Linear Wire Antennas

EE-4382/5306 - Antenna Engineering

Outline

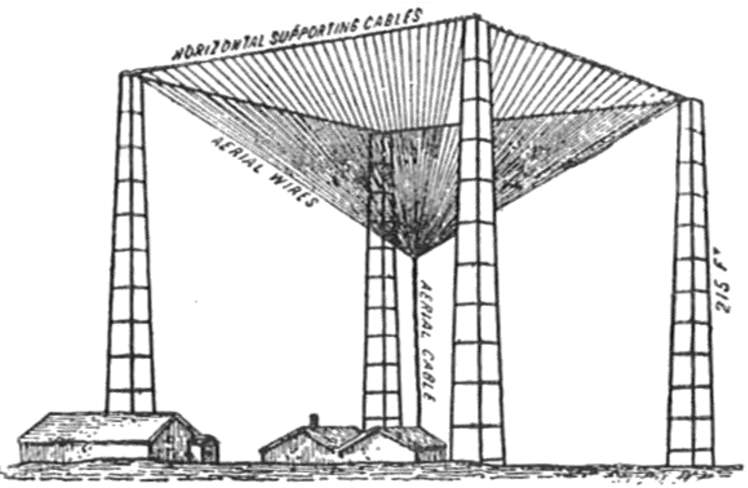
- Introduction
- Infinitesimal Dipole
- Small Dipole
- Finite Length Dipole
- Half-Wave Dipole
- Ground Effect

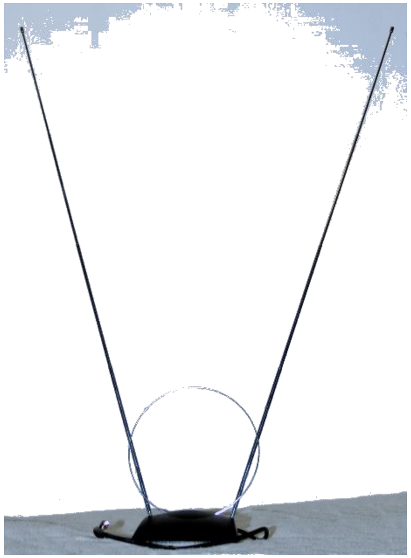
Constantine A. Balanis, *Antenna Theory: Analysis and Design* 4th Ed., Wiley, 2016.
Stutzman, Thiele, *Antenna Theory and Design* 3rd Ed., Wiley, 2012.

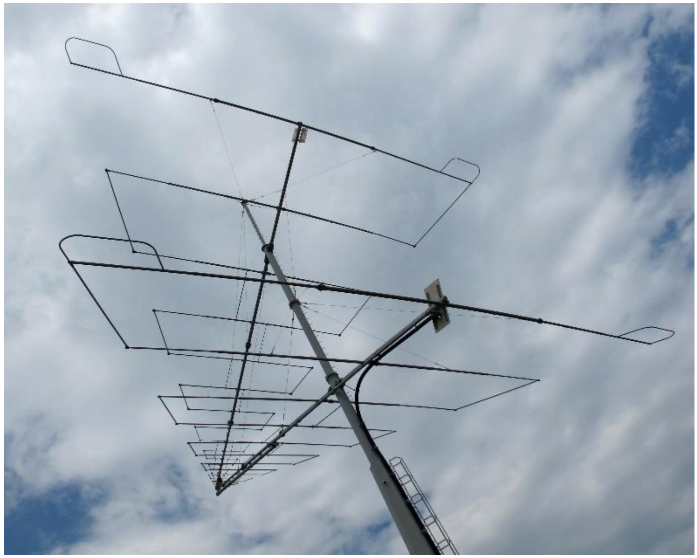
Introduction

Wire Antennas - Introduction

Wire antennas are the simplest, cheapest, and many times most effective antennas for many applications.
The easiness of configuration and analysis is why we begin with these types of antennas.







Infinitesimal Dipole

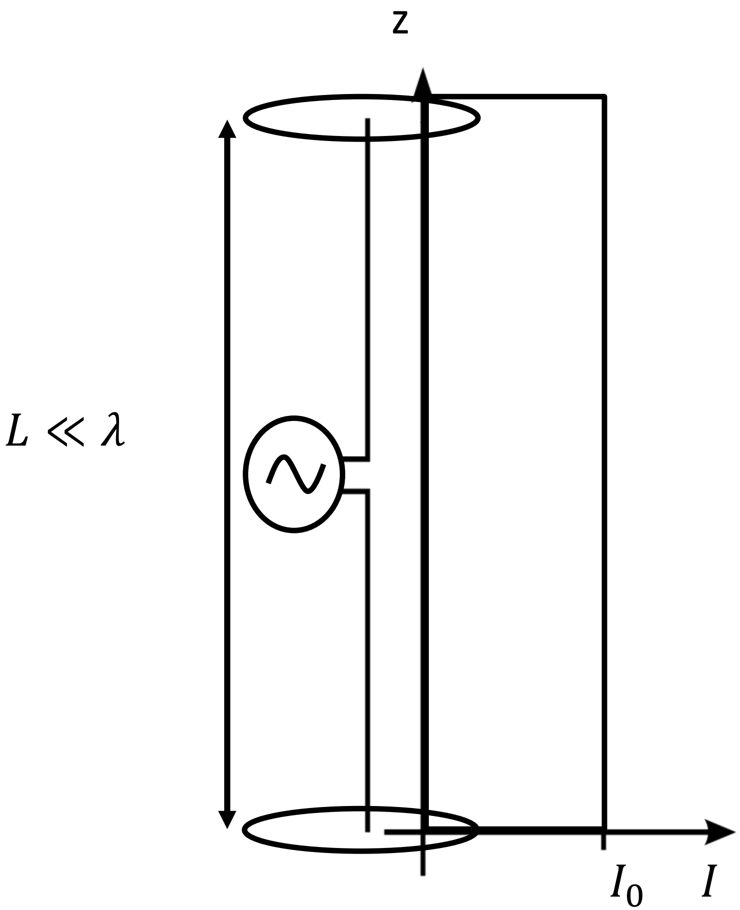
Infinitesimal Dipole

An infinitesimal dipole is in the order of $l \ll \lambda$, $a \ll \lambda$, where l is the length of the antenna, and a is the thickness.

The simplest antenna, it is just an open wire fed at the center with an alternating source.

To simplify the mathematical analysis, we will assume an infinitesimal vertical dipole placed along the z-axis, and the current is constant throughout the wire.

$$\mathbf{I}(z') = \hat{\boldsymbol{a}}_{\boldsymbol{z}} I_0$$



Infinitesimal Dipole

Electric Field Orientation

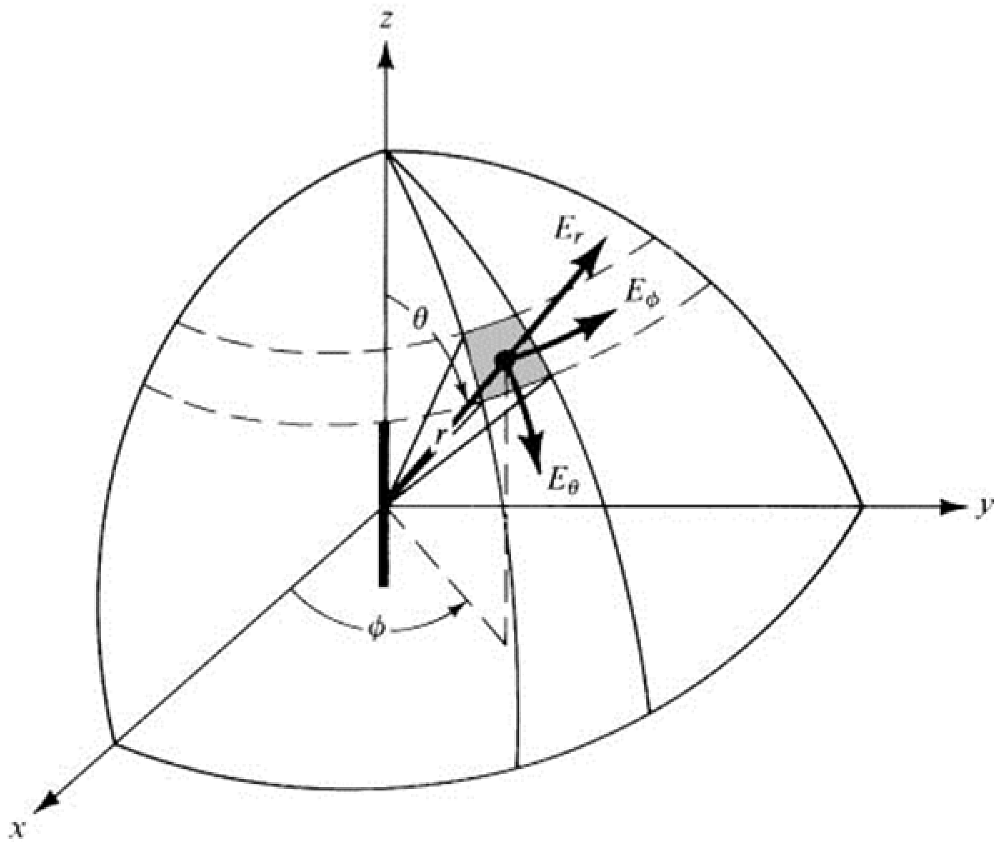


Fig. 4.1b

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Infinitesimal Dipole - Fields

For an electric current source, the magnetic field is equal to

$$H_\theta = H_r = 0$$

$$H_\phi = j\frac{kI_0 l \sin(\theta)}{4\pi r}\left[1 + \frac{1}{jkr}\right]e^{-jkr}$$

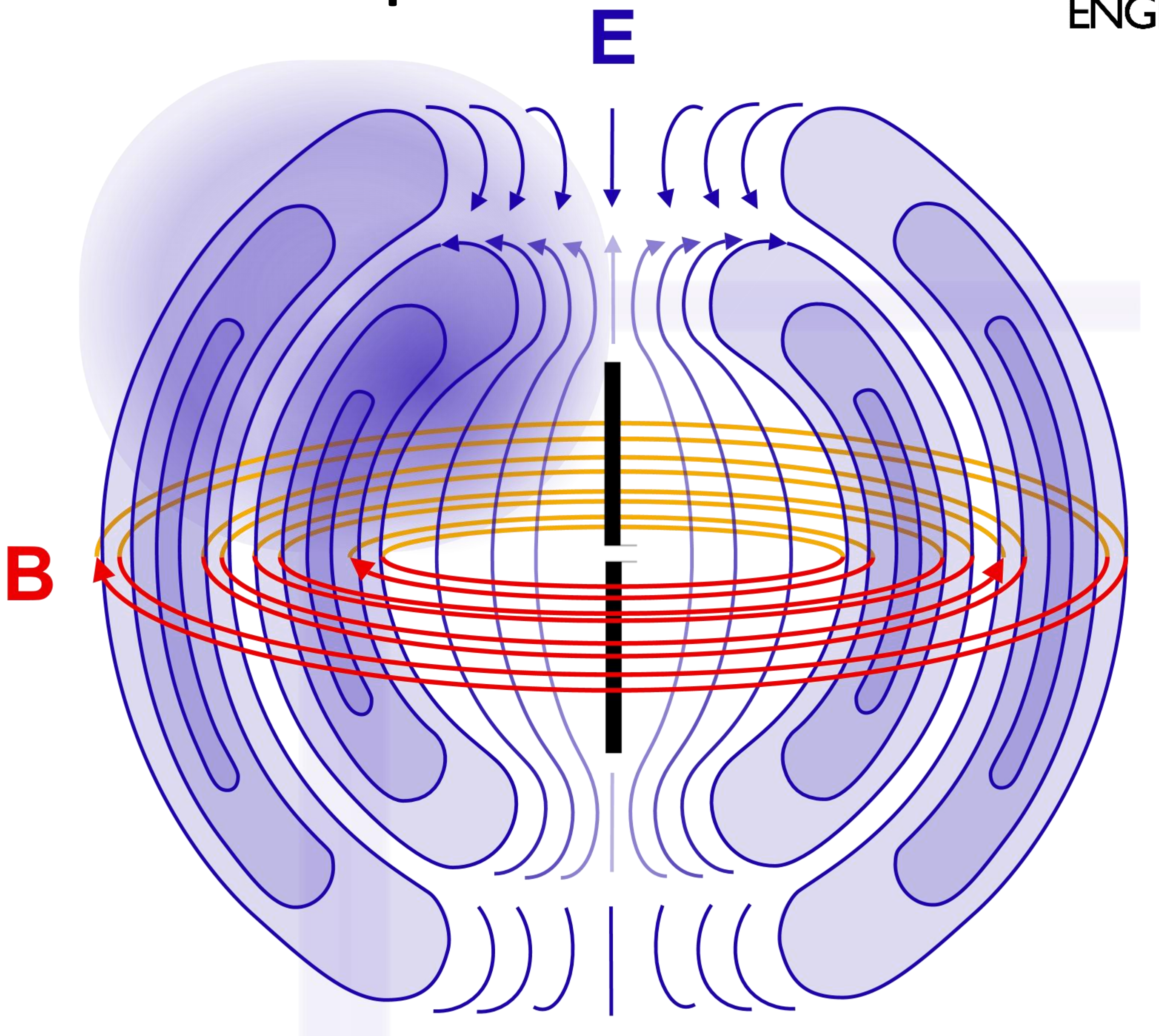
The electric field is equal to

$$E_\phi = 0$$

$$E_r = \eta\frac{I_0 l \cos(\theta)}{2\pi r^2}\left[1 + \frac{1}{jkr}\right]e^{-jkr}$$

$$E_\theta = j\eta\frac{kI_0 l \sin(\theta)}{4\pi r}\left[1 + \frac{1}{jkr} - \frac{1}{(kr)^2}\right]e^{-jkr}$$

Infinitesimal Dipole - Fields



Infinitesimal Dipole – Near-Field

At the **near-field region** $kr \ll 1$, the fields with higher order terms dominate:

$$E_\phi = H_\theta = H_r = 0$$

$$H_\phi = j\frac{kI_0 l\sin(\theta)}{4\pi r}\left[\cancel{1} + \frac{1}{jkr}\right]e^{-jkr} = \frac{I_0 l e^{-jkr}}{4\pi r^2}\sin(\theta)$$

$$E_r = \eta\frac{I_0 l\cos(\theta)}{2\pi r^2}\left[\cancel{1} + \frac{1}{jkr}\right]e^{-jkr} = -j\eta\frac{I_0 l e^{-jkr}}{2\pi k r^3}\cos(\theta)$$

$$E_\theta = j\eta\frac{kI_0 l\sin(\theta)}{4\pi r}\left[\cancel{1} + \cancel{\frac{1}{jkr}} - \frac{1}{(kr)^2}\right]e^{-jkr} = -j\eta\frac{I_0 l e^{-jkr}}{2\pi k r^3}\sin(\theta)$$

Infinitesimal Dipole - Far-field

At the **far-field region** $kr \gg 1$, the fields with lower order terms dominate:

$$E_\phi = H_\theta = H_r = 0$$

$$H_\phi = j\frac{kI_0 l \sin(\theta)}{4\pi r}\left[1 + \cancel{\frac{1}{jkr}}\right]e^{-jkr} = j\frac{kI_0 l e^{-jkr}}{4\pi r}\sin(\theta)$$

$$E_r = \eta\frac{I_0 l \cos(\theta)}{2\pi r^2}\left[\cancel{1} + \cancel{\frac{1}{jkr}}\right]e^{-jkr} \cong 0$$

$$E_\theta = j\eta\frac{kI_0 l \sin(\theta)}{4\pi r}\left[1 + \cancel{\frac{1}{jkr}} - \cancel{\frac{1}{(kr)^2}}\right]e^{-jkr} = j\eta\frac{kI_0 l e^{-jkr}}{4\pi r}\sin(\theta)$$

Infinitesimal Dipole – Power Density

Power Density:

For a lossless antenna, the real part of the input resistance is designated as radiation resistance.

Through the mechanism of radiation resistance, the power from guided waves transfers to free-space waves.

Recall the complex Poynting Vector:

$$\mathbf{W} = \frac{1}{2}(\mathbf{E} \times \mathbf{H}^*) = \frac{1}{2}(\hat{\boldsymbol{a}}_{\boldsymbol{r}} E_r + \hat{\boldsymbol{a}}_{\boldsymbol{\theta}} E_\theta) \times \left(\hat{\boldsymbol{a}}_{\boldsymbol{\phi}} H_\phi^*\right) = \frac{1}{2}(\hat{\boldsymbol{a}}_{\boldsymbol{r}} E_\theta H_\phi^* - \hat{\boldsymbol{a}}_{\boldsymbol{\theta}} E_r H_\phi^*)$$

Thus we get two components for power:

$$W_r = \frac{\eta}{8}\left|\frac{I_0 l}{\lambda}\right|^2 \frac{\sin^2(\theta)}{r^2}\left[1 - j\frac{1}{(kr)^3}\right]$$

$$W_\theta = j\eta \frac{k|I_0 l|^2 \cos(\theta)\sin(\theta)}{16\pi^2 r^3}\left[1 + \frac{1}{(kr)^2}\right]$$

Infinitesimal Dipole – Power Density

Power Density:

The complex power in the radial direction is obtained by integrating the components over the closed sphere:

$$P = \oiint_S \mathbf{W} \cdot d\boldsymbol{s} = \int_0^{2\pi} \int_0^{\pi} (\hat{\boldsymbol{a}}_{\boldsymbol{r}} W_r + \cancel{\hat{\boldsymbol{a}}_{\boldsymbol{\theta}} W_\theta}) \cdot \hat{\boldsymbol{a}}_{\boldsymbol{r}} r^2 \sin(\theta)\, d\theta d\phi$$

And we obtain

$$P = \eta \frac{\pi}{3} \left| \frac{I_0 l}{\lambda} \right|^2 \left[1 - j \frac{1}{(kr)^3} \right]$$

For real power in the **far field**, we only care about the first term. Reactive (imaginary power) is more intense in the near-field region, but does not contribute to the far-field power.

Infinitesimal Dipole – Power Density and Radiation Resistance

Power Density:

From the power calculated we obtain

$$P_{rad} = \eta \frac{\pi}{3} \left| \frac{I_0 l}{\lambda} \right|^2 = \frac{1}{2} |I_0|^2 R_r$$

Radiation Resistance:

Where R_r is the radiation resistance given by

$$R_r = \eta \frac{2\pi}{3} \left(\frac{l}{\lambda} \right)^2 = 80\pi^2 \left(\frac{l}{\lambda} \right)^2$$

Assumes antenna is in free-space

Infinitesimal Dipole – Directivity

We can calculate the directivity from the average power.

$$W_{rad} = W_{ave} = \frac{\eta}{8}\left|\frac{I_0 l}{\lambda}\right|^2 \frac{\sin^2(\theta)}{r^2} = \frac{\eta}{2}\left|\frac{kI_0 l}{4\pi}\right|^2 \frac{\sin^2(\theta)}{r^2}$$

$$U = r^2 W_{rad} = \frac{\eta}{8}\left|\frac{I_0 l}{\lambda}\right|^2 \sin^2(\theta)$$

$$D_0 = \frac{U_{max}}{U_0} = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi \frac{\eta}{8}\left|\frac{kI_0}{\lambda}\right|^2}{\eta \frac{\pi}{3}\left|\frac{I_0 l}{\lambda}\right|^2} = \boxed{\frac{3}{2}}$$

$$D_0 = \frac{3}{2} = 1.76\ \text{dBi}$$
$$U_n = \sin^2(\theta)$$

Infinitesimal Dipole – Radiation Pattern

3-D Radiation Pattern of Infinitesimal Dipole

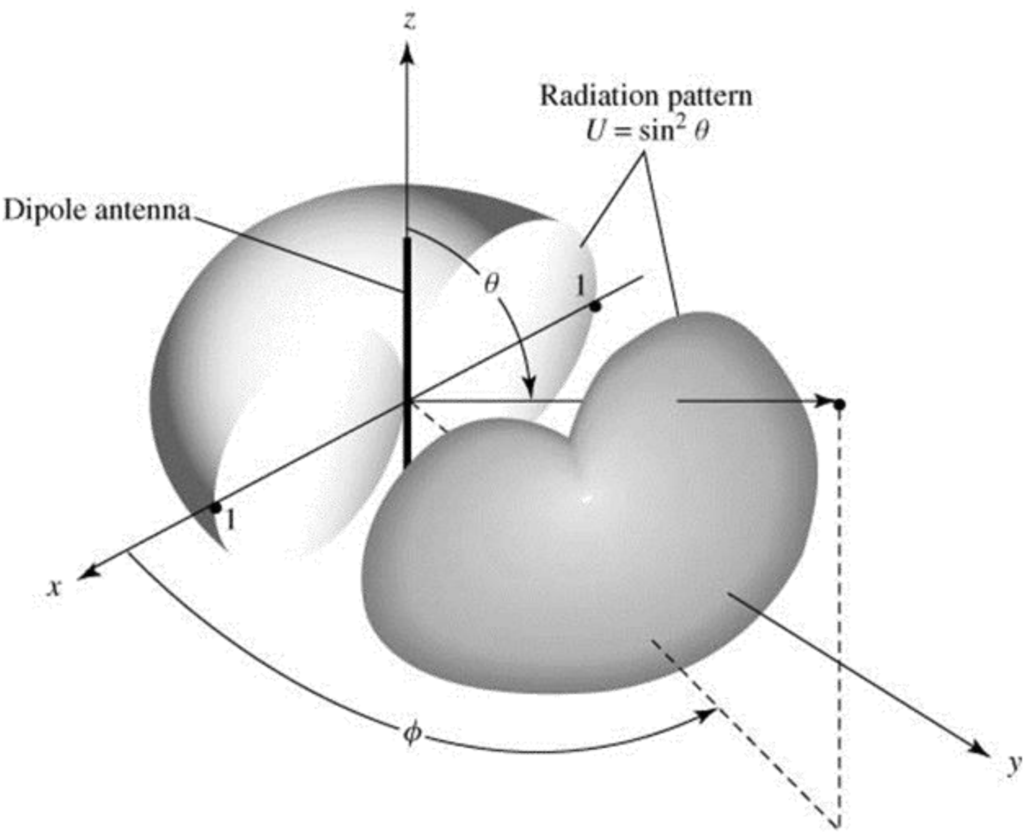


Fig. 4.3

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Infinitesimal Dipole – Radiation Resistance Example

Example 4.1 (page 150 Balanis)

Find the radiation resistance of an infinitesimal dipole whose length is $l = \frac{\lambda}{50}$

$$R_r = 80\pi^2 \left(\frac{l}{\lambda}\right)^2 = 80\pi^2 \left(\frac{1}{50}\right)^2 = \boxed{0.316\ \Omega}$$

Short Dipole

Short (Small) Dipole

The short dipole is a more practical, 'real' antenna, usually has lengths from $\frac{\lambda}{50}$ to $\frac{\lambda}{10}$.

A better representation of current distribution of wire antennas is the triangular distribution.

$\ell << \lambda$

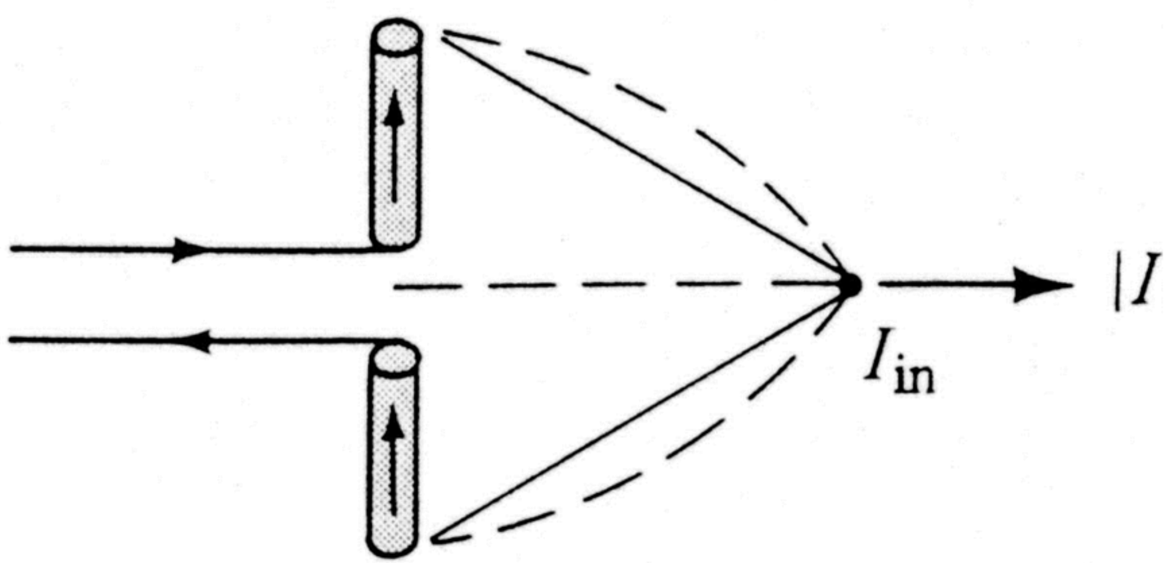
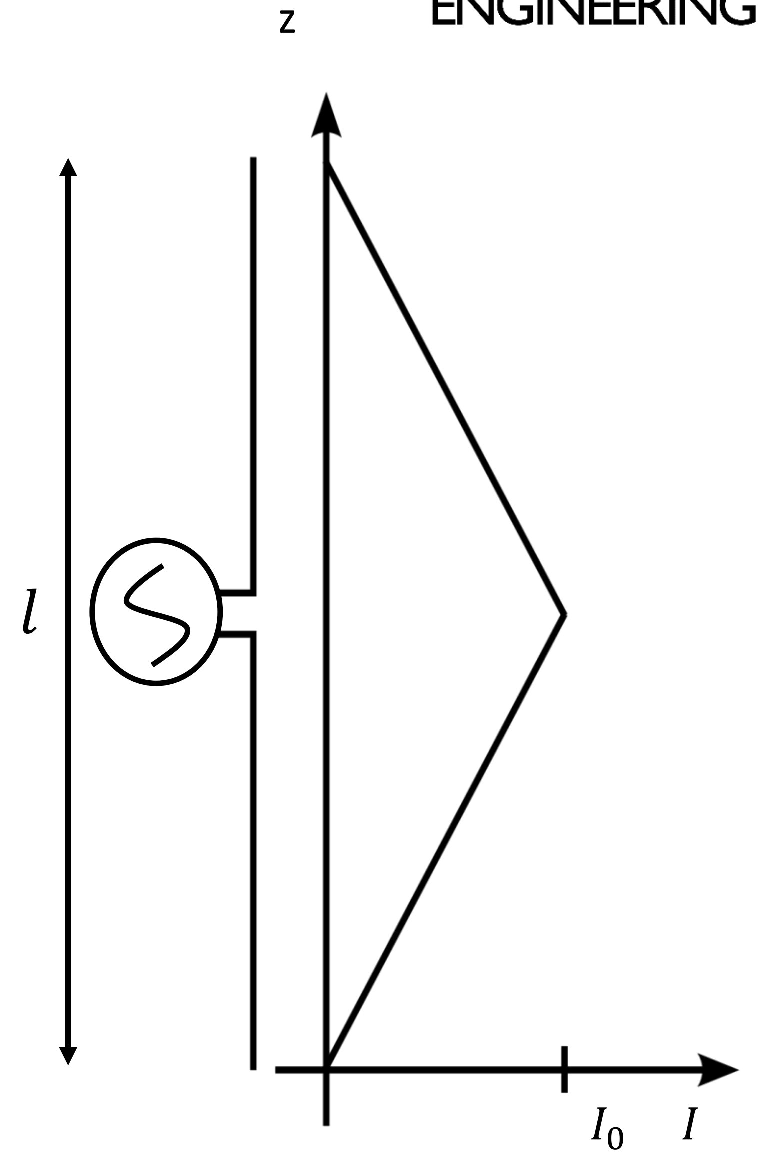


Fig. 1.16a

Chapter 1
Antennas

Short Dipole Current Distribution

$$\mathbf{I}_e(x, y, z) = \begin{cases} \hat{\boldsymbol{a}}_z I_0 \left(1 - \frac{2}{l} z\right), & 0 \le z \le \frac{l}{2} \\ \hat{\boldsymbol{a}}_z I_0 \left(1 + \frac{2}{l} z\right), & -\frac{l}{2} \le z \le 0 \end{cases}$$



Short Dipole Geometry

Dipole and Geometry

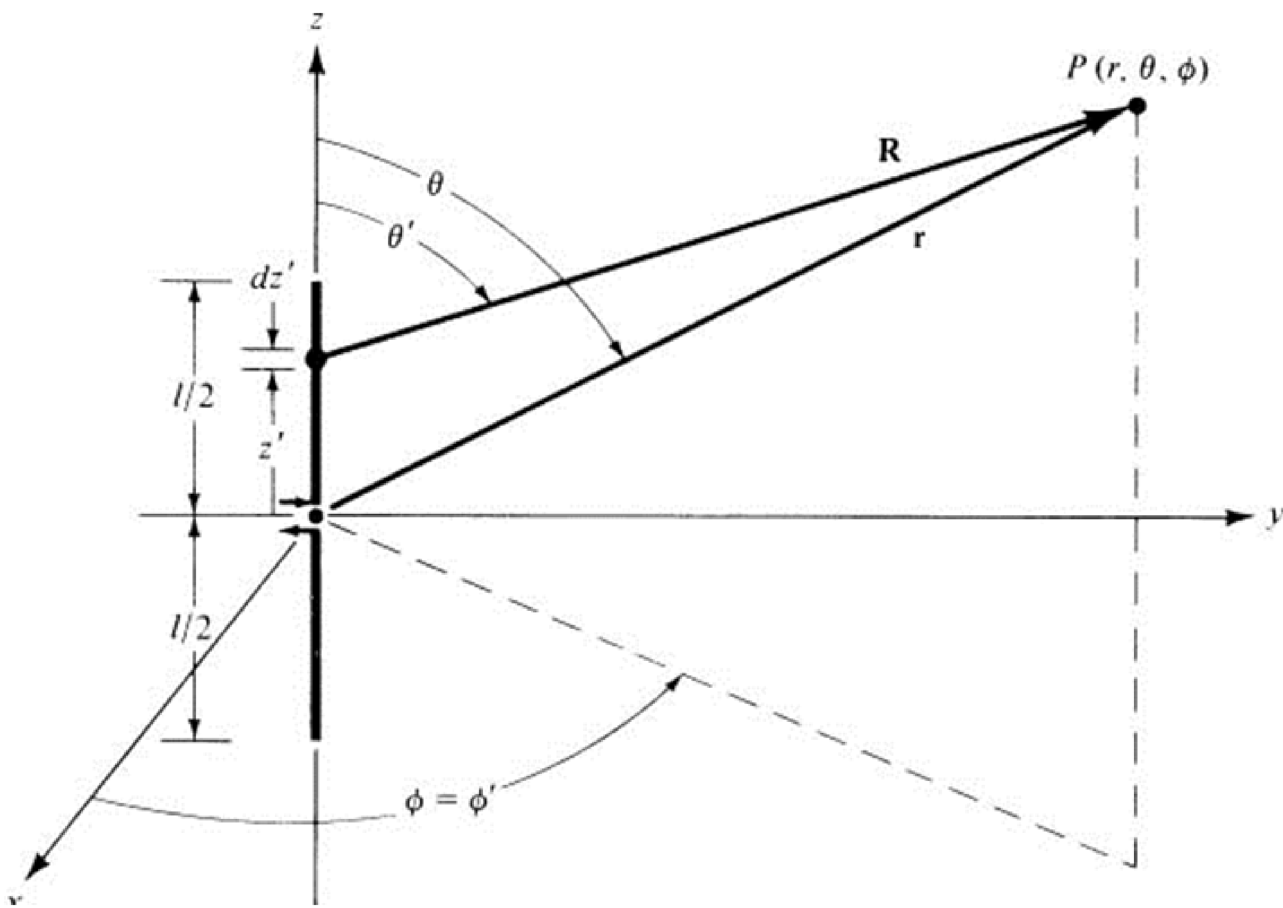


Fig. 4.4a

Far-Field E- and H- Fields

For an electric current source, the magnetic field is equal to

$$H_r = H_\theta = 0$$

$$H_\phi = j\frac{kI_0 l}{8\pi r}e^{-jkr}\sin(\theta)$$

The electric field is equal to

$$E_r = E_\phi = 0$$

$$E_\theta = j\eta\frac{kI_0 l}{8\pi r}e^{-jkr}\sin(\theta)$$

The fields of the short dipole are one-half of the infinitesimal dipole

Short Dipole - Directivity and Radiation Resistance

The radiation pattern is the same for the short and infinitesimal dipole. Since the directivity is controlled by the power pattern of the antenna, it is also the same as the infinitesimal dipole.

$$D_0 = \frac{3}{2} = 1.76\ \mathrm{dBi}$$
$$U_n = \sin^2(\theta)$$

The power radiated by the short dipole is one-fourth $\left(\frac{1}{4}\right)$ of the infinitesimal dipole.

$$R_r = 20\pi^2 \left(\frac{l}{\lambda}\right)^2$$

Short Dipole – Radiation Pattern

3-D Radiation Pattern of Infinitesimal Dipole

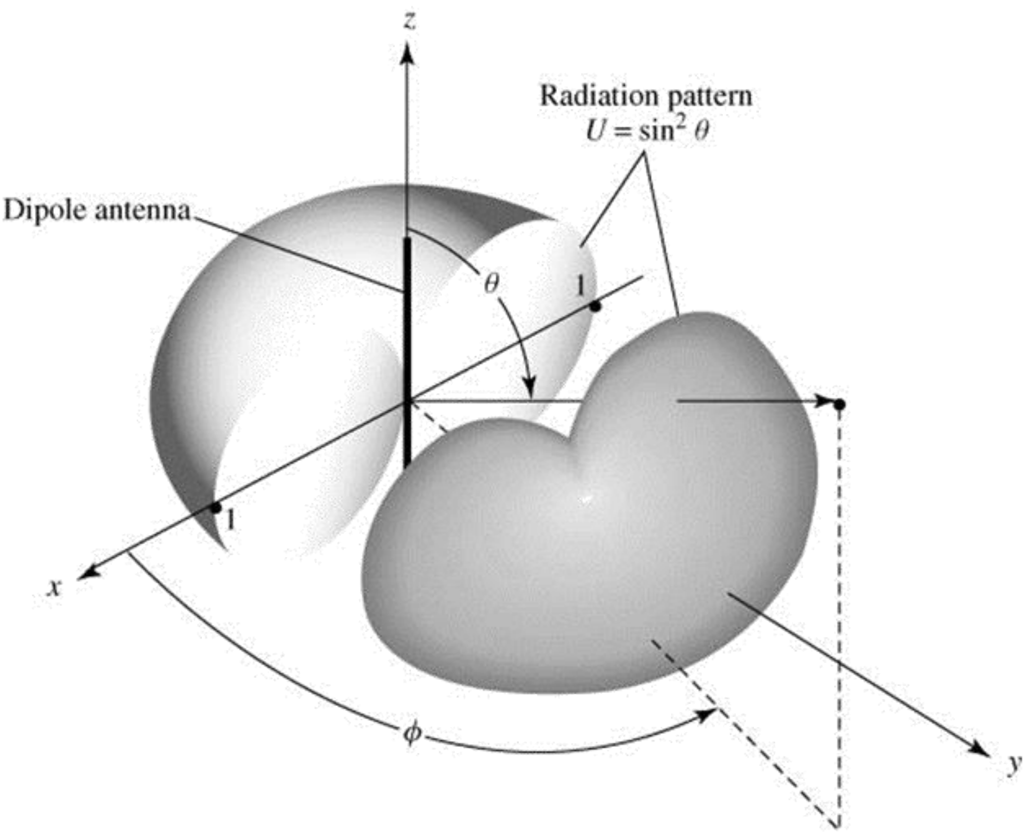


Fig. 4.3

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