf{D}^h_{y'}\boldsymbol{\mu}_{zz}^{-1}\mathbf{D}^e_{y'}\right) & \mathbf{D}^h_{y'}\boldsymbol{\mu}_{zz}^{-1}\mathbf{D}^e_{x'} \end{bmatrix}$

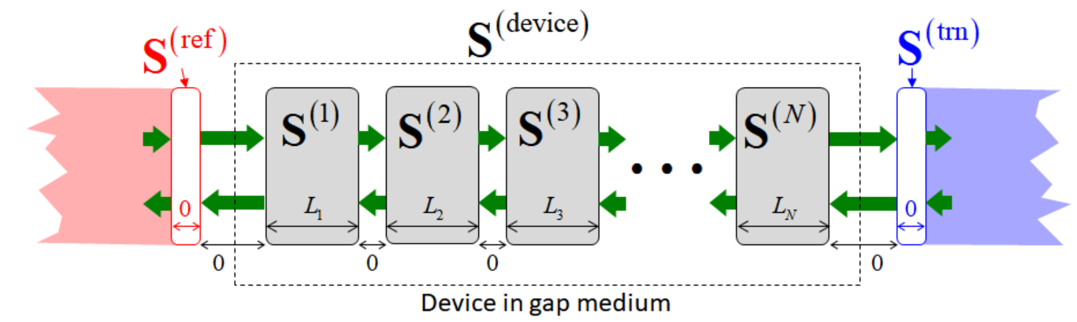
S-MATRIX ALGORITHM

$$\mathbf{S}_{11}^{(AB)} = \mathbf{S}_{11}^{(A)} + \mathbf{S}_{12}^{(A)}\left[\mathbf{I} - \mathbf{S}_{11}^{(B)}\mathbf{S}_{22}^{(A)}\right]^{-1}\mathbf{S}_{11}^{(B)}\mathbf{S}_{21}^{(A)} \qquad \mathbf{S}_{12}^{(AB)} = \mathbf{S}_{12}^{(A)}\left[\mathbf{I} - \mathbf{S}_{11}^{(B)}\mathbf{S}_{22}^{(A)}\right]^{-1}\mathbf{S}_{12}^{(B)}$$

$$\mathbf{S}_{22}^{(AB)} = \mathbf{S}_{22}^{(B)} + \mathbf{S}_{21}^{(B)}\left[\mathbf{I} - \mathbf{S}_{22}^{(A)}\mathbf{S}_{11}^{(B)}\right]^{-1}\mathbf{S}_{22}^{(A)}\mathbf{S}_{12}^{(B)} \qquad \mathbf{S}_{21}^{(AB)} = \mathbf{S}_{21}^{(B)}\left[\mathbf{I} - \mathbf{S}_{22}^{(A)}\mathbf{S}_{11}^{(B)}\right]^{-1}\mathbf{S}_{21}^{(A)}$$

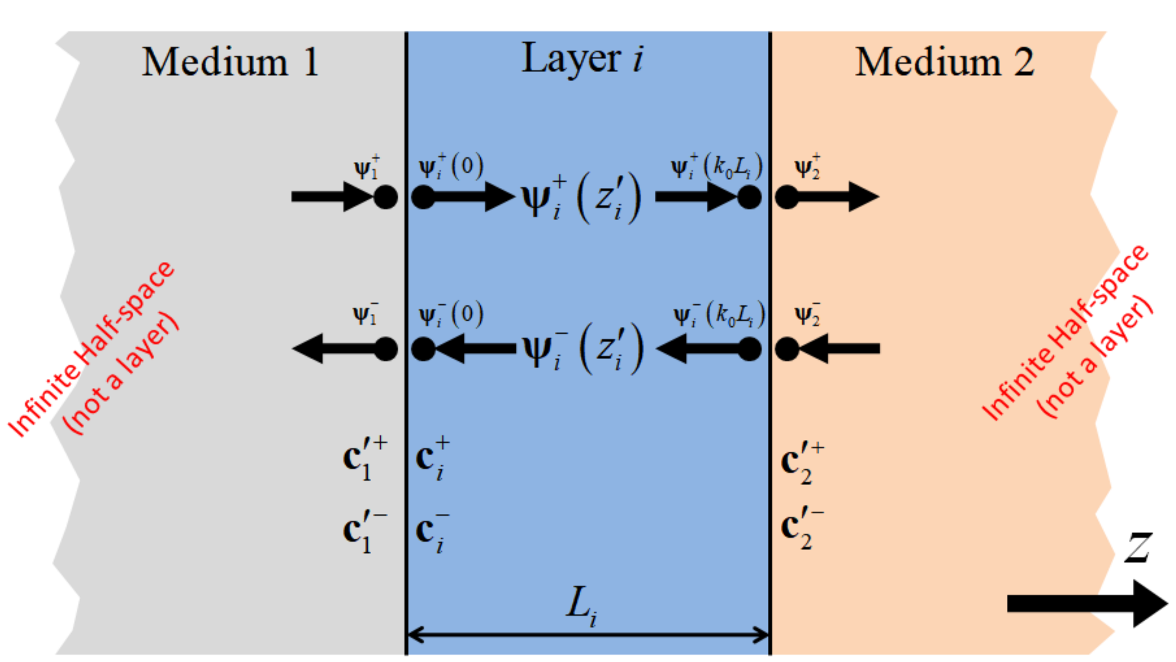
Redheffer star product

$$\mathbf{S}^{(AB)} = \mathbf{S}^{(A)} \otimes \mathbf{S}^{(B)}$$



$$\mathbf{S}^{(\text{global})} = \mathbf{S}^{(\text{ref})} \otimes \underbrace{\left[\mathbf{S}^{(1)} \otimes \mathbf{S}^{(2)} \otimes \cdots \otimes \mathbf{S}^{(N)}\right]}_{\mathbf{S}^{(\text{device})}} \otimes \mathbf{S}^{(\text{trn})}$$

GENERAL S-MATRIX



$\boldsymbol{\psi}_i^{\pm}(z) \equiv$ field within i^{th} layer

$\mathbf{c}_i^{\pm} \equiv$ mode coefficients inside i^{th} layer

$\mathbf{c}_i'^{\pm} \equiv$ mode coefficients outside i^{th} layer

$$\mathbf{S}_{11}^{(i)} = \left(\mathbf{A}_{i1} - \mathbf{X}_i\mathbf{B}_{i2}\mathbf{A}_{i2}^{-1}\mathbf{X}_i\mathbf{B}_{i1}\right)^{-1}\left(\mathbf{X}_i\mathbf{B}_{i2}\mathbf{A}_{i2}^{-1}\mathbf{X}_i\mathbf{A}_{i1} - \mathbf{B}_{i1}\right)$$

$$\mathbf{S}_{12}^{(i)} = \left(\mathbf{A}_{i1} - \mathbf{X}_i\mathbf{B}_{i2}\mathbf{A}_{i2}^{-1}\mathbf{X}_i\mathbf{B}_{i1}\right)^{-1}\mathbf{X}_i\left(\mathbf{A}_{i2} - \mathbf{B}_{i2}\mathbf{A}_{i2}^{-1}\mathbf{B}_{i2}\right)$$

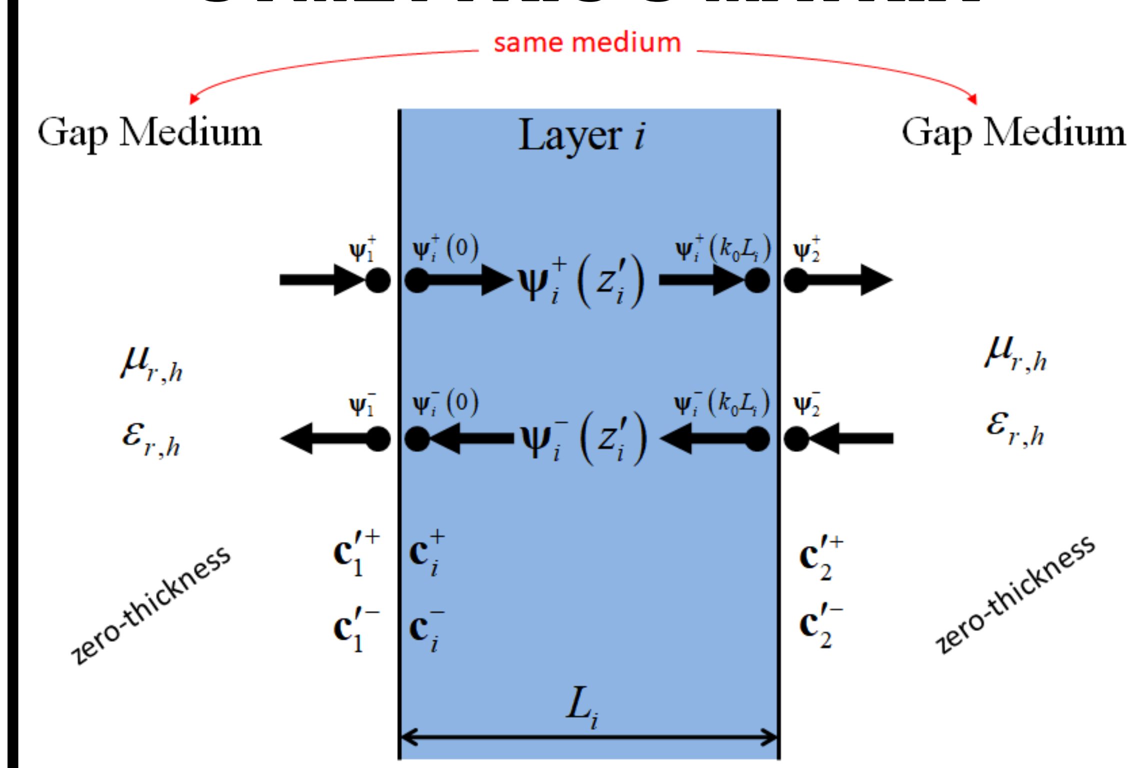
$$\mathbf{S}_{21}^{(i)} = \left(\mathbf{A}_{i2} - \mathbf{X}_i\mathbf{B}_{i1}\mathbf{A}_{i1}^{-1}\mathbf{X}_i\mathbf{B}_{i2}\right)^{-1}\mathbf{X}_i\left(\mathbf{A}_{i1} - \mathbf{B}_{i1}\mathbf{A}_{i1}^{-1}\mathbf{B}_{i1}\right)$$

$$\mathbf{S}_{22}^{(i)} = \left(\mathbf{A}_{i2} - \mathbf{X}_i\mathbf{B}_{i1}\mathbf{A}_{i1}^{-1}\mathbf{X}_i\mathbf{B}_{i2}\right)^{-1}\left(\mathbf{X}_i\mathbf{B}_{i1}\mathbf{A}_{i1}^{-1}\mathbf{X}_i\mathbf{A}_{i2} - \mathbf{B}_{i2}\right)$$

$$\mathbf{A}_{ij} = \mathbf{W}_i^{-1}\mathbf{W}_j + \mathbf{V}_i^{-1}\mathbf{V}_j \qquad \mathbf{X}_i = e^{\boldsymbol{\lambda}_i k_0 L_i}$$

$$\mathbf{B}_{ij} = \mathbf{W}_i^{-1}\mathbf{W}_j - \mathbf{V}_i^{-1}\mathbf{V}_j$$

SYMETTRIC S-MATRIX



$$\mathbf{S}_{11}^{(i)} = \left(\mathbf{A}_i - \mathbf{X}_i\mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{X}_i\mathbf{B}_i\right)^{-1}\left(\mathbf{X}_i\mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{X}_i\mathbf{A}_i - \mathbf{B}_i\right)$$

$$\mathbf{S}_{12}^{(i)} = \left(\mathbf{A}_i - \mathbf{X}_i\mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{X}_i\mathbf{B}_i\right)^{-1}\mathbf{X}_i\left(\mathbf{A}_i - \mathbf{B}_i\mathbf{A}_i^{-1}\mathbf{B}_i\right)$$

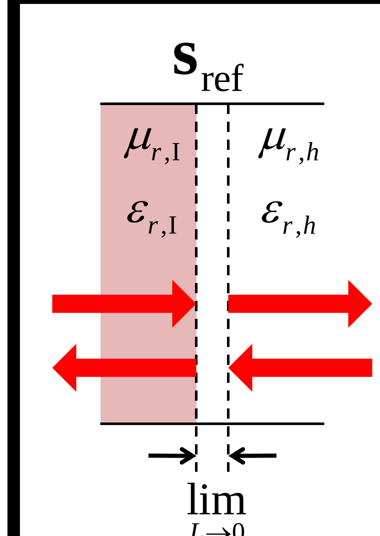
$$\mathbf{S}_{21}^{(i)} = \mathbf{S}_{12}^{(i)} \qquad \mathbf{S}_{22}^{(i)} = \mathbf{S}_{11}^{(i)}$$

- Layers are symmetric so the scattering matrix elements have redundancy.
- Scattering matrix equations are simplified.
- Fewer calculations.
- Less memory storage.

$$\mathbf{A}_i = \mathbf{W}_i^{-1}\mathbf{W}_h + \mathbf{V}_i^{-1}\mathbf{V}_h \qquad \mathbf{X}_i = e^{\boldsymbol{\lambda}_i k_0 L_i}$$

$$\mathbf{B}_i = \mathbf{W}_i^{-1}\mathbf{W}_h - \mathbf{V}_i^{-1}\mathbf{V}_h$$

REFLECTION SIDE S-MATRIX



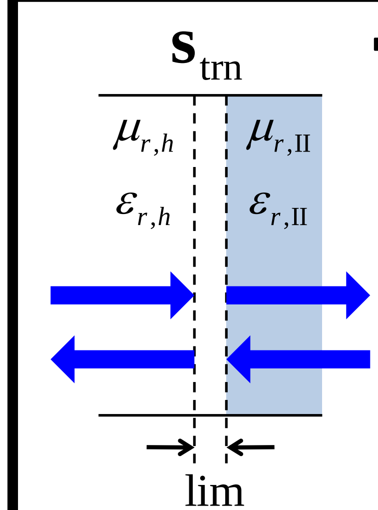
$$\mathbf{S}_{11}^{(\text{ref})} = -\mathbf{A}_{\text{ref}}^{-1}\mathbf{B}_{\text{ref}} \qquad \mathbf{A}_{\text{ref}} = \mathbf{W}_h^{-1}\mathbf{W}_{\text{ref}} + \mathbf{V}_h^{-1}\mathbf{V}_{\text{ref}}$$

$$\mathbf{S}_{12}^{(\text{ref})} = 2\mathbf{A}_{\text{ref}}^{-1} \qquad \mathbf{B}_{\text{ref}} = \mathbf{W}_h^{-1}\mathbf{W}_{\text{ref}} - \mathbf{V}_h^{-1}\mathbf{V}_{\text{ref}}$$

$$\mathbf{S}_{21}^{(\text{ref})} = 0.5\left(\mathbf{A}_{\text{ref}} - \mathbf{B}_{\text{ref}}\mathbf{A}_{\text{ref}}^{-1}\mathbf{B}_{\text{ref}}\right)$$

$$\mathbf{S}_{22}^{(\text{ref})} = \mathbf{B}_{\text{ref}}\mathbf{A}_{\text{ref}}^{-1}$$

TRANSMISSION SIDE S-MATRIX



$$\mathbf{S}_{11}^{(\text{trn})} = \mathbf{B}_{\text{trn}}\mathbf{A}_{\text{trn}}^{-1}$$

$$\mathbf{S}_{12}^{(\text{trn})} = 0.5\left(\mathbf{A}_{\text{trn}} - \mathbf{B}_{\text{trn}}\mathbf{A}_{\text{trn}}^{-1}\mathbf{B}_{\text{trn}}\right)$$

$$\mathbf{S}_{21}^{(\text{trn})} = 2\mathbf{A}_{\text{trn}}^{-1} \qquad \mathbf{A}_{\text{trn}} = \mathbf{W}_h^{-1}\mathbf{W}_{\text{trn}} + \mathbf{V}_h^{-1}\mathbf{V}_{\text{trn}}$$

$$\mathbf{S}_{22}^{(\text{trn})} = -\mathbf{A}_{\text{trn}}^{-1}\mathbf{B}_{\text{trn}} \qquad \mathbf{B}_{\text{trn}} = \mathbf{W}_h^{-1}\mathbf{W}_{\text{trn}} - \mathbf{V}_h^{-1}\mathbf{V}_{\text{trn}}$$