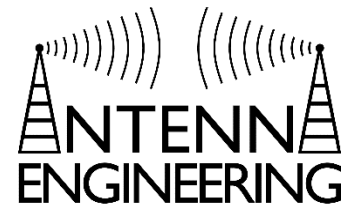


# Antenna Arrays

*EE-4382/5306 - Antenna Engineering*

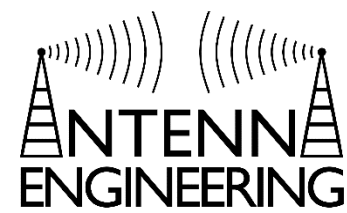
# Outline



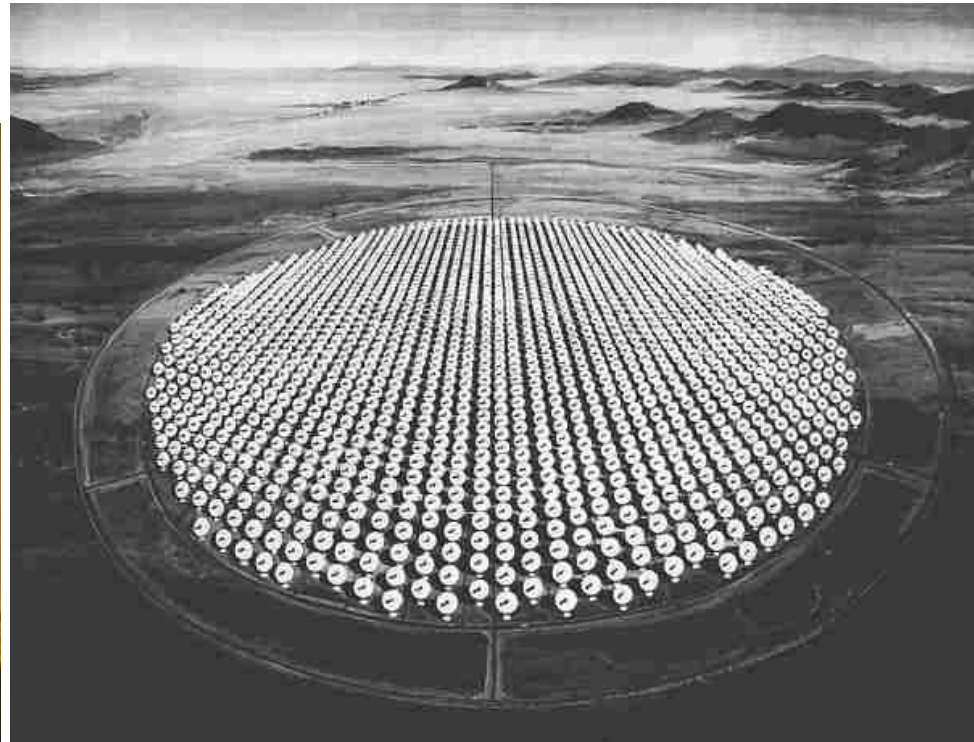
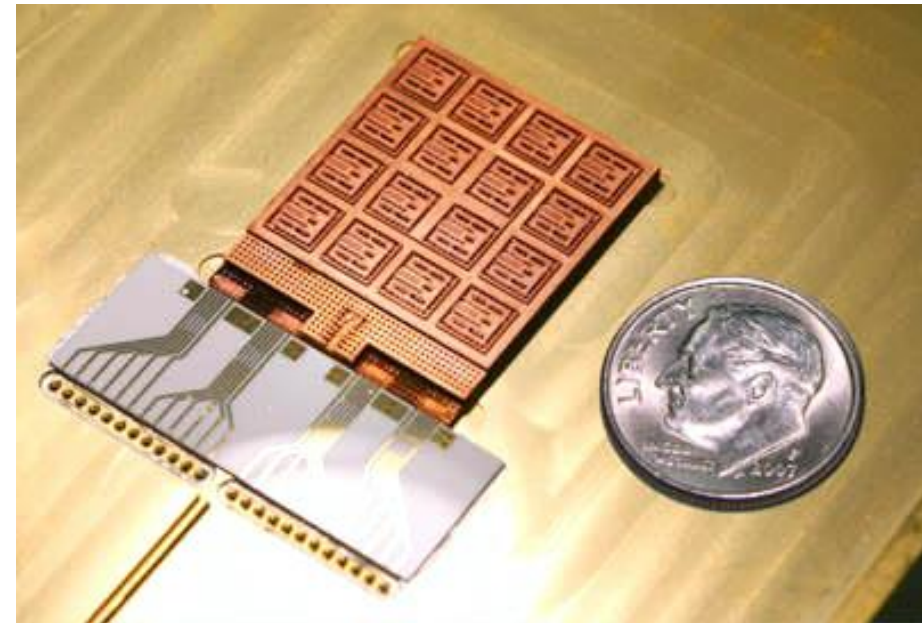
- Introduction
- Two Element Array
- Rectangular-to-Polar Graphical Solution
- N-Element Linear Array: Uniform Spacing and Amplitude
  - Theory of N-Element Linear Array
  - Rectangular to Polar Graphical Solution
  - Broadside Array
  - Ordinary End-Fire Array
  - Phased Array
  - Hansen-Woodyard End-Fire Array
- N-Element Linear Array: Directivity
- Design Procedure
- Radio Observatory Antenna Arrays

# Introduction

# Antenna Arrays - Introduction

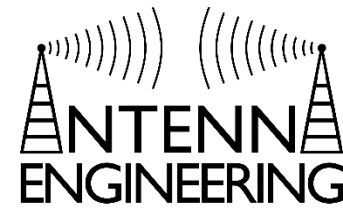


Antenna arrays are a configuration of multiple radiating elements in a geometrical order. Antenna arrays are an efficient way to freely change the pattern of an antenna, making it more directive and therefore increasing the gain. Electronically adjusting the excitation of individual elements leads to a phased (scanning) array, which enables greater degrees of freedom.



Linear Antenna Arrays

# Antenna Arrays - Introduction



In an array of identical radiating elements, there are at least five factors that can be controlled to shape the overall pattern:

1. The geometrical configuration of the array (linear, circular, rectangular, elliptical, etc.)
2. The relative displacement between the elements
3. The excitation amplitude of the individual elements
4. The excitation phase of the individual elements
5. The relative pattern of the individual elements

# Two-Element Array

# Two-Element Array

Two infinitesimal dipoles are placed along the z-axis. The total field radiated assuming no mutual coupling, is equal to the sum of the two elements. In the y-z plane:

$$E_t = E_1 + E_2$$

$$= \hat{\mathbf{a}}_{\theta} j\eta \frac{kI_0 l}{4\pi} \left[ \frac{e^{-j\left[kr_1 - \left(\frac{\beta}{2}\right)\right]}}{r_1} \cos(\theta_1) + \frac{e^{-j\left[kr_2 - \left(\frac{\beta}{2}\right)\right]}}{r_2} \cos(\theta_2) \right]$$

Where the  $\beta$  is the difference in the phase excitation between elements.

Assuming far-field observations:

$$\theta_1 \cong \theta_2 \cong \theta$$

$$r_1 \cong r - \frac{d}{2} \cos(\theta)$$

$$r_2 \cong r + \frac{d}{2} \cos(\theta)$$

$$r_1 \cong r_2 \cong r$$

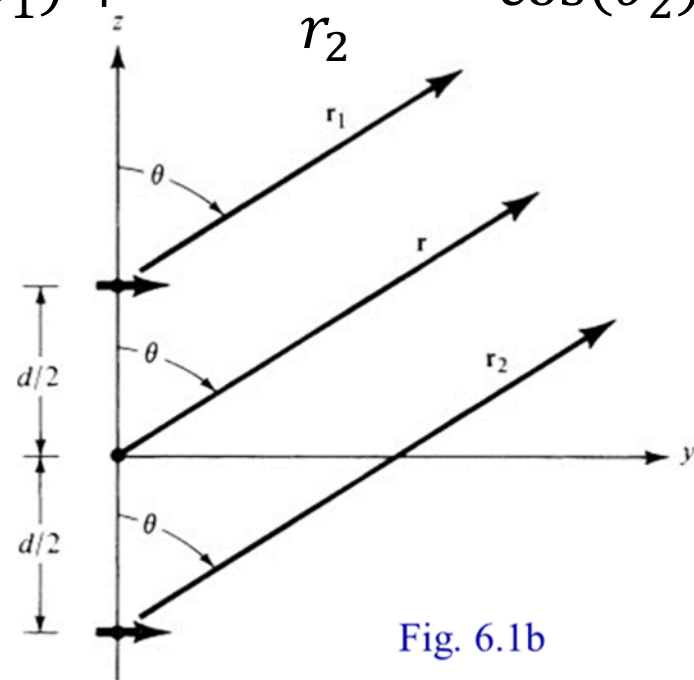
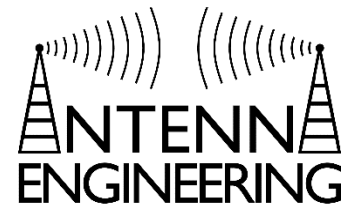


Fig. 6.1b

(b) Far-field observations

# Two-Element Array



Assuming far-field observations, the total field becomes

$$E_t = \hat{a}_\theta j\eta \frac{kI_0 l e^{-jkr}}{4\pi r} \cos(\theta) \left[ e^{+j[kd \cos(\theta) + \beta]/2} + e^{+j[kd \cos(\theta) + \beta]/2} \right]$$

$$E_t = \underbrace{\hat{a}_\theta j\eta \frac{kI_0 l e^{-jkr}}{4\pi r} \cos(\theta)}_{\text{Field of single element}} \underbrace{\left\{ 2 \cos \left[ \frac{1}{2} (kd \cos(\theta) + \beta) \right] \right\}}_{\text{Array Factor}}$$

$$AF = 2 \cos \left[ \frac{1}{2} (kd \cos(\theta) + \beta) \right]$$

$$(AF)_n = \cos \left[ \frac{1}{2} (kd \cos(\theta) + \beta) \right]$$

$$E(\text{total}) = [E(\text{single element at ref. point})] \times [\text{array factor}]$$

# Two-Element Array - Examples

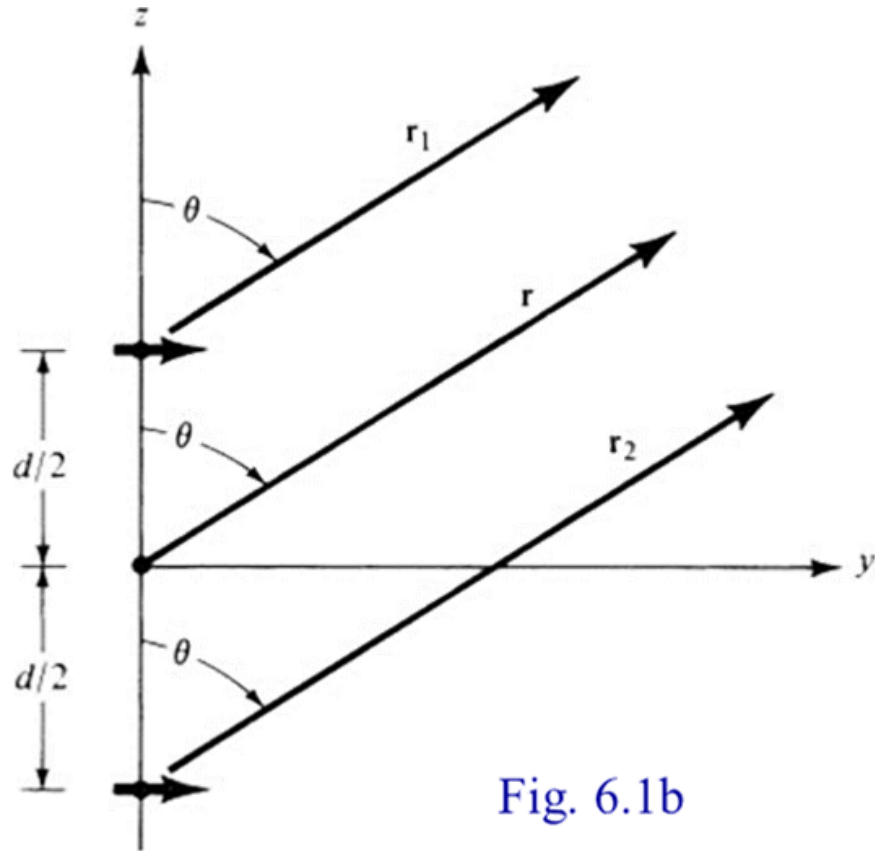
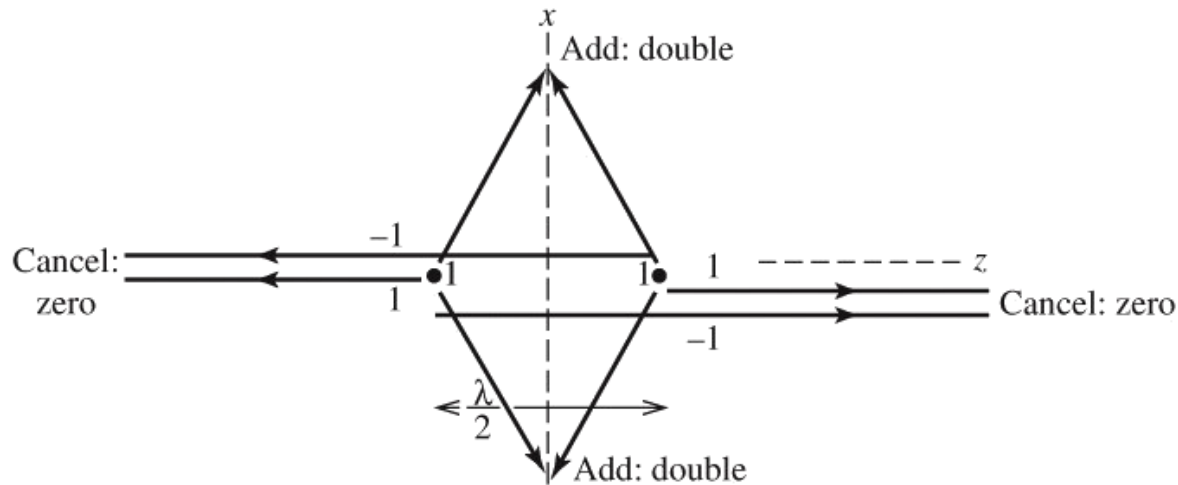


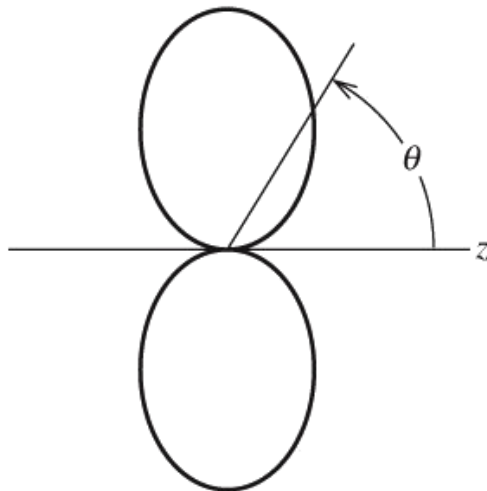
Fig. 6.1b

Given the array shown for two identical isotropic sources, find the total field when  $d = \lambda/2$  and  $\beta = 0$ .

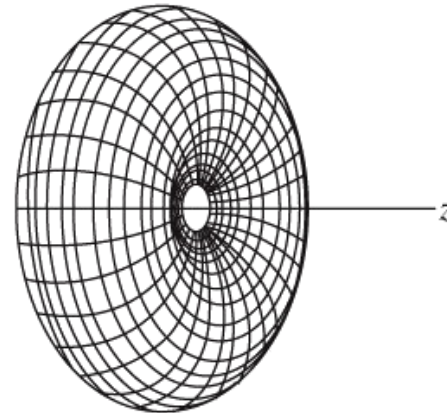
# Two-Element Array - Examples



(a) Inspection method.



(b) Polar plot of the array factor  
 $f(\theta) = \cos[(\pi/2) \cos \theta]$ .



(c) 3D polar pattern.

# Two-Element Array - Examples

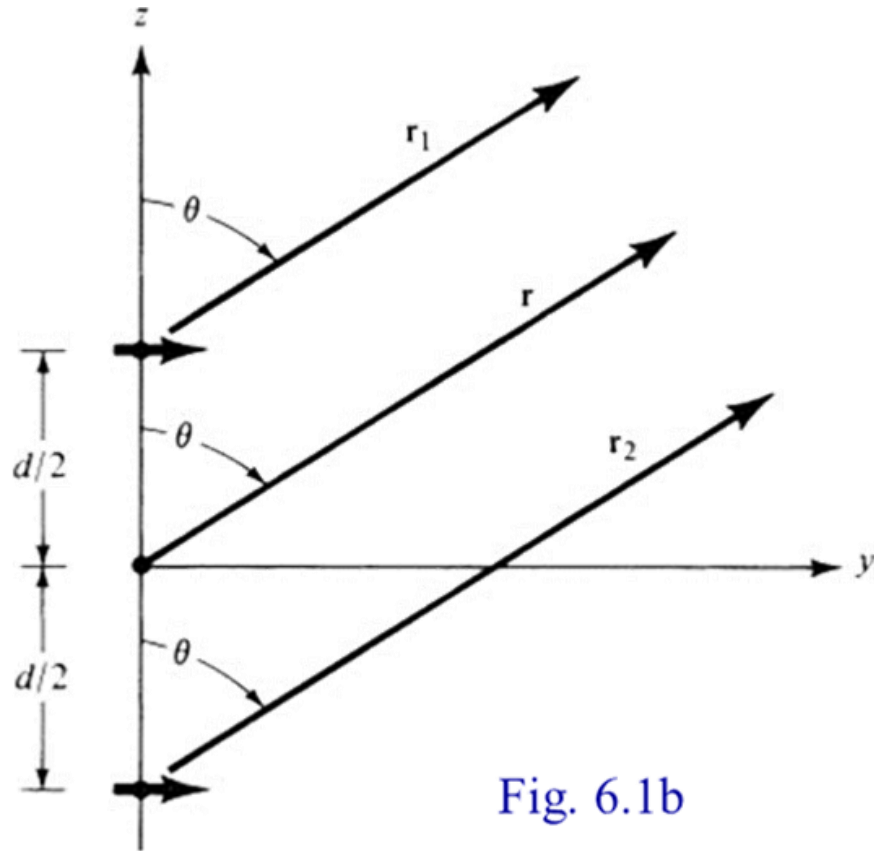
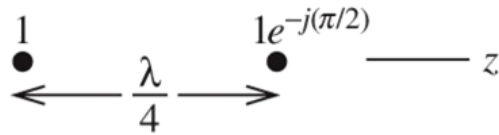


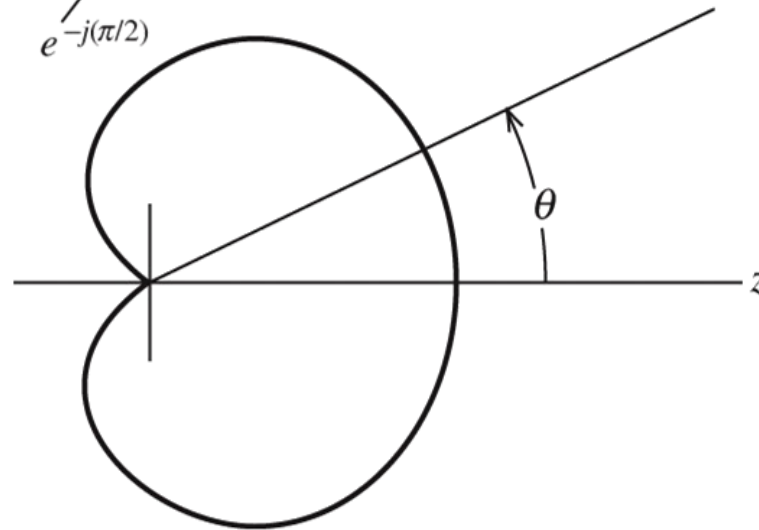
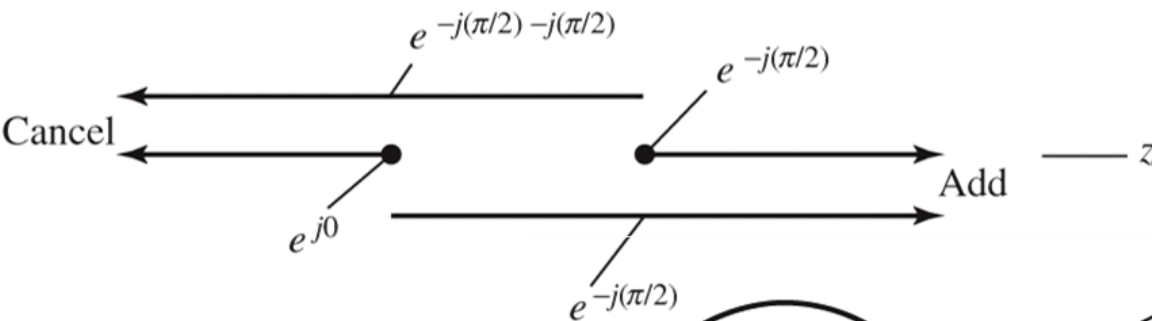
Fig. 6.1b

Given the array shown for two identical isotropic sources, find the normalized total field when  $d = \lambda/4$  and  $\beta = -90^\circ$ .

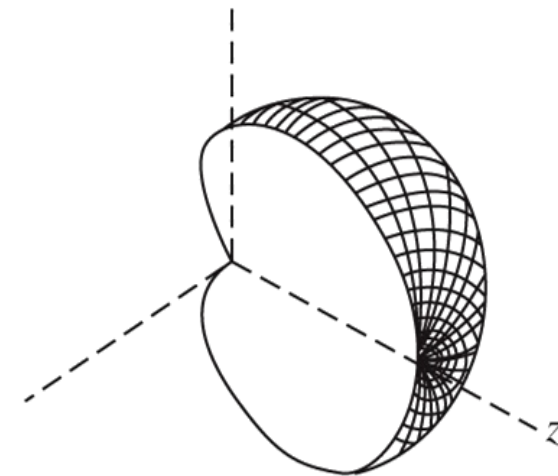
# Two-Element Array - Examples



(a) Array configuration.



(c) Polar plot of the array factor  
 $f(\theta) = \cos[(\pi/4)(\cos\theta - 1)]$ .



(d) 3D polar pattern.

# Two-Element Array - Examples

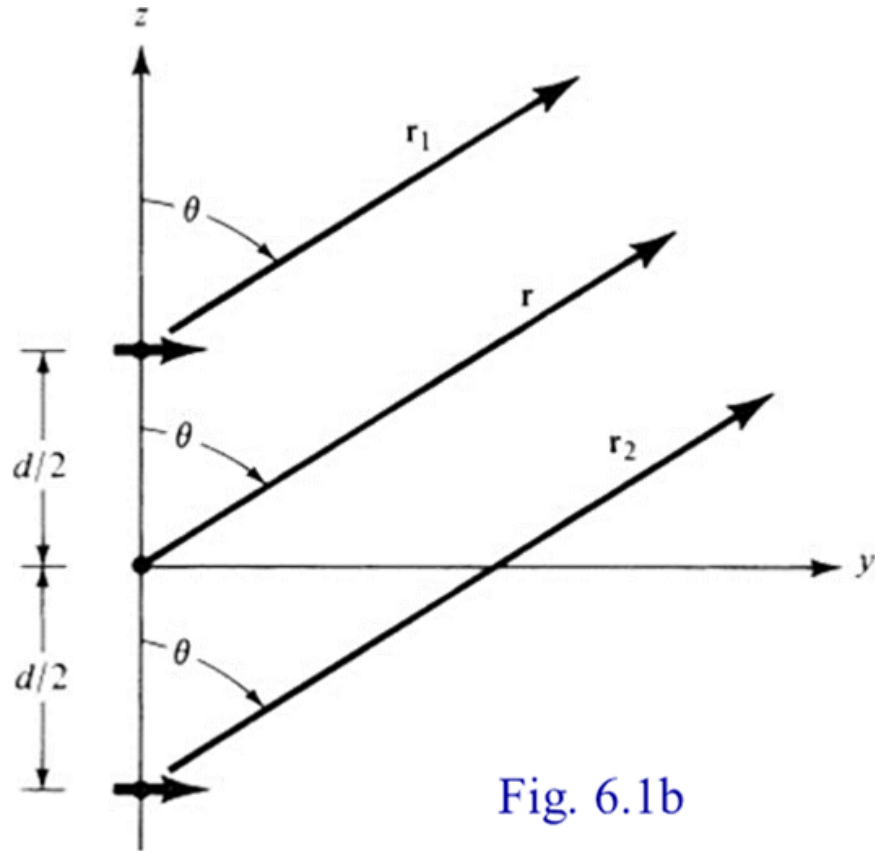
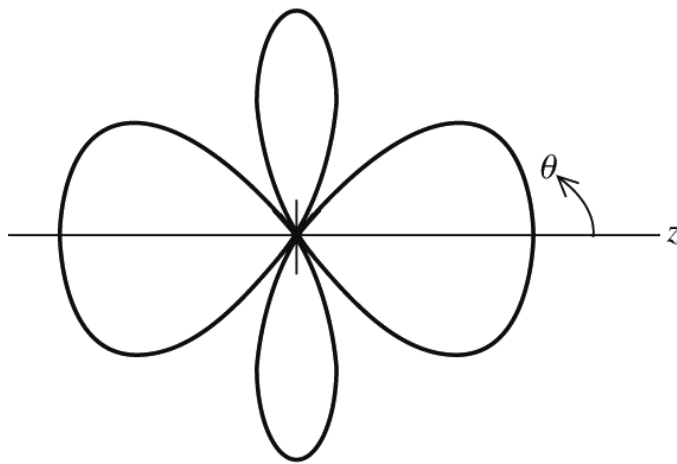
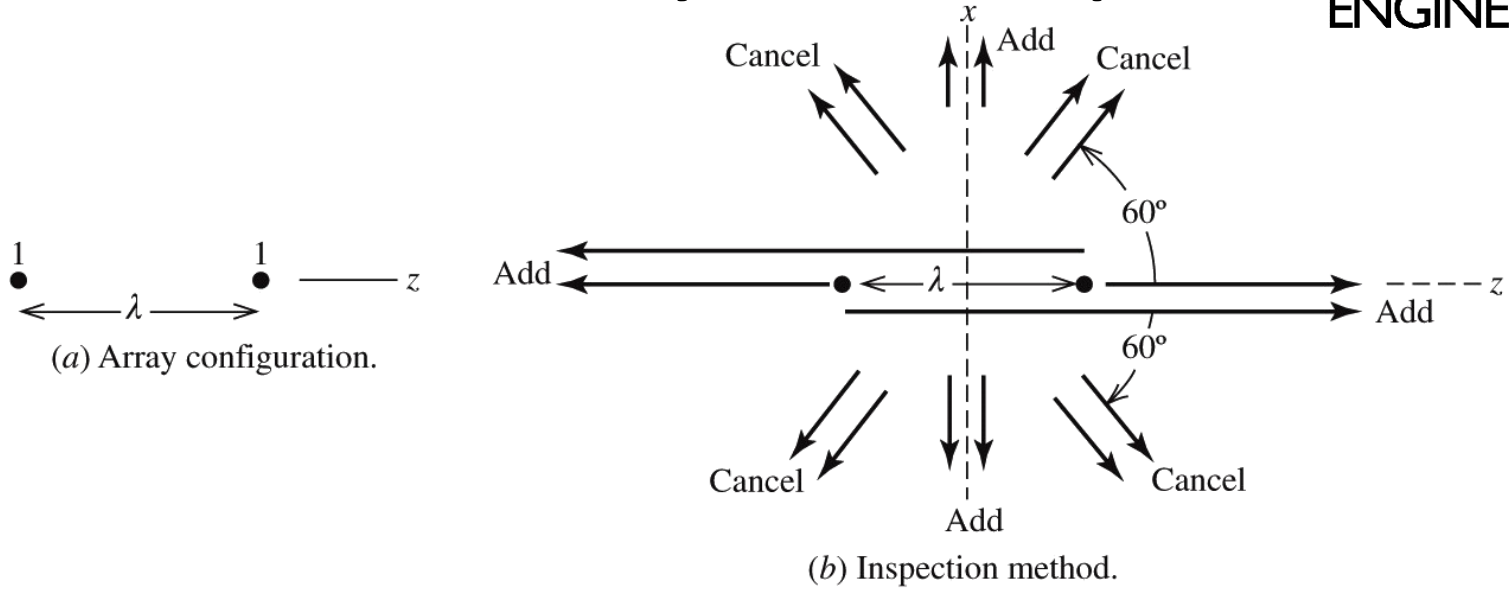


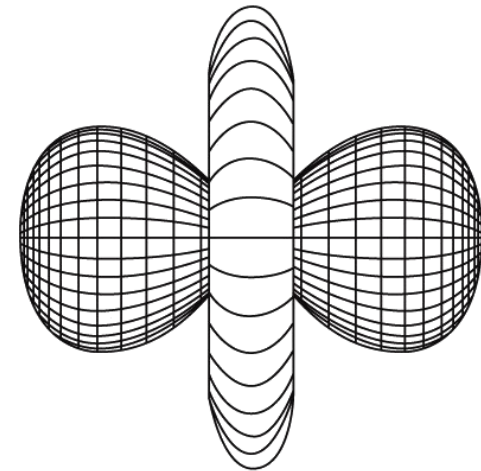
Fig. 6.1b

Given the array shown for two identical isotropic sources, find the normalized total field when  $d = \lambda$  and  $\beta = 0^\circ$ .

# Two-Element Array – Examples



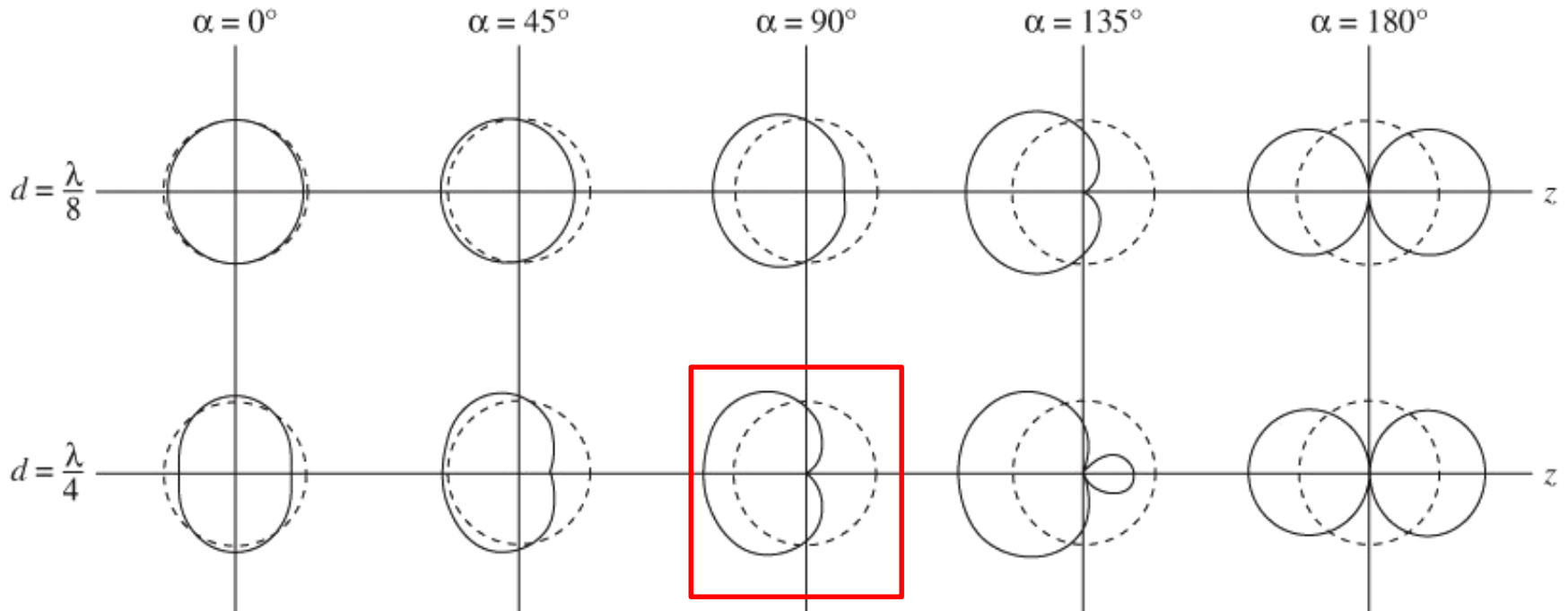
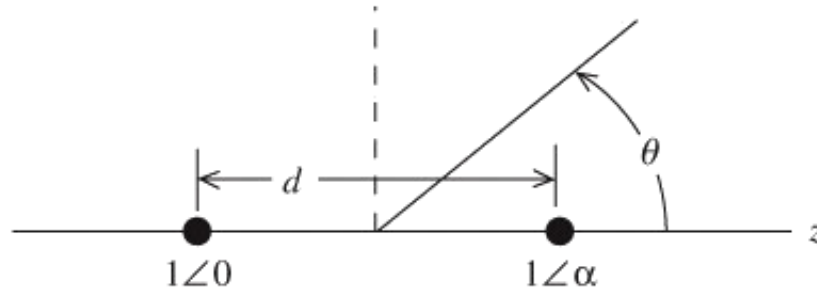
(c) Polar plot of array factor magnitude  $|f(\theta) = |\cos(\pi \cos \theta)|$ .



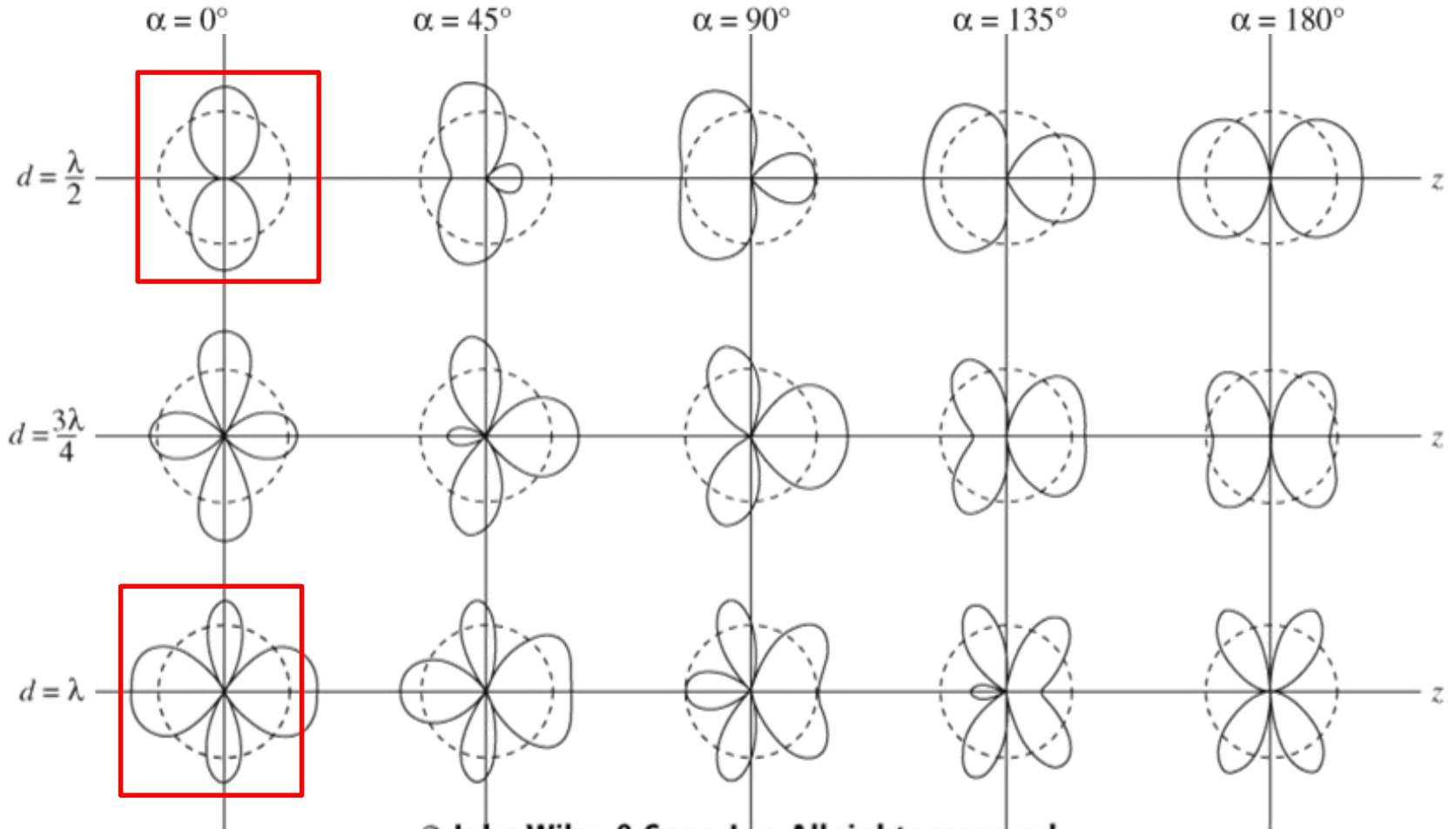
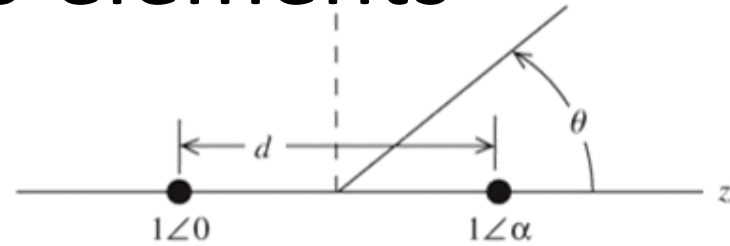
(d) Three-dimensional plot.

Figure 8-3

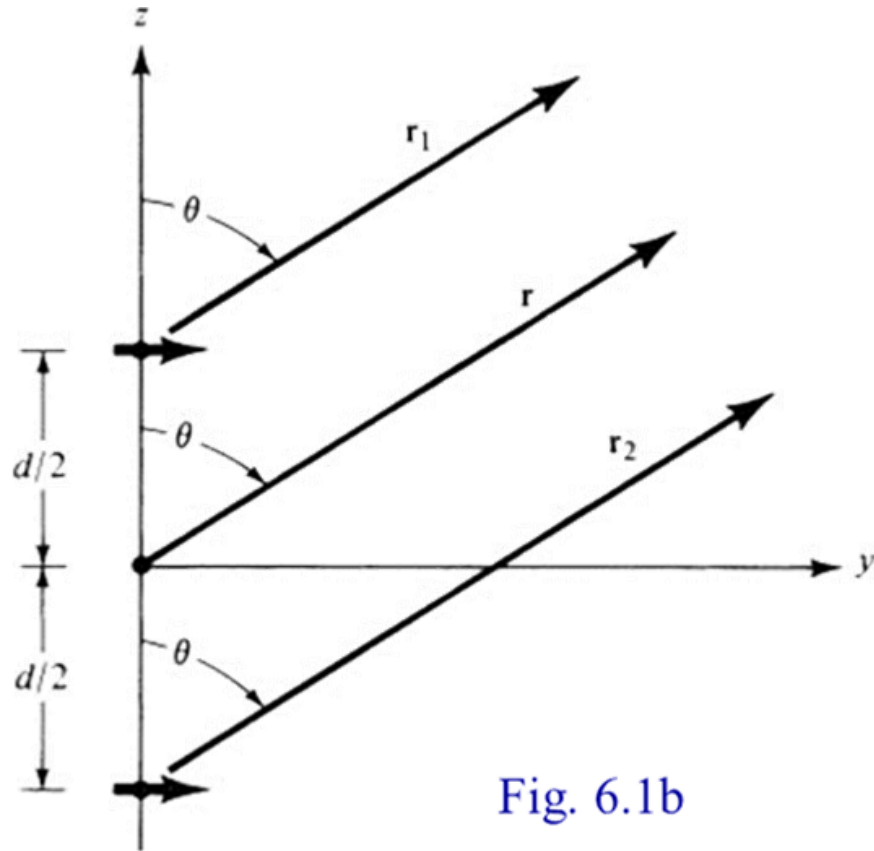
# Isotropic Point Sources – Array Factor for two elements



# Isotropic Point Sources – Array Factor for two elements



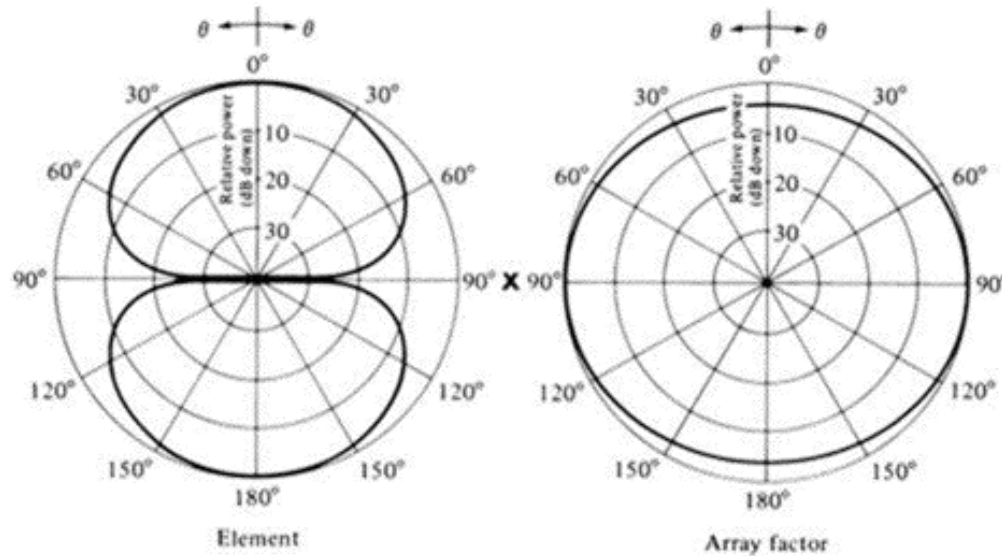
# Two-Element Array - Examples



Given the array shown for two identical infinitesimal dipoles, find by the nulls of the total field when  $d = \lambda/4$  and

- a.  $\beta = 0$
- b.  $\beta = +\pi/2$
- c.  $\beta = -\pi/2$

# Two-Element Array - Examples



$$\beta = 0, d = \lambda / 4$$

$$\Rightarrow \theta_n = 90^\circ$$

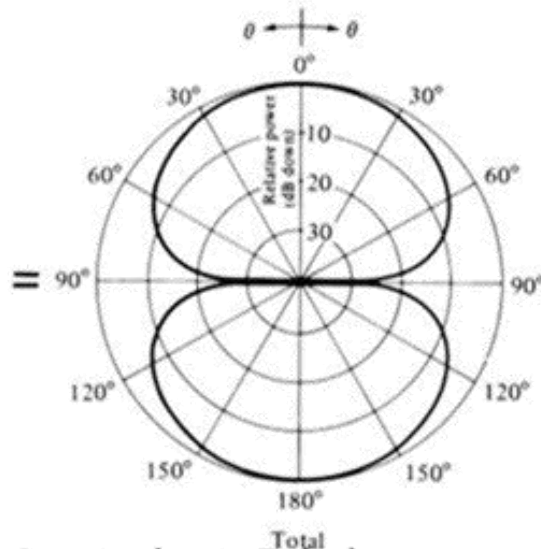
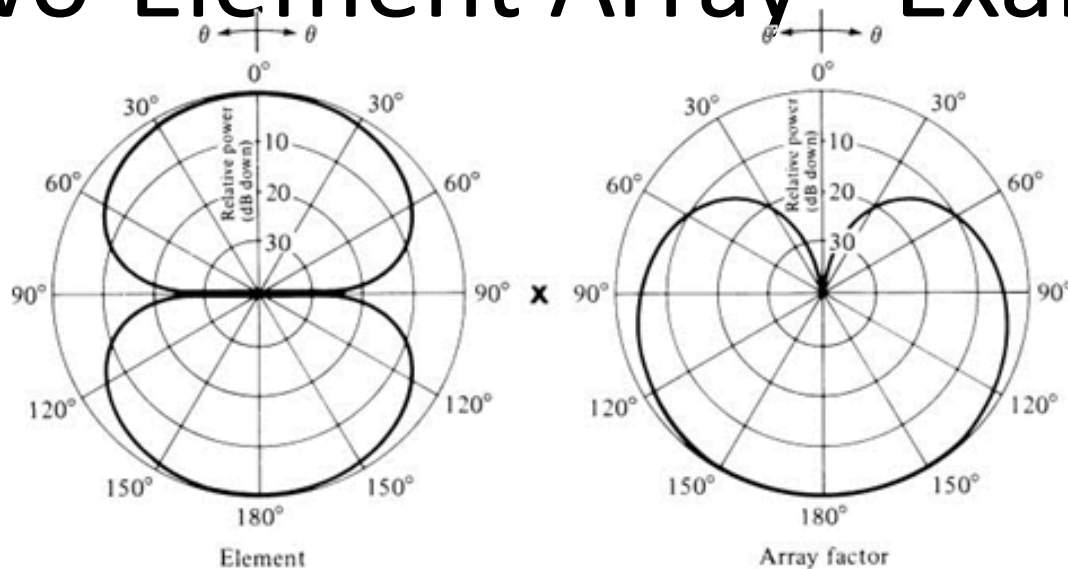


Fig. 6.3

# Two-Element Array - Examples



$$\underline{\beta = +90^\circ, d = \lambda/4}$$

$$\Rightarrow \theta_n = 0^\circ, 90^\circ$$

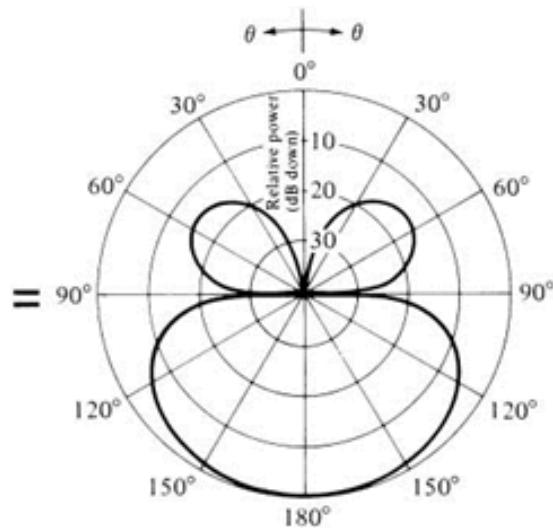
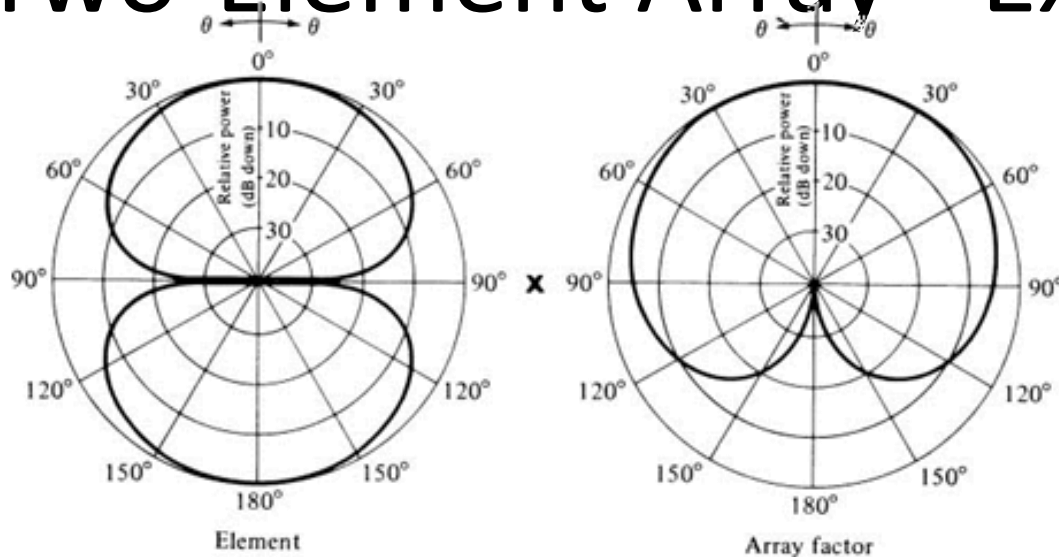


Fig. 6.4a

# Two-Element Array - Examples



$$\underline{\beta = -90^\circ, d = \lambda/4}$$

$$\Rightarrow \theta_n = 0^\circ, 180^\circ$$

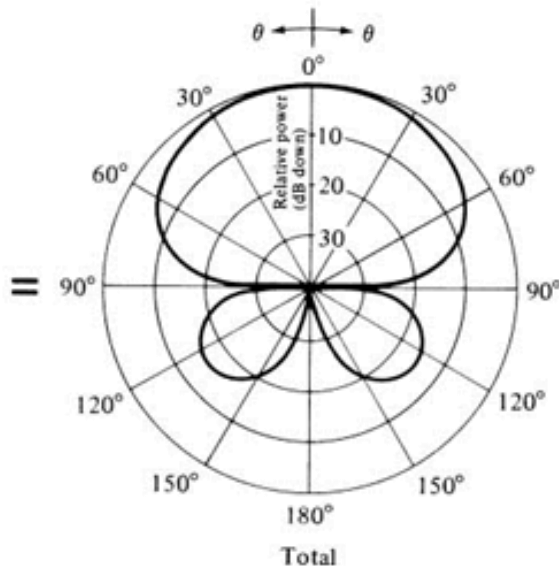
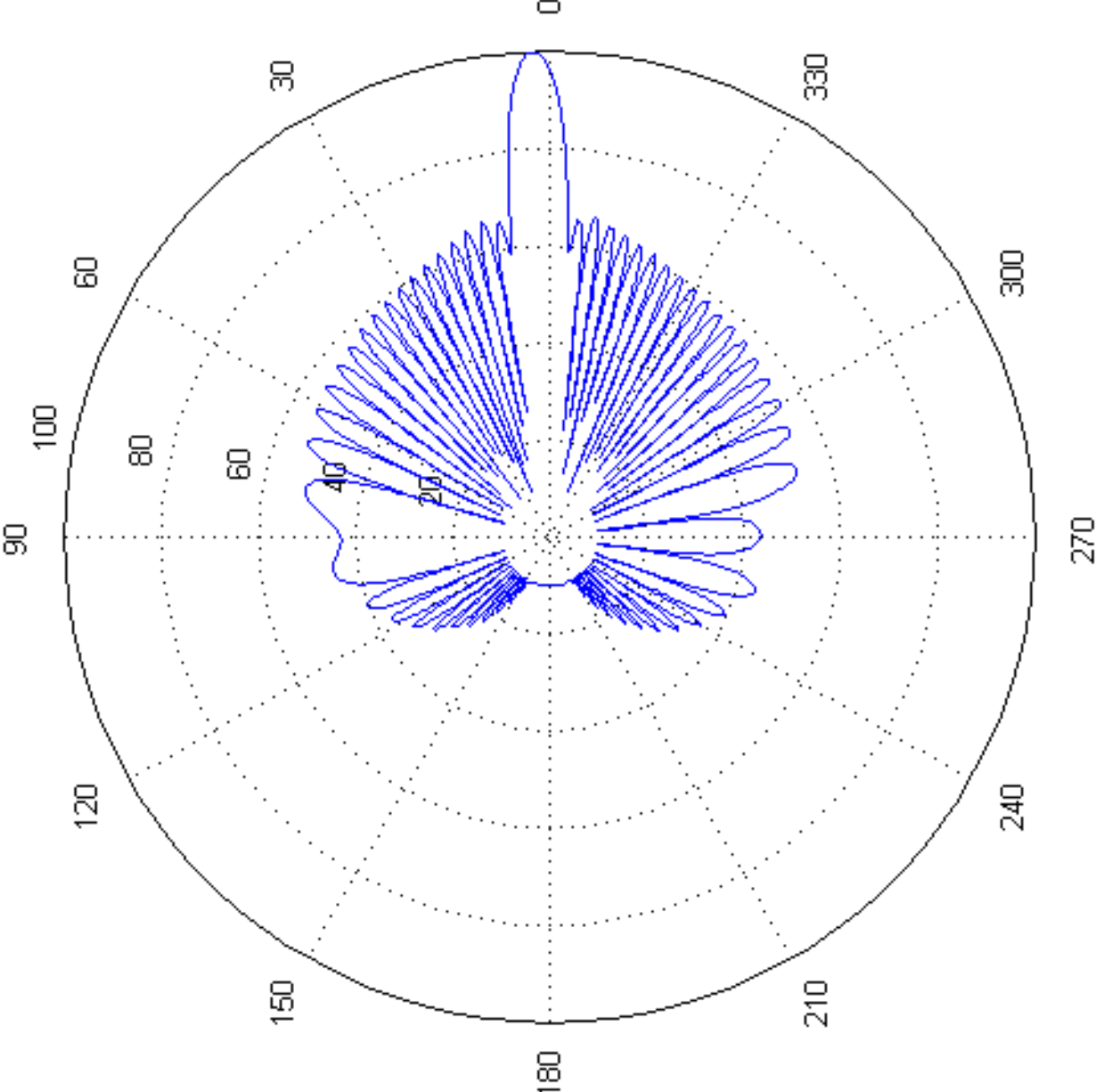
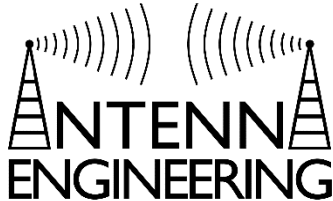


Fig. 6.4b

# Antenna Array – Scanning Array



# N-Element Linear Array: Uniform Amplitude and Spacing

# Linear Array: Uniform Amplitude and Spacing

An uniform array is an array of elements, all with identical magnitude, and each with a progressive phase.

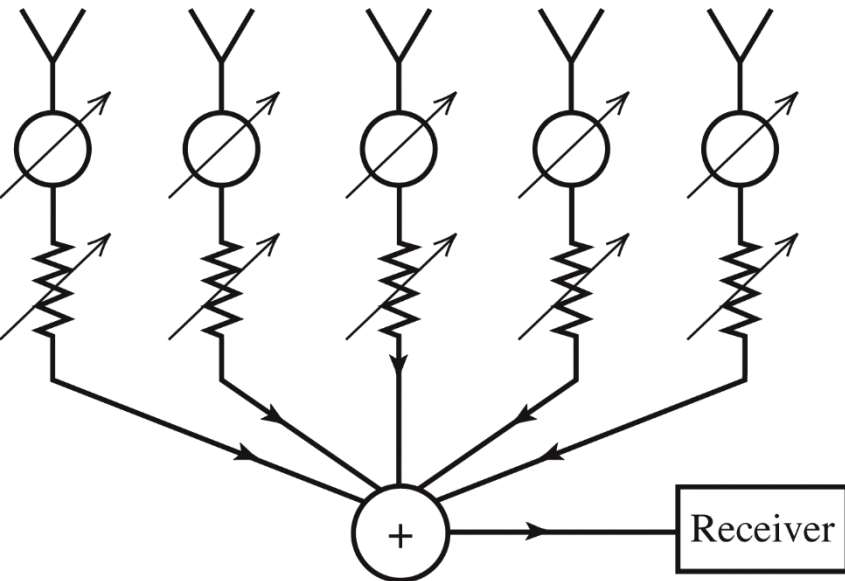


Figure 8-1  
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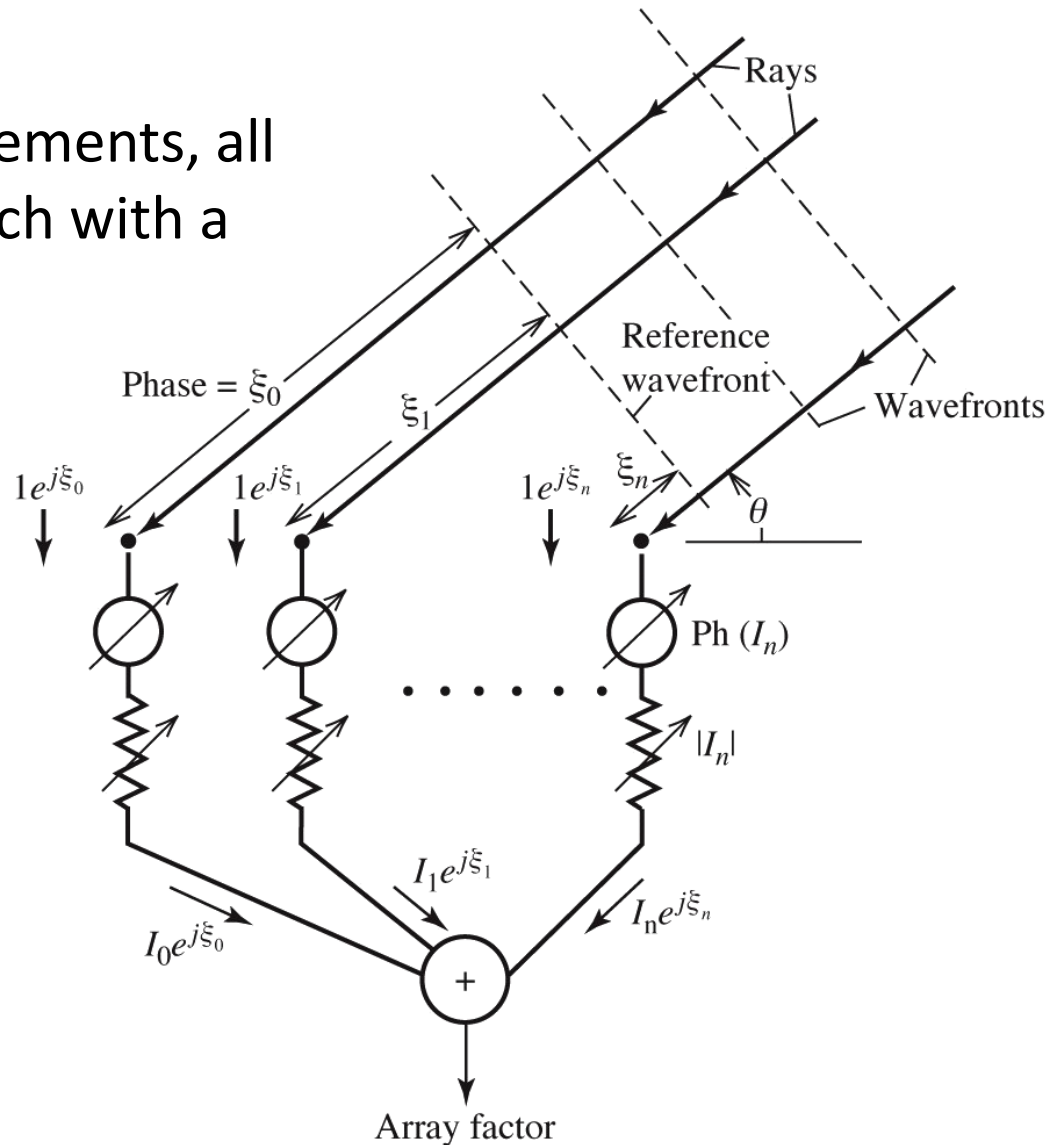
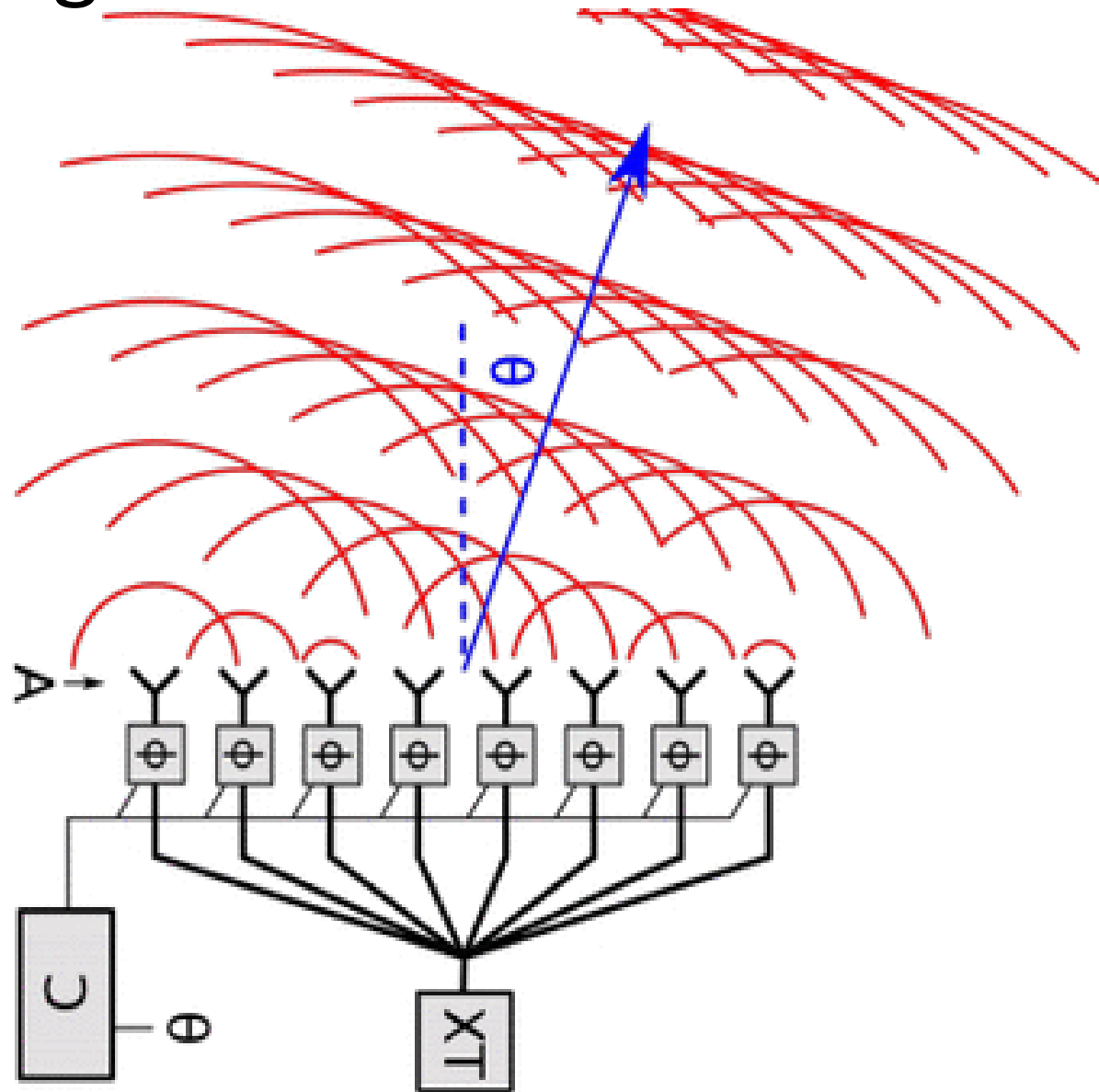
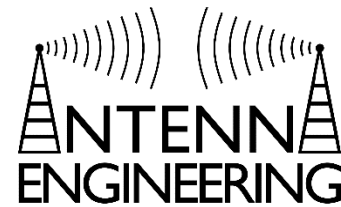


Figure 8-2  
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# Linear Array: Uniform Amplitude and Spacing



# Linear Array: Uniform Amplitude and Spacing



$$AF = 1 + e^{+j(kd \cos(\theta) + \beta)} + e^{+j2(kd \cos(\theta) + \beta)} + \dots + e^{+j(N-1)(kd \cos(\theta) + \beta)}$$

$$AF = \sum_{n=1}^N e^{+j(n-1)(kd \cos(\theta) + \beta)}$$

$$AF = \sum_{n=1}^N e^{+j(n-1)\Psi}$$

$$\Psi = kd \cos(\theta) + \beta$$

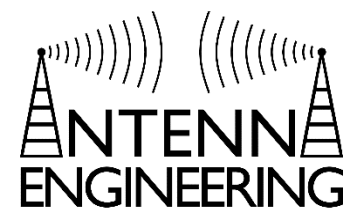
Another useful expression is the closed form expression of the array factor.

Multiply by  $e^{j\Psi}$   $(AF)e^{j\Psi} = e^{j\Psi} + e^{j2\Psi} + e^{j3\Psi} + \dots + e^{jN\Psi}$

Subtract AF summation  $AF(e^{j\Psi} - 1) = (-1 + e^{jN\Psi})$

Simplify 
$$AF = \frac{e^{jN\Psi}}{e^{j\Psi} - 1} = e^{j\left[\frac{N-1}{2}\right]\Psi} \left[ \frac{e^{j\left(\frac{N}{2}\right)\Psi} - e^{-j\left(\frac{N}{2}\right)\Psi}}{e^{j\left(\frac{1}{2}\right)\Psi} - e^{-j\left(\frac{1}{2}\right)\Psi}} \right] = e^{j\left[\frac{N-1}{2}\right]\Psi} \left[ \frac{\sin\left(\frac{N}{2}\Psi\right)}{\sin\left(\frac{1}{2}\Psi\right)} \right]$$

# Linear Array: Uniform Amplitude and Spacing



$$AF = \left[ \frac{\sin\left(\frac{N}{2}\Psi\right)}{\sin\left(\frac{1}{2}\Psi\right)} \right] \cong \left[ \frac{\sin\left(\frac{N}{2}\Psi\right)}{\frac{\Psi}{2}} \right]$$

$$AF_n = \frac{1}{N} \left| \frac{\sin\left(\frac{N}{2}\Psi\right)}{\frac{\Psi}{2}} \right| \cong \left| \frac{\sin\left(\frac{N}{2}\Psi\right)}{\frac{N}{2}\Psi} \right| \text{ for small values of } \Psi$$

$$\Psi = kd \cos(\theta) + \beta$$

The nulls are given by setting the array factor to 0.

$$\sin\left(\frac{N}{2}\Psi\right) = 0 \gg \frac{N}{2}\Psi \Big|_{\theta=\theta_n} = \pm n\pi \gg \theta_n = \cos^{-1} \left[ \frac{\lambda}{2\pi d} \left( -\beta \pm \frac{2n}{N}\pi \right) \right]$$

$$n = 1, 2, 3, \dots (\text{null})$$

$$n \neq N, 2N, 3N, \dots (\text{maximum})$$

The number of nulls that can exist will be a function of the element separation  $d$  and phase excitation difference  $\beta$ .

# Linear Array: Rectangular Plot

First main maximum occurs when

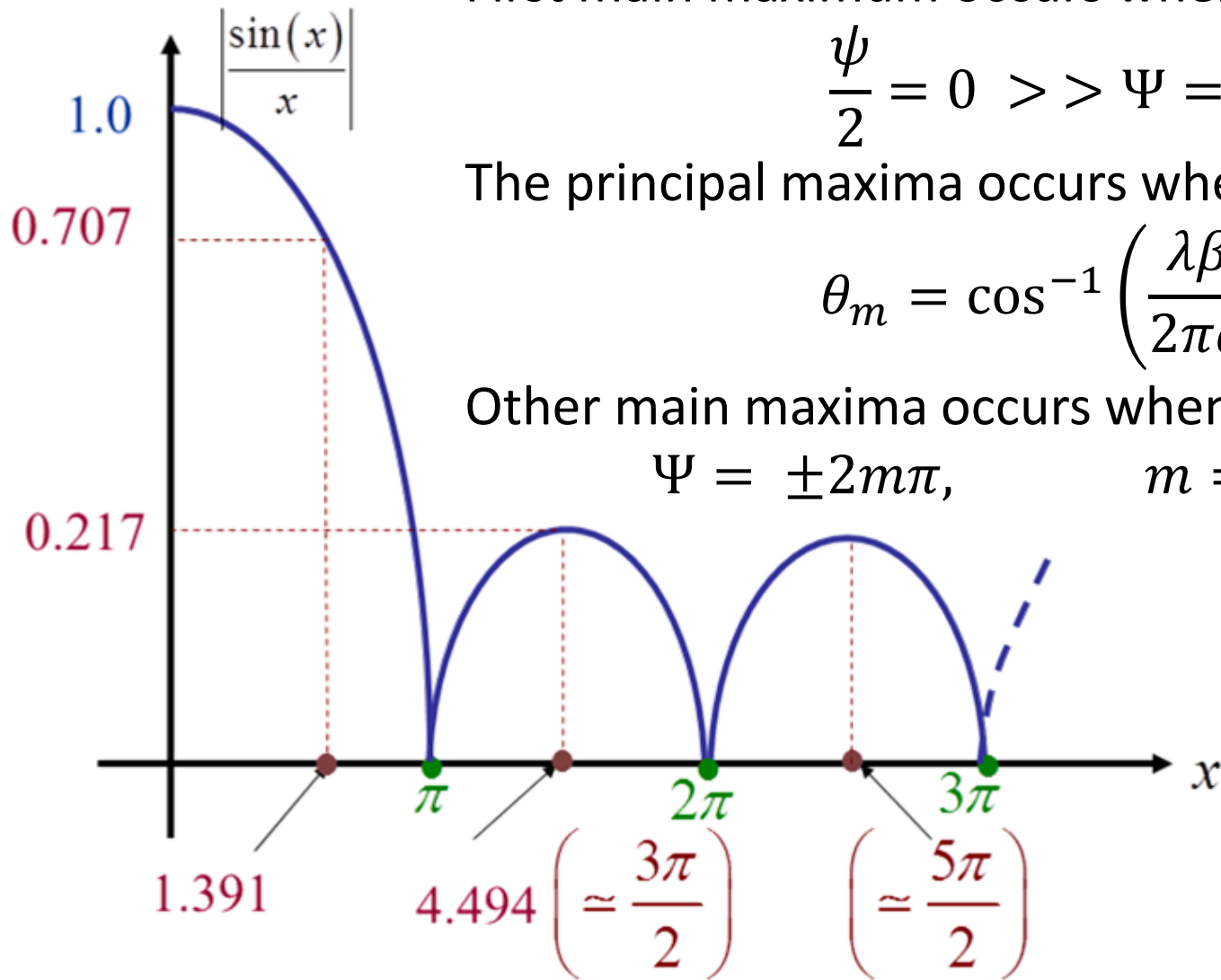
$$\frac{\psi}{2} = 0 \gg \Psi = 0$$

The principal maxima occurs when

$$\theta_m = \cos^{-1} \left( \frac{\lambda \beta}{2\pi d} \right)$$

Other main maxima occurs when

$$\Psi = \pm 2m\pi, \quad m = 1, 2, 3, \dots$$



# Linear Array: Rectangular Plot

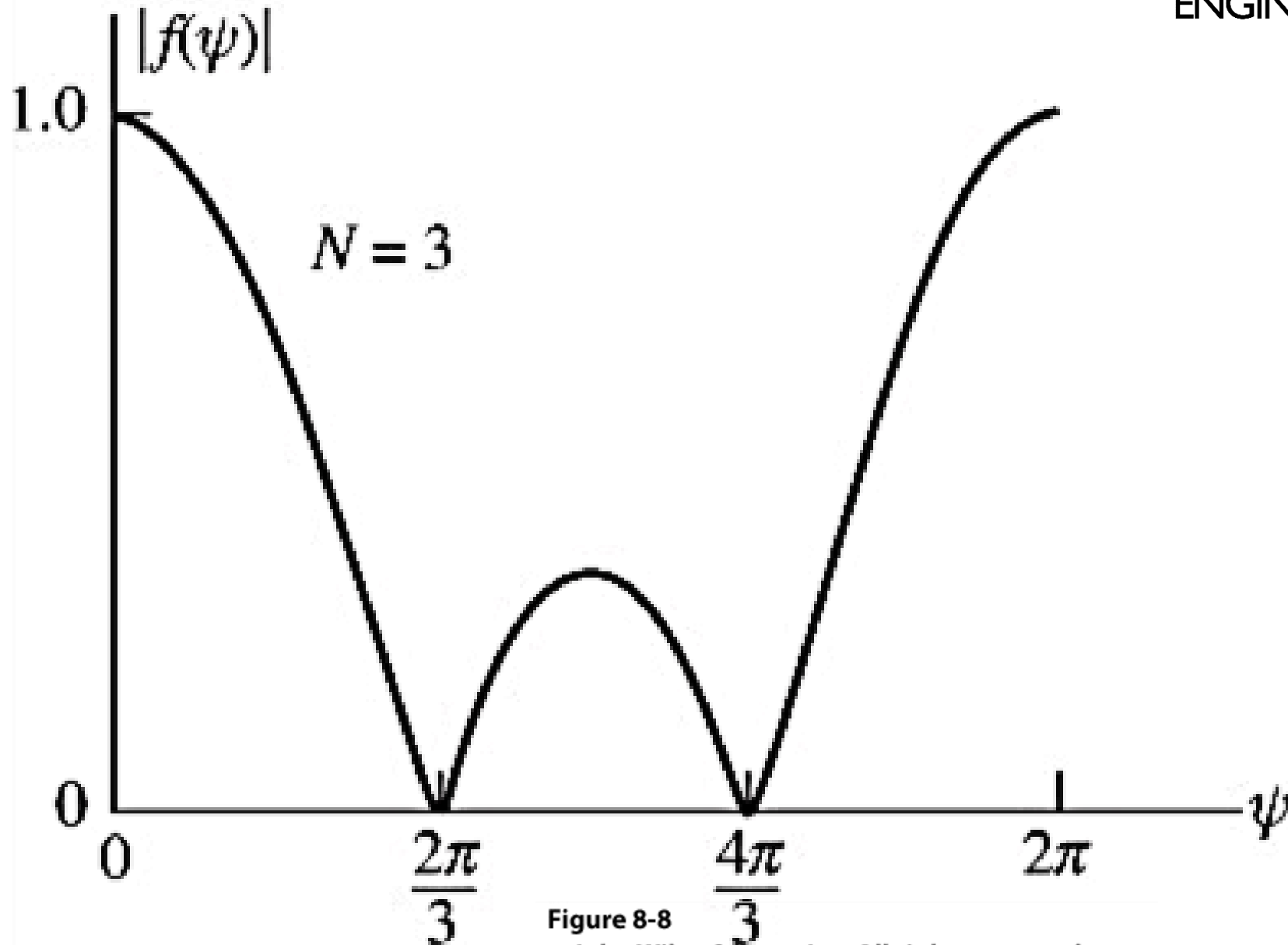


Figure 8-8

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# Linear Array: Rectangular Plot

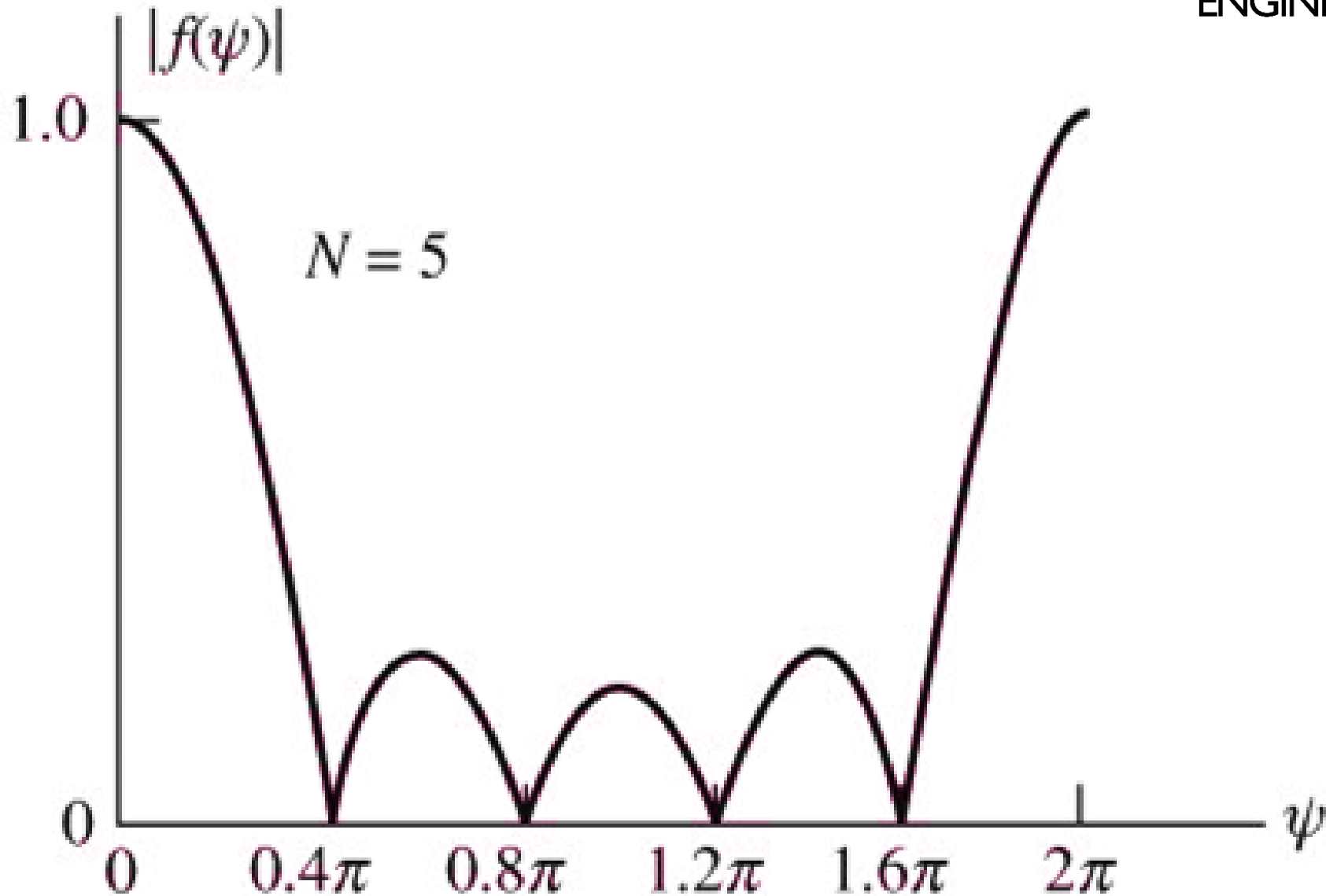


Figure 8-8

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# Linear Array: Rectangular Plot

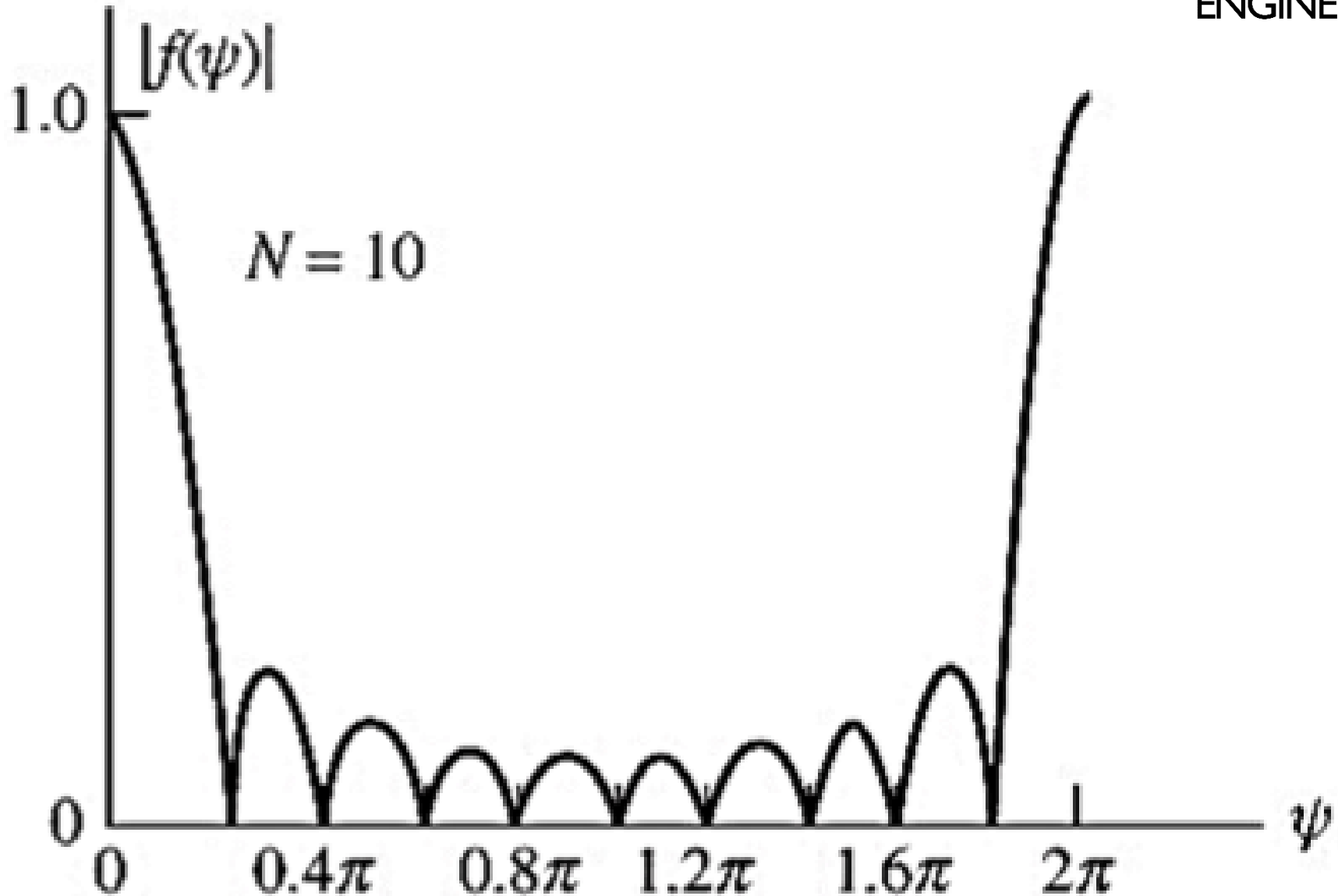
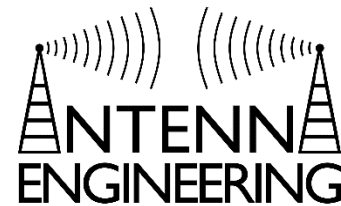


Figure 8-8

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# Linear Array: Rectangular Plot



Observations for rectangular plots of linear arrays with elements that are equally spaced, uniformly excited:

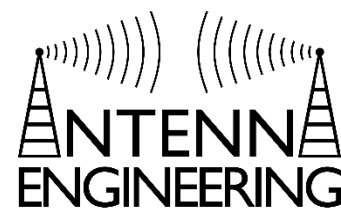
1. As  $N$  increases, the main lobe narrows
2. As  $N$  increases, there are more side lobes in one period of  $f(\Psi)$ . In fact, the number of full lobes (one main lobe and the side lobes) in one period of  $f(\Psi)$  equals  $N - 1$ . There are  $N - 2$  side lobes in each period.
3. The minor lobes are of width  $2\pi/N$  in the variable  $\Psi$  and the major lobes are twice this width.
4. The side lobe peaks decrease with increasing  $N$ .
5.  $|f(\Psi)|$  is symmetric about  $\pi$ .

Figure 8-8

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# Rectangular to Polar Graphical Solution

# Rectangular to Polar Graphical Solution



In antenna theory, many solutions are of the form

$$f(\zeta) = f(C \cos(\gamma) + \delta)$$

Where  $C$  and  $\delta$  are constants and  $\gamma$  is a variable. The approximate array factor of an  $N$ -element, uniform amplitude linear array is a  $\frac{\sin(\zeta)}{\zeta}$  where

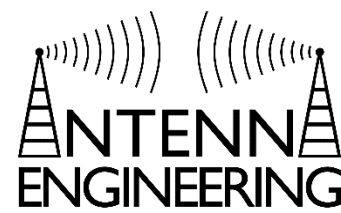
$$\zeta = C \cos(\gamma) + \delta = \frac{N}{2} \Psi = \frac{N}{2} (kd \cos(\theta) + \beta)$$

$$C = \frac{N}{2} kd$$

$$\delta = \frac{N}{2} \beta$$

The  $f(\zeta)$  function can be plotted in rectilinear coordinates, and transferred to a polar graph.

# Rectangular to Polar Graphical Solution



The procedure that must be followed in the construction of the polar graph is as follows:

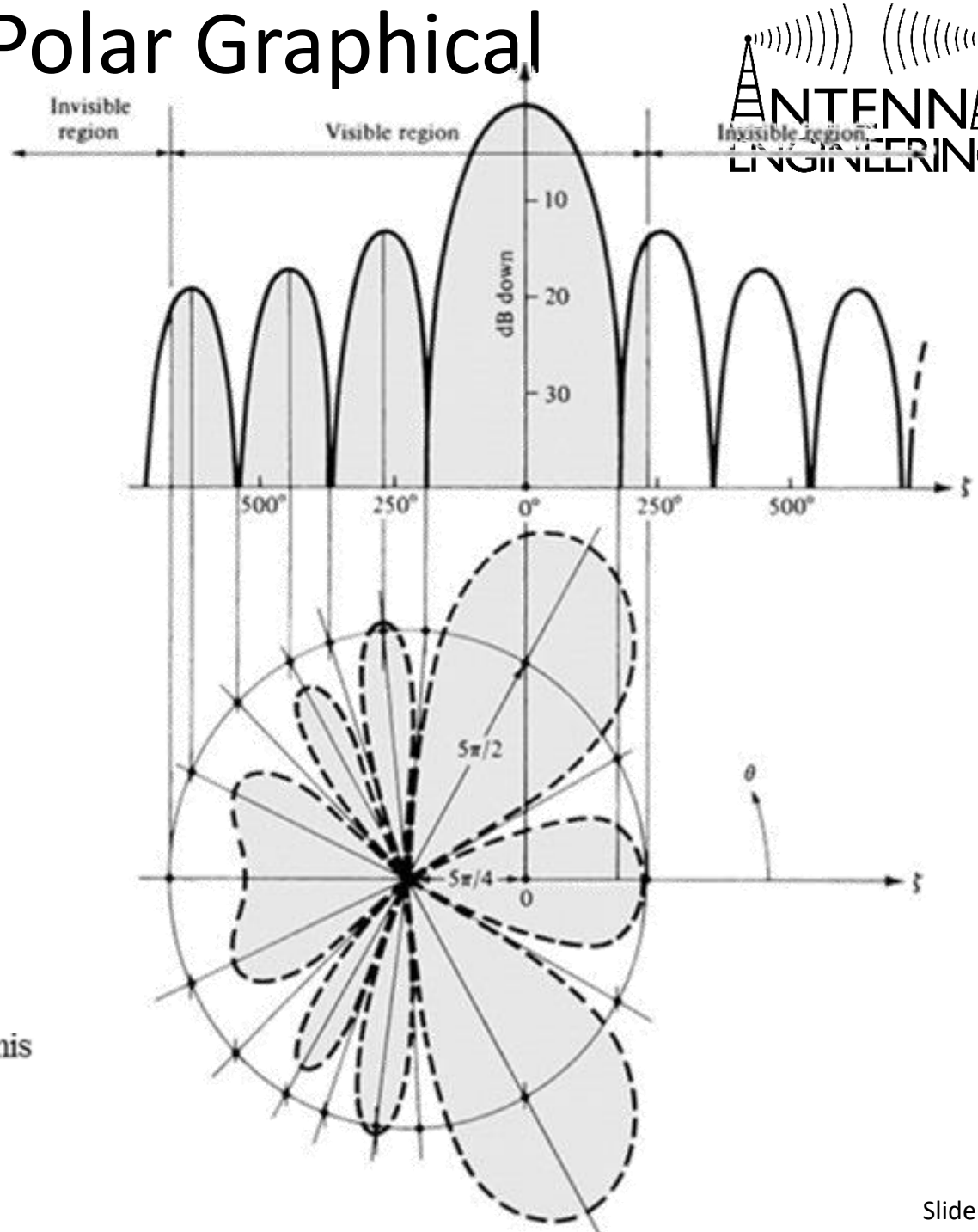
1. Plot, using rectilinear coordinates, the function  $|f(\zeta)|$ .
2. a) Draw a circle with radius  $C$  and its center on the abscissa at  $\zeta = \delta$ 
  - b) Draw vertical lines to the abscissa so that they will intersect the circle.
  - c) From the center of the circle, draw radial lines through the points of the circle intersected by the vertical lines.
  - d) Along radial lines, mark off corresponding magnitudes from the linear plot.
  - e) Connect all points to form a continuous graph.

# Rectangular to Polar Graphical Solution



$$\text{Radius} = C = \frac{N}{2}kd$$

$$\text{Center} = \zeta = \delta = \frac{N}{2}\beta$$

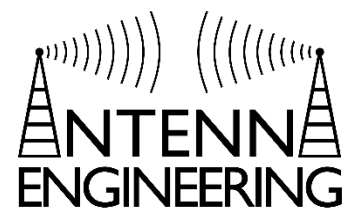


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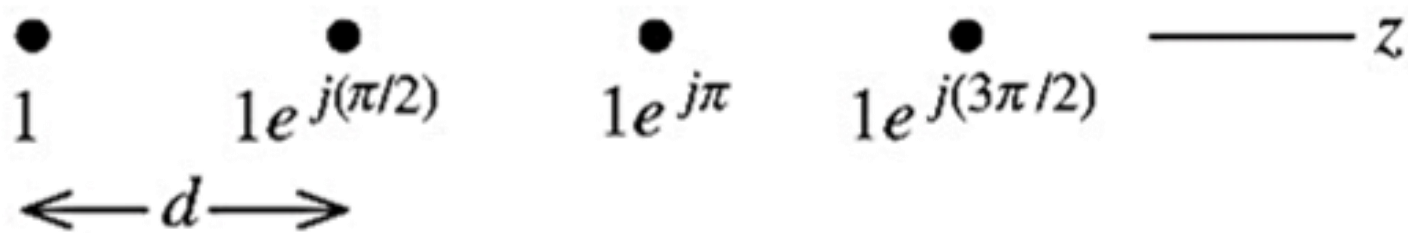
## Chapter 6

*Arrays: Linear, Planar, & Circular*

# Four element linear array - Example



Find and plot the array factor of a four-element, uniformly excited, equally spaced array. The spacing is  $\lambda/2$  and  $90^\circ$  interelement phasing (i.e.  $\beta = \pi/2$ ).



# Four element linear array - Example

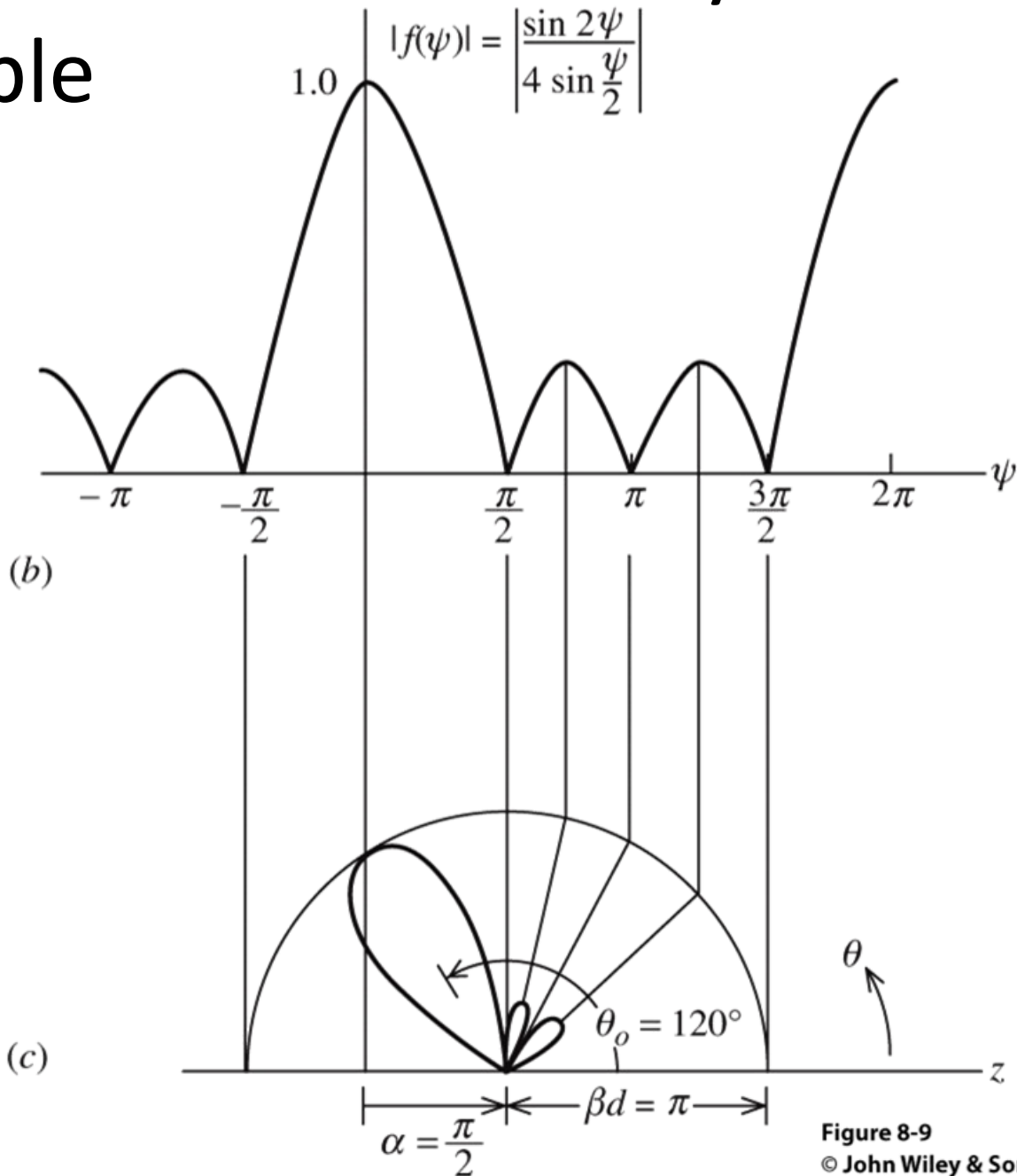
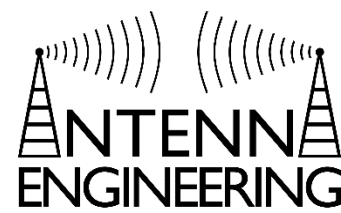


Figure 8-9

# Two element linear array - Examples



Find and plot the array factor of a two-element, isotropic, equally spaced array with distance  $d = \lambda/2$  and uniform phase excitation  $\alpha = 0^\circ$

Find and plot the same array factor of a two-element, isotropic, equally spaced array with distance  $d = \lambda/2$  but with phase excitation  $\alpha = 180^\circ$

Find and plot the same array factor of a two-element, isotropic, equally spaced array with distance  $d = \lambda/4$  but with phase excitation  $\alpha = -90^\circ$

# Two element linear array - Examples

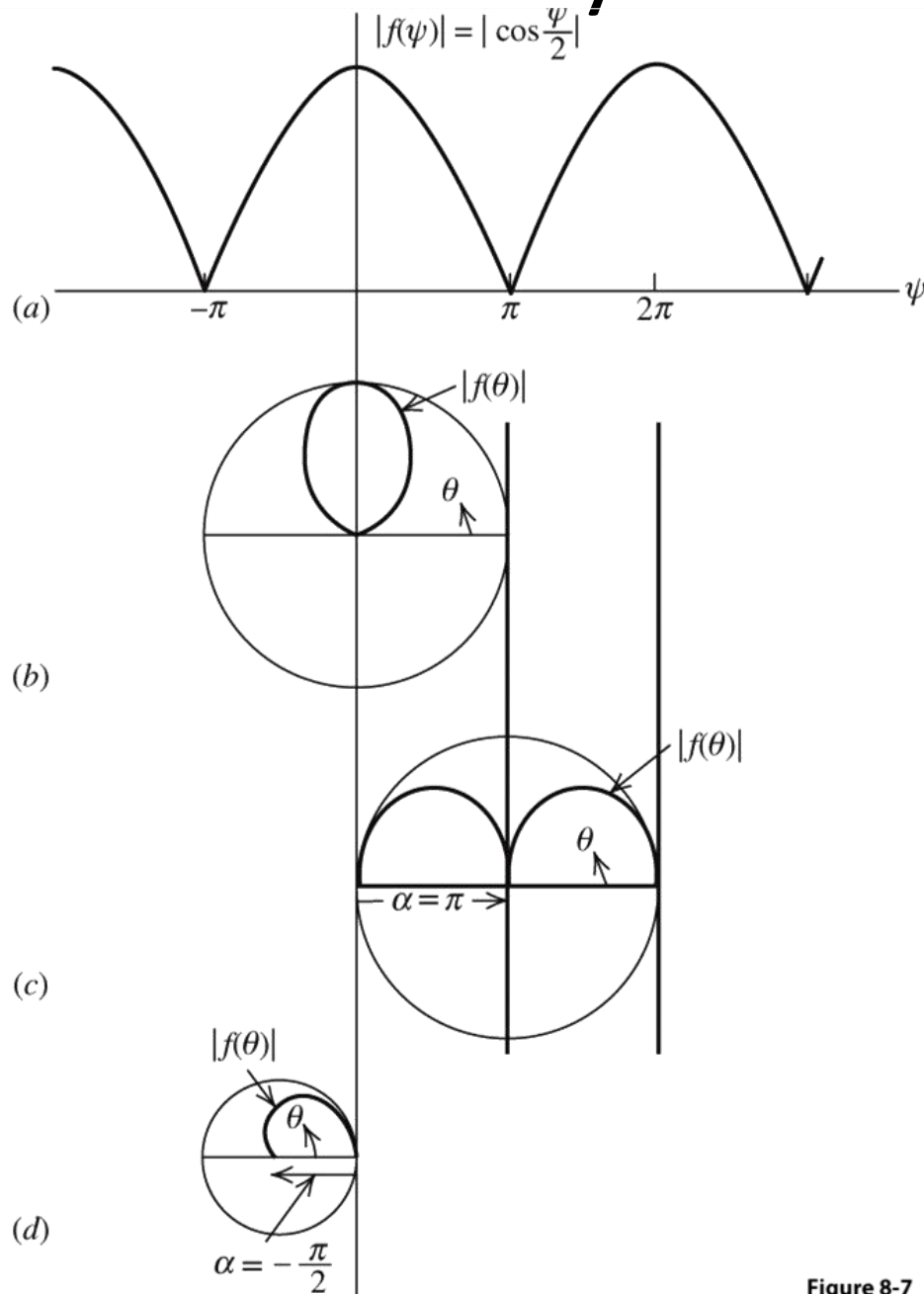
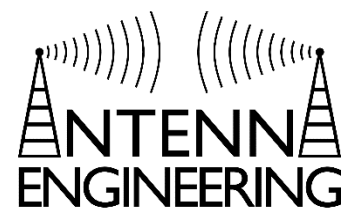


Figure 8-7

# Five element Endfire Linear Array - Examples



Find and plot the array factor of a five-element, isotropic, equally spaced array with distance  $d = 0.45\lambda$  and uniform phase excitation  $\alpha = 0.9\pi$

Find and plot the array factor of a five-element, isotropic, equally spaced array with distance  $d = 0.5\lambda$  and uniform phase excitation  $\alpha = \pi$

# Five element linear array - Examples

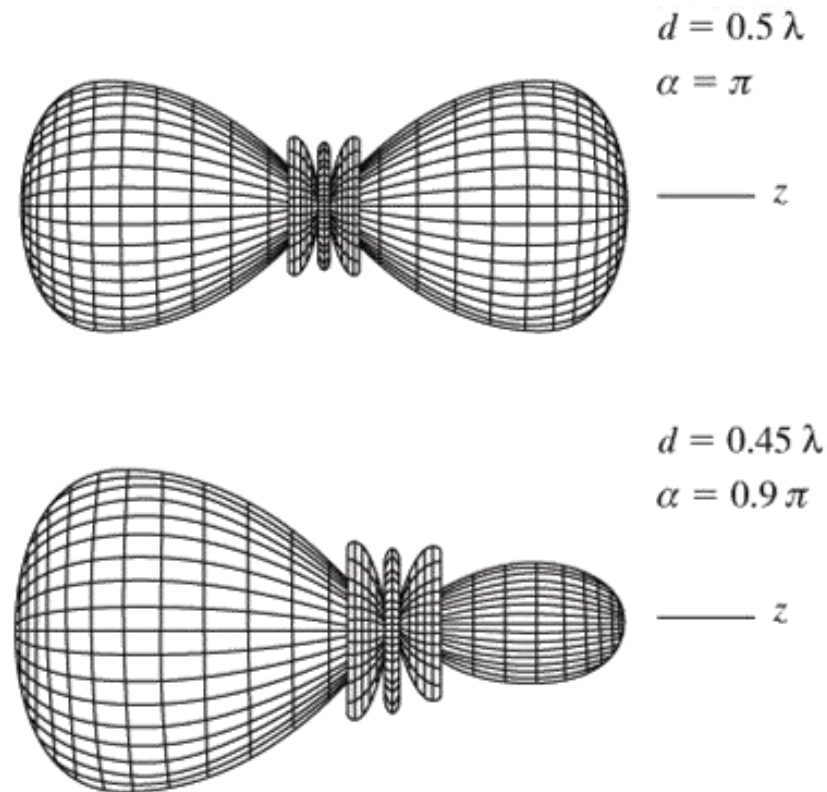
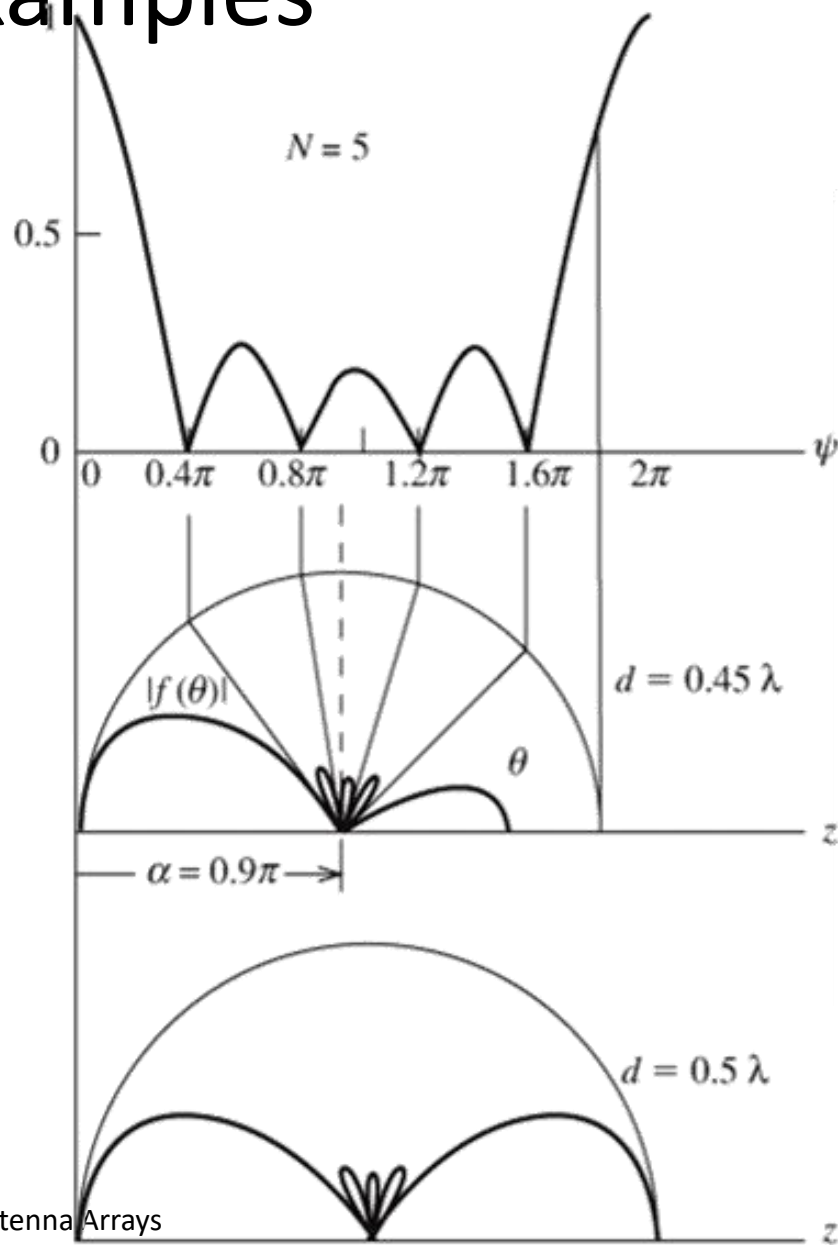


Figure 8-10  
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# Broadside Array

In many applications it is desirable to have the maximum radiation of an array directed normal to the axis of the array ( $\theta = 90^\circ$ ). To optimize this design, both the maxima of the single element and the array factor should be both directed toward  $\theta = 90^\circ$ .

Recall the maximum of the array factor occurs when

$$\Psi = kd \cos(\theta) + \beta = 0$$

Since it is desired to have the first maximum directed toward  $\theta = 90^\circ$

$$\Psi = kd \cos(\theta) + \beta \Big|_{\theta=90^\circ} = \beta = 0$$

To have the maximum of the array factor in an uniform linear array directed to the broadside to the axis, all elements need to have the same phase excitation.

# Broadside Array

To ensure that there are no other maxima in other directions (grating lobes), the separation between the elements should not be equal to multiples of a wavelength ( $d \neq n\lambda, n = 1, 2, 3, \dots$ ) when  $\beta = 0$ .

If  $d = n\lambda, n = 1, 2, 3$ , and  $\beta = 0$ , then

$$\Psi = kd \cos(\theta) + \beta \Big|_{\substack{d=n\lambda \\ \beta=0 \\ n=1,2,3,\dots}} = 2\pi n \cos(\theta) \Big|_{\theta=0,\pi} = \pm 2n\pi \text{ **avoid this!**}$$

To avoid any grating lobes, the largest spacing between the elements should be **less** than one wavelength ( $d = \lambda$ )

# Broadside Array

Broadside ( $d = \lambda / 4$ )  $N = 10$

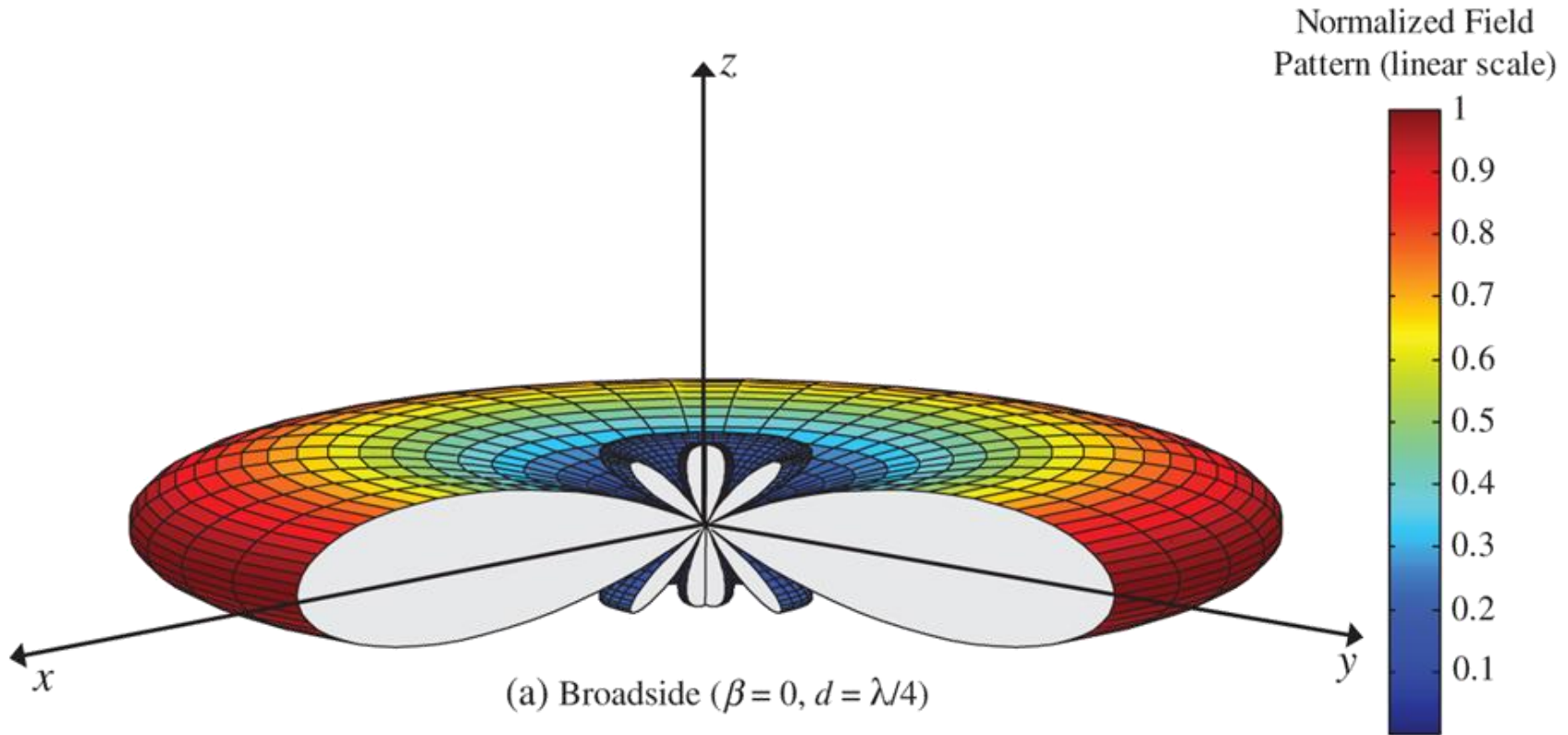
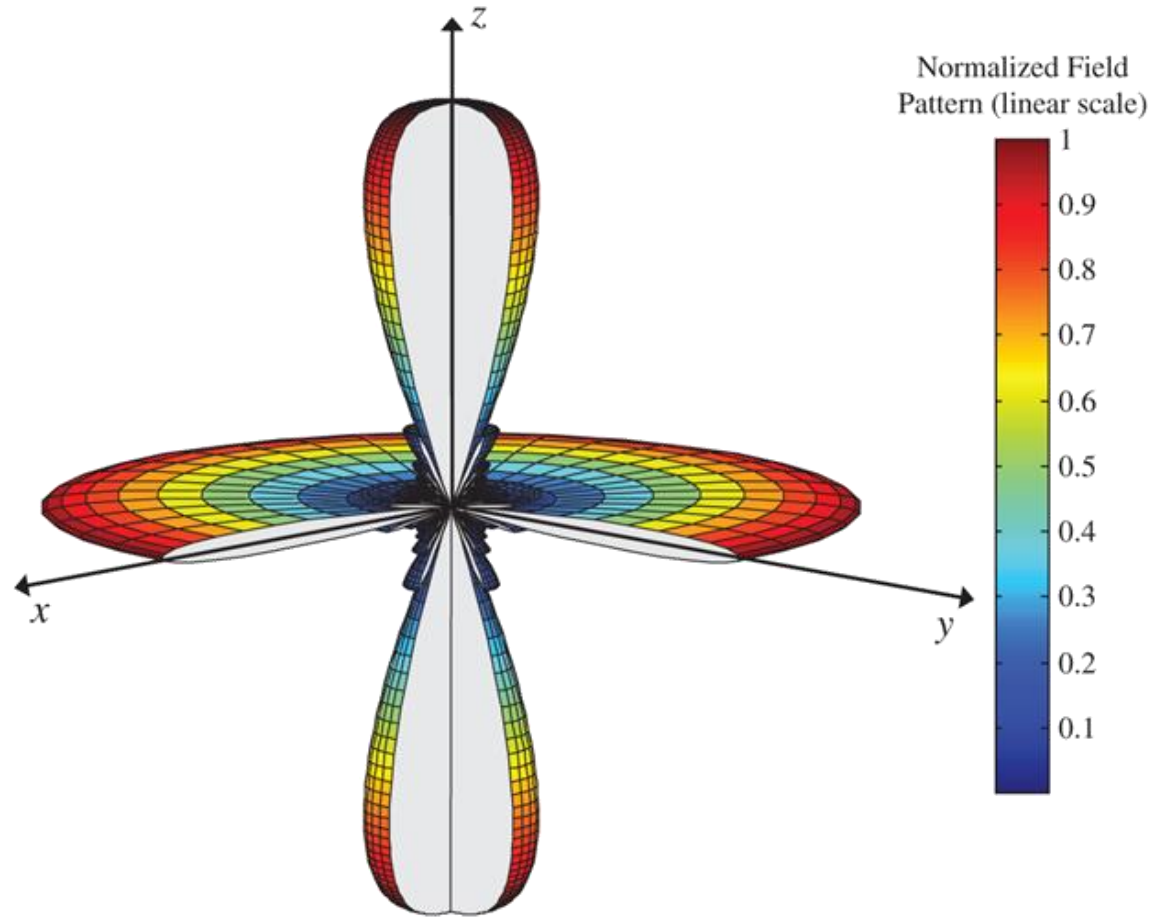


Fig. 6.6a

# Broadside Array

Broadside/End-Fire ( $d = \lambda, N = 10$ )

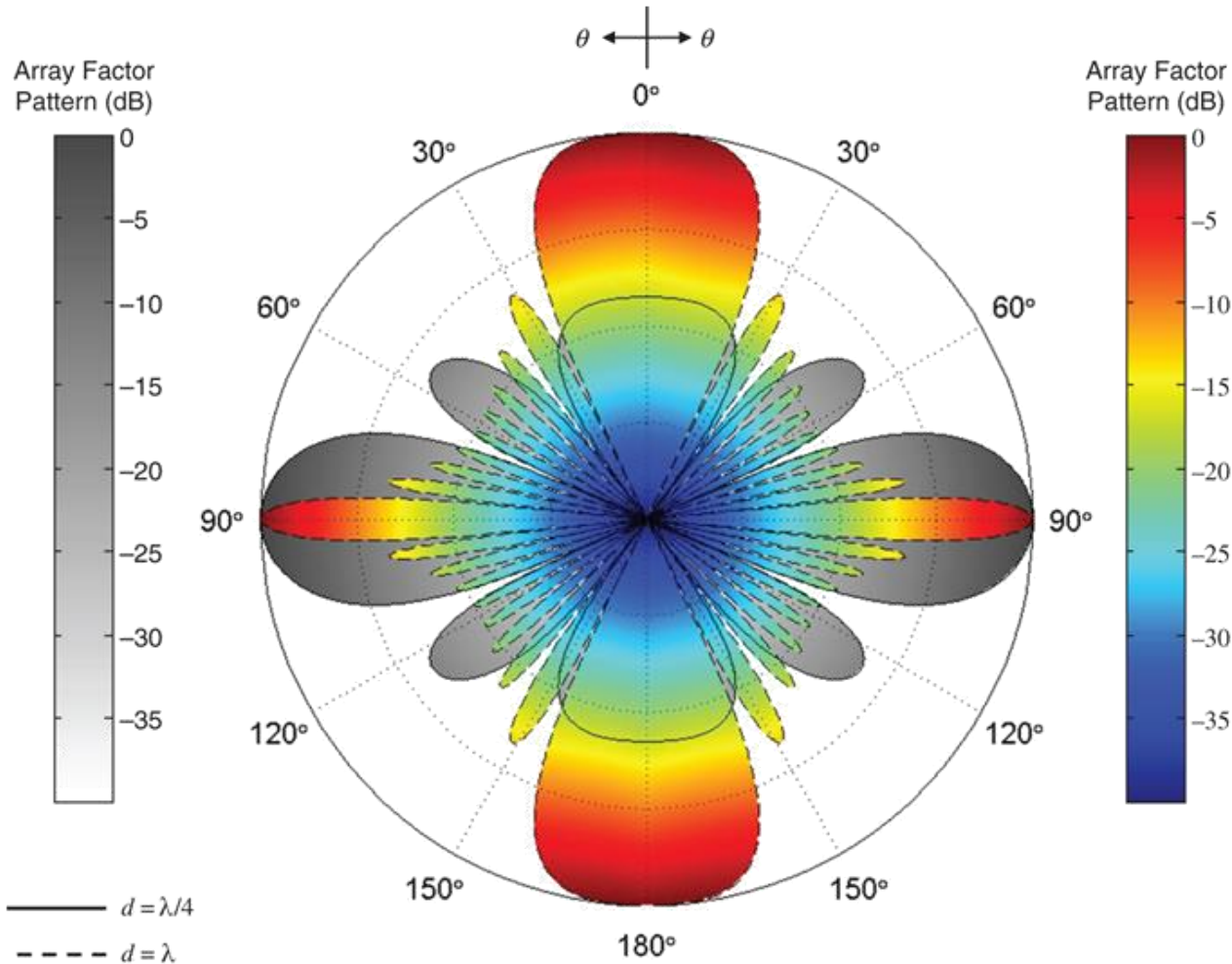


(b) Broadside/end-fire ( $\beta = 0, d = \lambda$ )

Fig. 6.6b

# Broadside Array

## Broadside Array ( $N = 10$ )



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Fig. 6.7

Chapter 6  
Arrays: Linear, Planar, & Circular

# Broadside Array

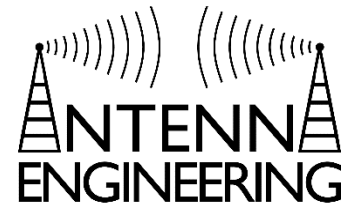
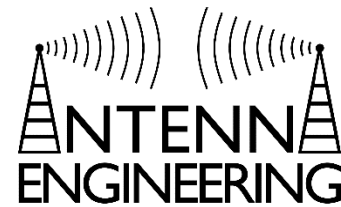


Table 6.1 NULLS, MAXIMA, HALF-POWER POINTS, AND MINOR LOBE MAXIMA FOR UNIFORM AMPLITUDE BROADSIDE ARRAYS

NULLS	$\theta_n = \cos^{-1} \left( \pm \frac{n \lambda}{N d} \right)$ $n = 1, 2, 3, \dots$ $n \neq N, 2N, 3N, \dots$
MAXIMA	$\theta_m = \cos^{-1} \left( \pm \frac{m \lambda}{d} \right)$ $m = 0, 1, 2, \dots$
HALF-POWER POINTS	$\theta_h \approx \cos^{-1} \left( \pm \frac{1.391 \lambda}{\pi N d} \right)$ $\pi d / \lambda \ll 1$
MINOR LOBE MAXIMA	$\theta_s \approx \cos^{-1} \left[ \pm \frac{\lambda}{2d} \left( \frac{2s + 1}{N} \right) \right]$ $s = 1, 2, 3, \dots$ $\pi d / \lambda \ll 1$

# Ordinary End-Fire Array



Instead of having the maximum radiation broadside to the axis of an array, it may be desirable to direct it along the axis of the array (end-fire). Sometimes it may be desirable that it radiates toward only one direction ( $\theta = 0^\circ, 180^\circ$ )

For the maximum toward  $\theta = 0^\circ$ :

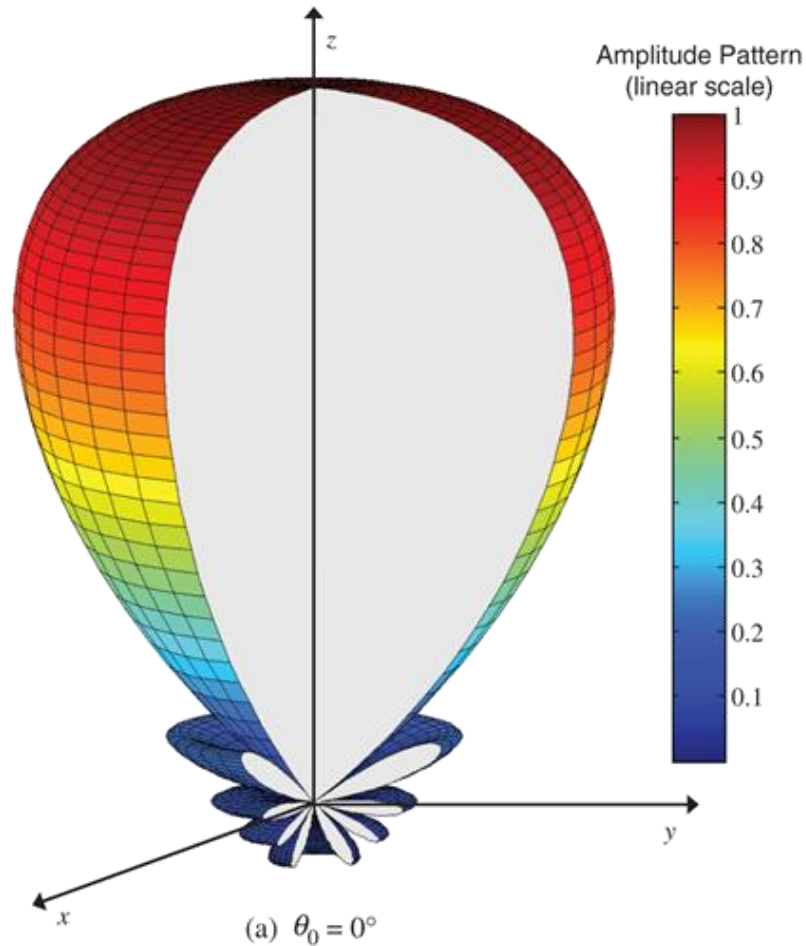
$$\Psi = kd \cos(\theta) + \beta \Big|_{\theta=0^\circ} = kd + \beta = 0 \gg \boxed{\beta = -kd}$$

For the maximum toward  $\theta = 180^\circ$ :

$$\Psi = kd \cos(\theta) + \beta \Big|_{\theta=180^\circ} = -kd + \beta = 0 \gg \boxed{\beta = kd}$$

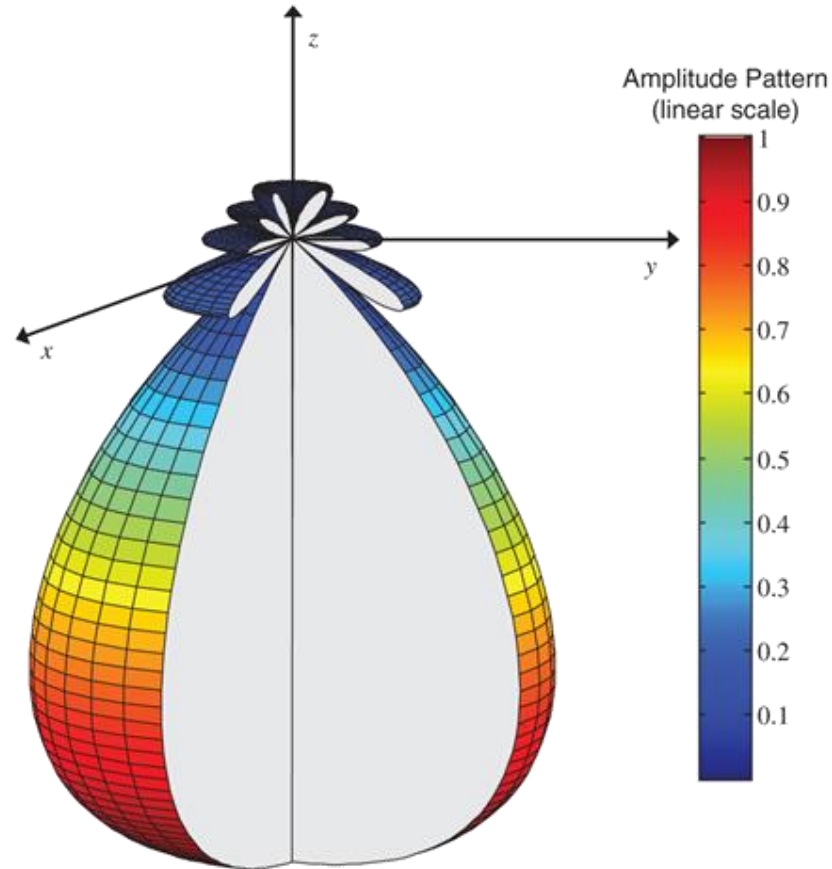
# Ordinary End-Fire Array

End-Fire ( $d = \lambda / 4$ )  $N = 10$



# Ordinary End-Fire Array

End-Fire ( $d = \lambda / 4$ )  $N = 10$



(b)  $\theta_0 = 180^\circ$

# Ordinary End-Fire Array

End-Fire ( $d = \lambda / 4$ )  $N = 10$

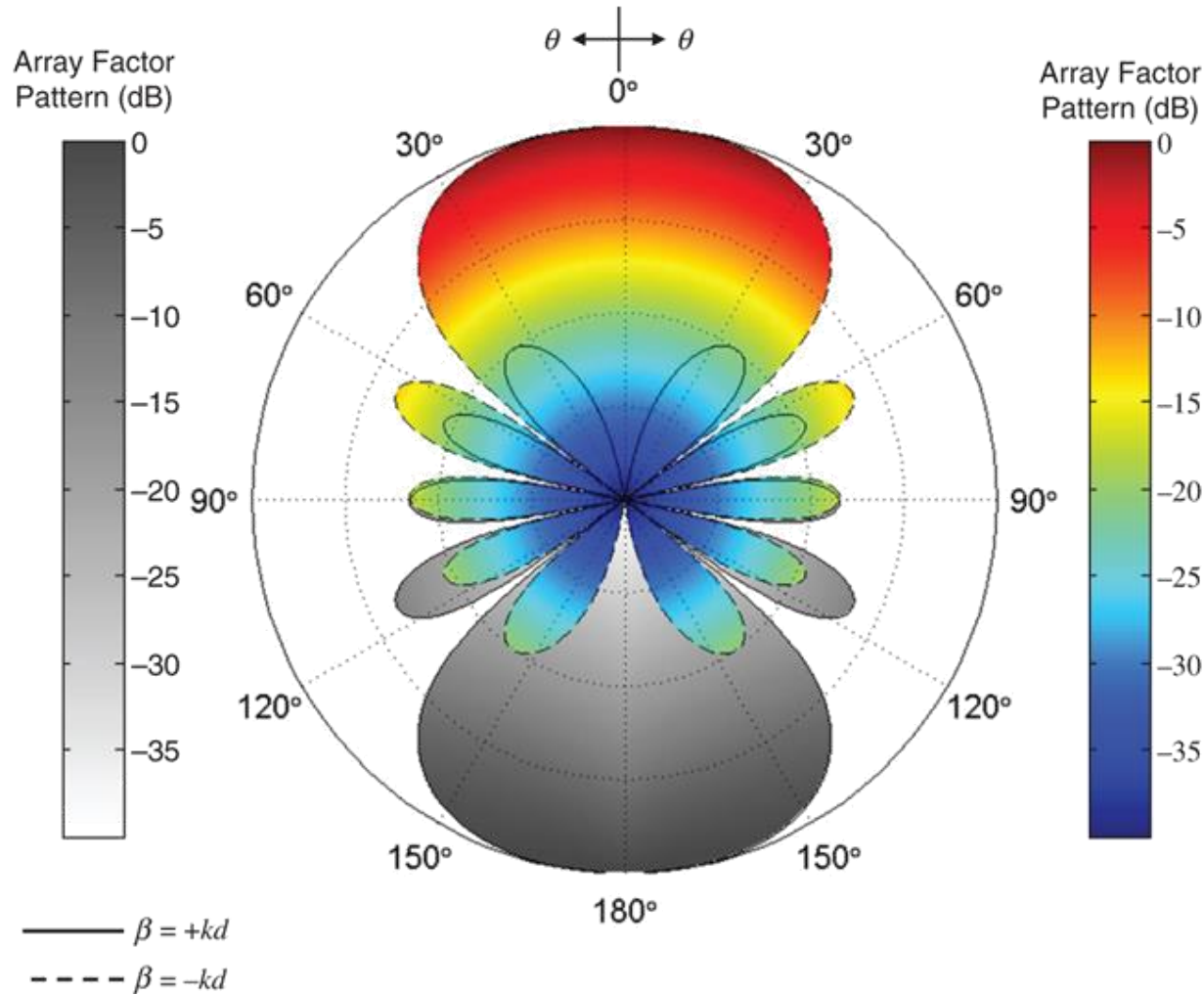
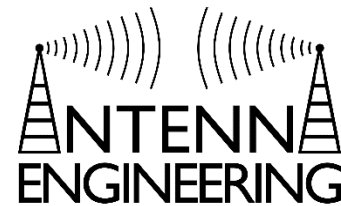


Fig. 6.9

# Ordinary End-Fire Array

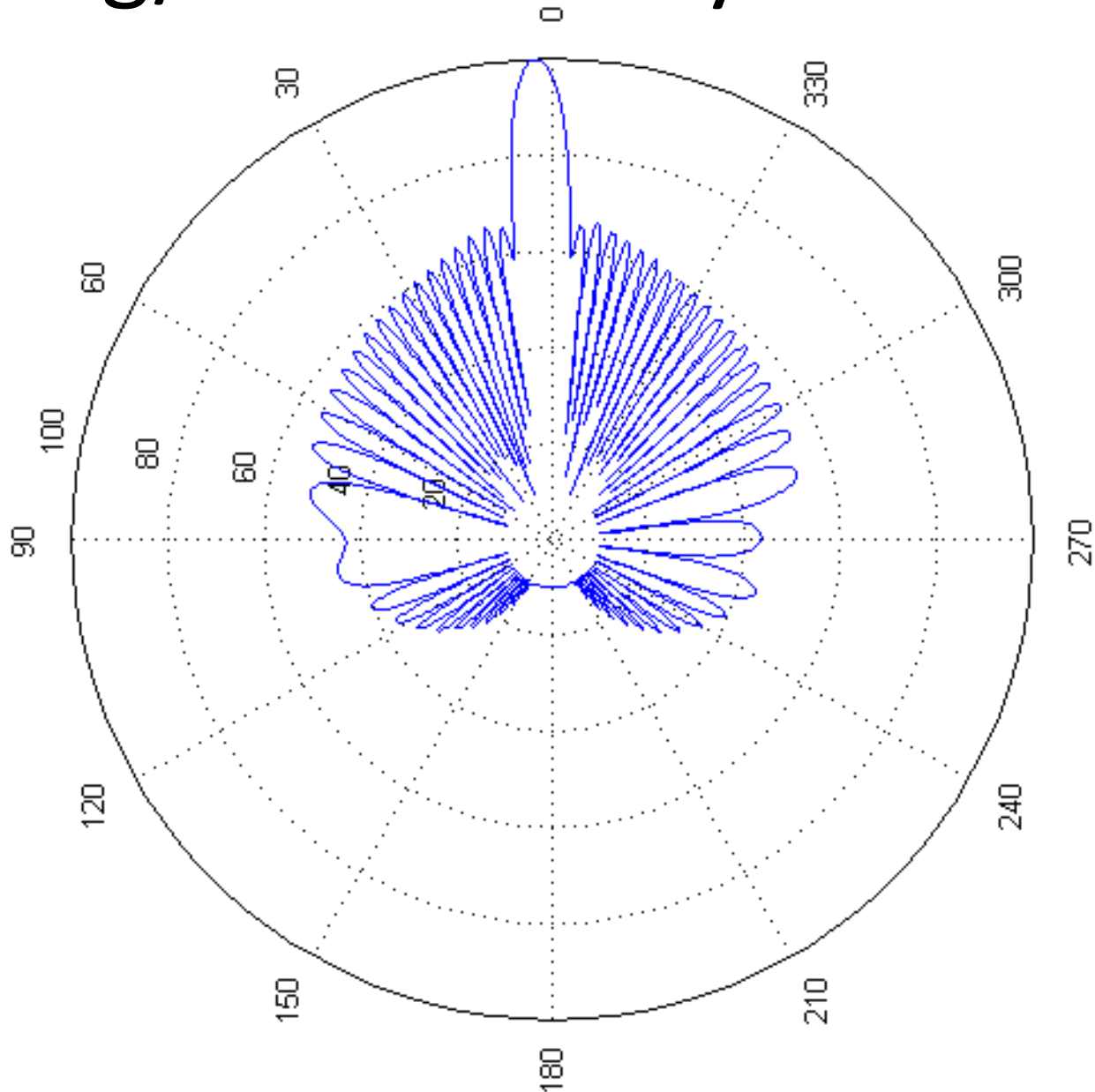


1.  $d < \lambda/2$ : End-fire only in one direction  
( $\theta = 0^\circ$  or  $180^\circ$ )
2.  $d = \lambda/2$ : End-fire in both directions  
( $\theta = 0^\circ$  &  $180^\circ$ )
3.  $\lambda/2 < d < \lambda$ : End-fire in one direction  
( $\theta = 0^\circ$  or  $180^\circ$ ) and maximum  
toward two other directions
4.  $d = \lambda = n\lambda$ : End-fire in both directions  
( $\theta = 0^\circ$  &  $180^\circ$ ) and broadside

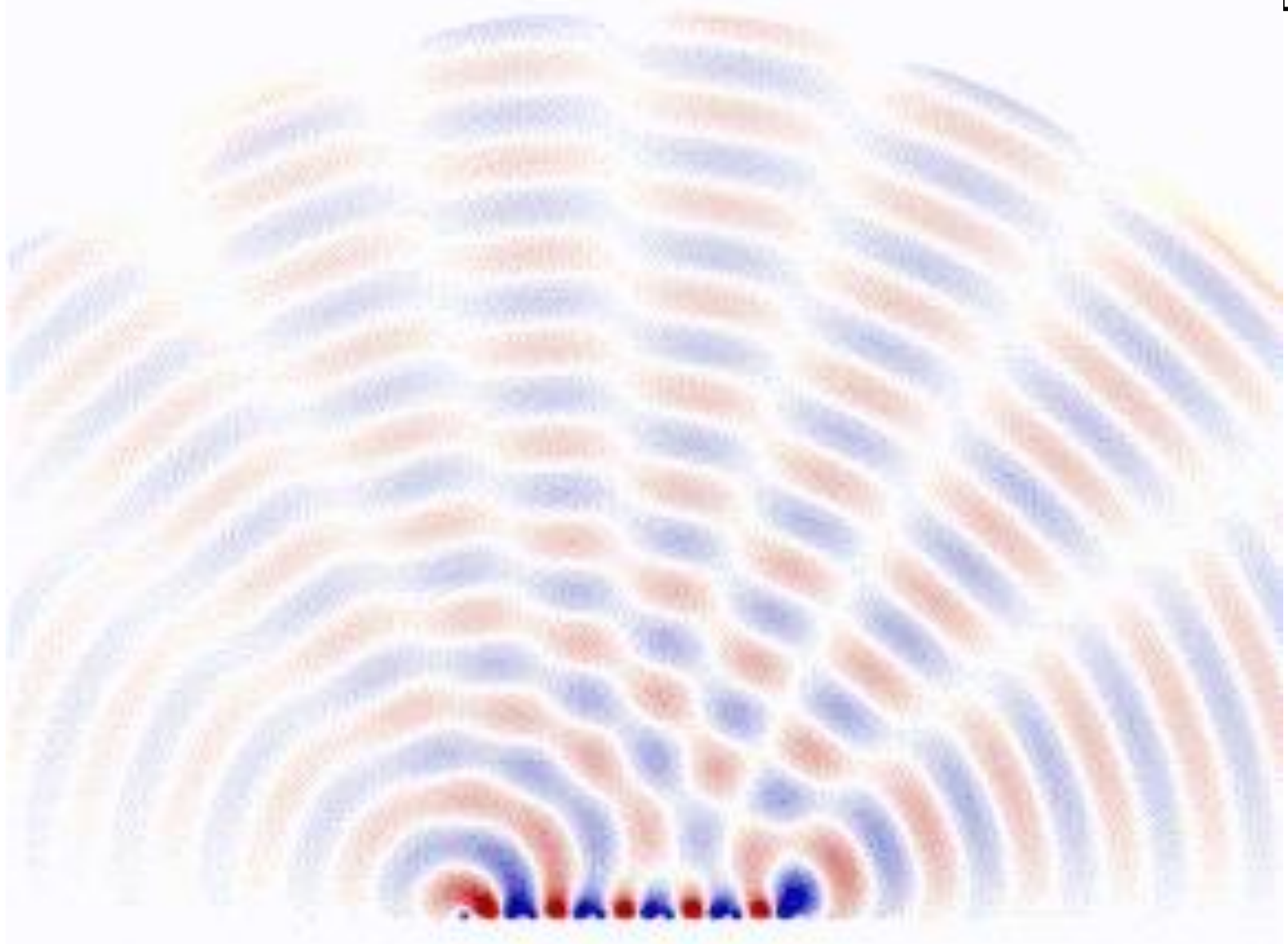
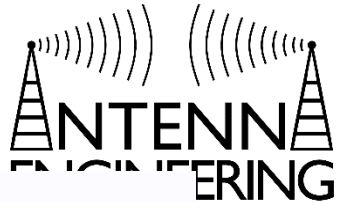
**Table 6.7** Maximum Element Spacing  $d_{max}$  to Maintain Either One or Two Amplitude Maxima of a Linear Array

Array	Distribution	Type	Direction of Maximum	Element Spacing
Linear	Uniform	Broadside	$\theta_o = 90^\circ$ only $\theta_o = 0^\circ, 90^\circ, 180^\circ$ simultaneously	$d_{max} < \lambda$ $d = \lambda$
Linear	Uniform	Ordinary End-Fire	$\theta_o = 0^\circ$ only $\theta_o = 180^\circ$ only $\theta_o = 0^\circ, 90^\circ, 180^\circ$ simultaneously	$d_{max} < \lambda/2$ $d_{max} < \lambda/2$ $d = \lambda$
Linear	Uniform	Hansen-Woodyard End-Fire	$\theta_o = 0^\circ$ only $\theta_o = 180^\circ$ only	$d = \lambda/4$ $d = \lambda/4$
Linear	Uniform	Scanning	$\theta_o = \theta_{max}$ $0 < \theta_o < 180^\circ$	$d_{max} < \lambda$
Linear	Nonuniform	Binomial	$\theta_o = 90^\circ$ only $\theta_o = 0^\circ, 90^\circ, 180^\circ$ simultaneously	$d_{max} < \lambda$ $d = \lambda$
Linear	Nonuniform	Dolph-Tschebyscheff	$\theta_o = 90^\circ$ only $\theta_o = 0^\circ, 90^\circ, 180^\circ$ simultaneously	$d_{max} \leq \frac{\lambda}{\pi} \cos^{-1} \left( -\frac{1}{z_o} \right)$ $d = \lambda$
Planar	Uniform	Planar	$\theta_o = 0^\circ$ only $\theta_o = 0^\circ, 90^\circ$ and $180^\circ$ ; $\phi_o = 0^\circ, 90^\circ, 180^\circ, 270^\circ$ simultaneously	$d_{max} < \lambda$ $d = \lambda$

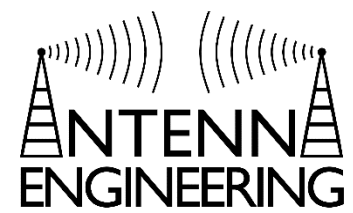
# Scanning/Phased Array



# Scanning/Phased Array



# Scanning/Phased Array



# Hansen-Woodyard End-Fire Array

We discussed the conditions to have an ordinary end-fire array in the previous sections.

In order to enhance the directivity of an end-fire array without destroying any of the other characteristics, Hansen and Woodyard proposed in 1938 proposed that the required phase shift between closely spaced elements of a very long array should be

For the maximum toward  $\theta = 0^\circ$ :

$$\beta = -\left(kd + \frac{2.92}{N}\right) \cong -\left(kd + \frac{\pi}{N}\right)$$

For the maximum toward  $\theta = 180^\circ$ :

$$\beta = -\left(kd + \frac{2.92}{N}\right) \cong +\left(kd + \frac{\pi}{N}\right)$$

For both directions, spacing should be

$$d = \left(\frac{N-1}{N}\right)\frac{\lambda}{4} \cong \frac{\lambda}{4} \text{ for large } N$$

# Hansen-Woodyard End-Fire Array

$$N = 10$$

$$d = \lambda / 4$$

$$\beta = -(kd + \pi/N)$$

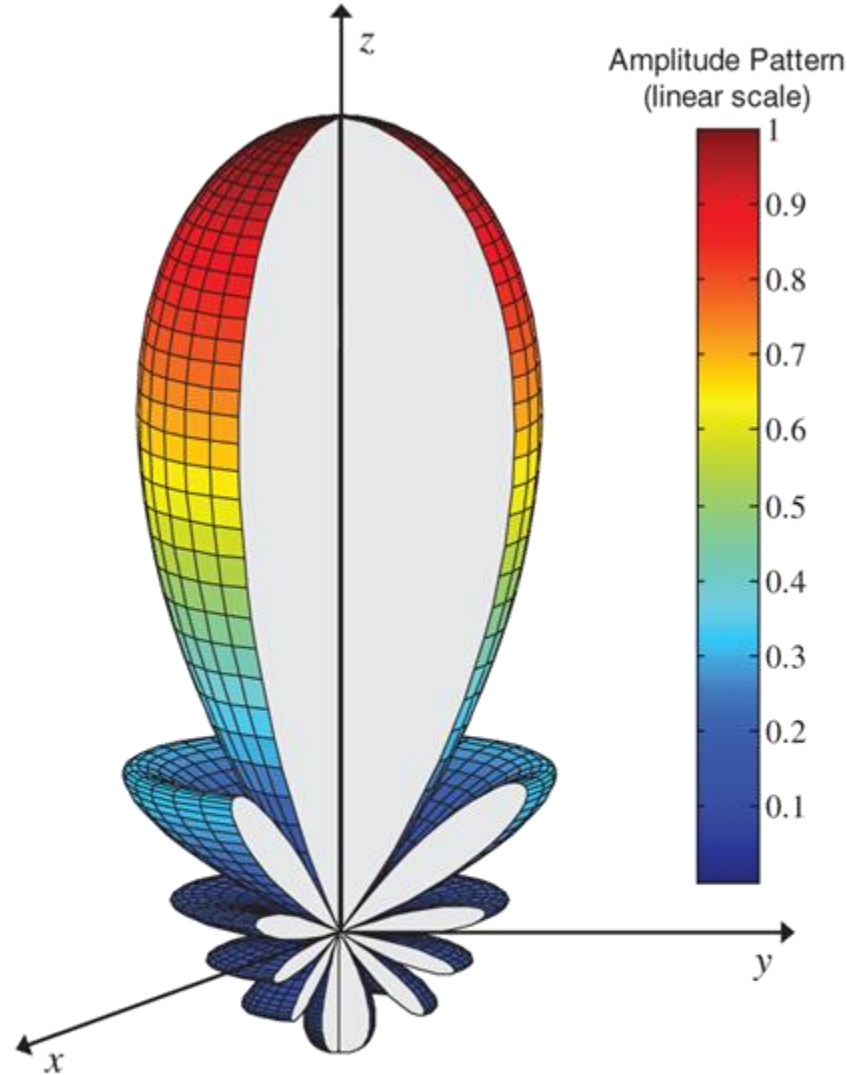


Fig. 6.13(b)

(b) Hansen-Woodyard

# Hansen-Woodyard End-Fire Array

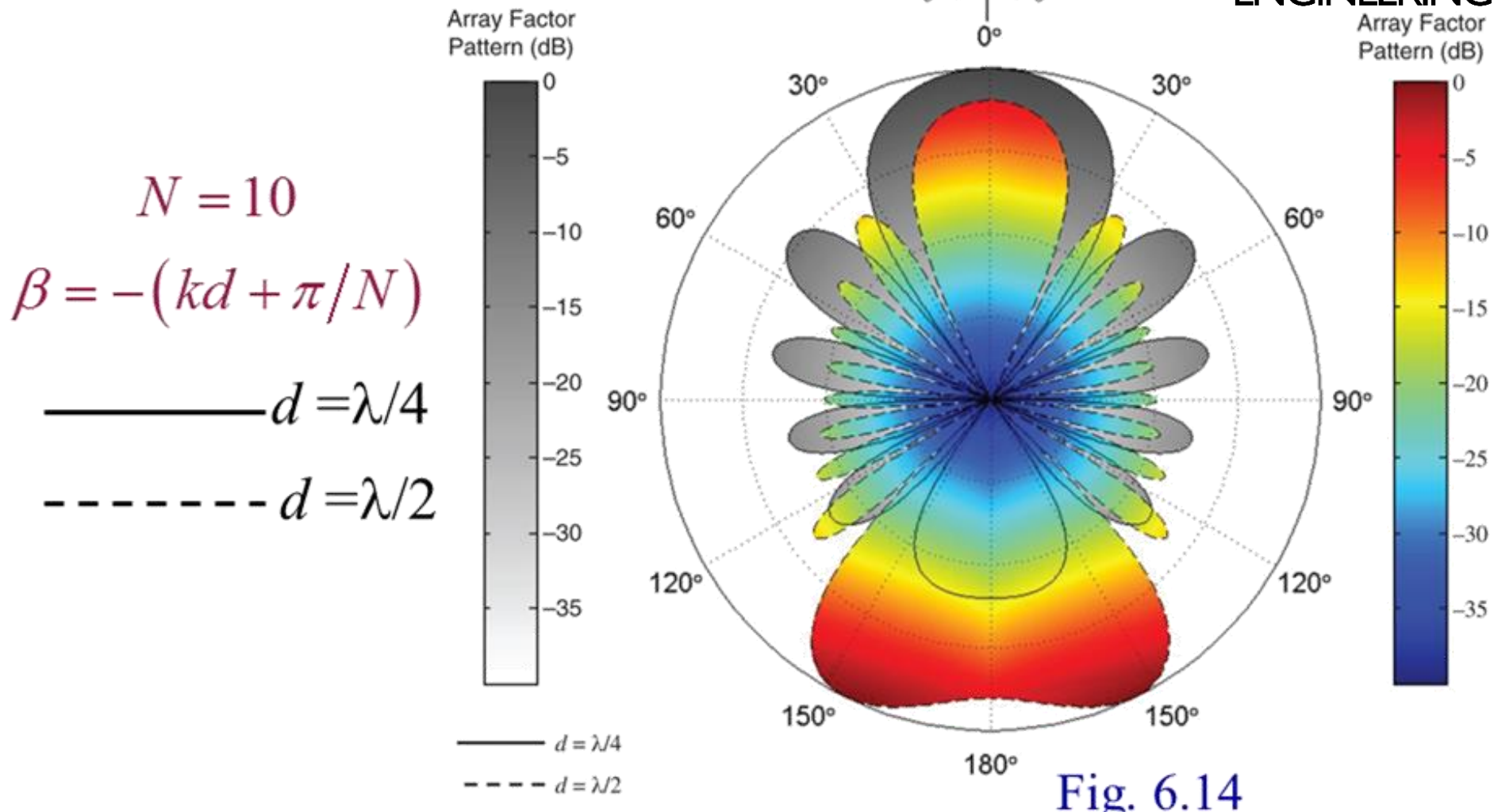
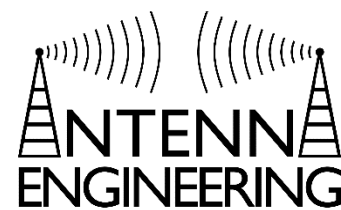


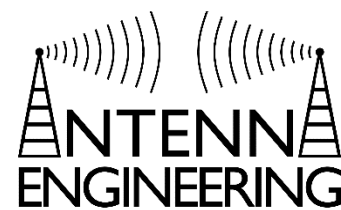
Fig. 6.14

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Chapter 6  
*Arrays: Linear, Planar, & Circular*

# Linear Arrays - Summary

## Linear Arrays



### 1. Broadside

$$(\theta_m = 90^\circ)$$

### 2. Ordinary End-Fire

$$(\theta_m = 0^\circ, 180^\circ)$$

### 3. Phased (Scanning)

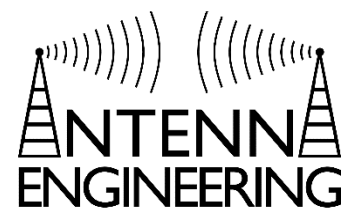
$$(0^\circ \leq \theta_m \leq 180^\circ)$$

### 4. Hansen-Woodyard End-fire

$$(\theta_m = 0^\circ, 180^\circ)$$

# Linear Arrays - Summary

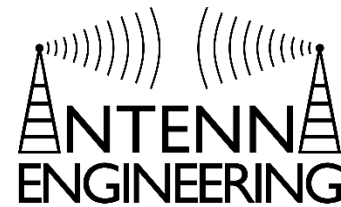
## Linear Array Phase Excitation



Array Type	Phase Excitation ( $\beta$ )
Broadside	$\beta = 0, \pm 2n\pi, n = 0, 1, ..$
Ordinary End-Fire	$\beta = \mp kd$ (- for $\theta = 0^\circ$ , + for $\theta = 180^\circ$ )
Hansen-Woodyard End-Fire	$\beta = \mp \left( kd + \frac{2.94}{N} \right) \approx \mp \left( kd + \frac{\pi}{N} \right)$ (- for $\theta = 0^\circ$ , + for $\theta = 180^\circ$ )
Scanning ( $\theta = \theta_o$ )	$\beta = kd \cos \theta_o$

# N-Element Linear Arrays: Directivity

# Antenna Array Directivity



For a linear antenna array, determine total length by

$$L = (N - 1)d$$

For a large **broadside** array ( $L \gg d$ ), directivity reduces to

$$D_0 \cong 2N \left( \frac{d}{\lambda} \right) = 2 \left( 1 + \frac{L}{d} \right) \left( \frac{d}{\lambda} \right) \cong 2 \left( \frac{L}{\lambda} \right)$$

For a large ordinary end-fire array ( $L \gg d$ ), directivity reduces to

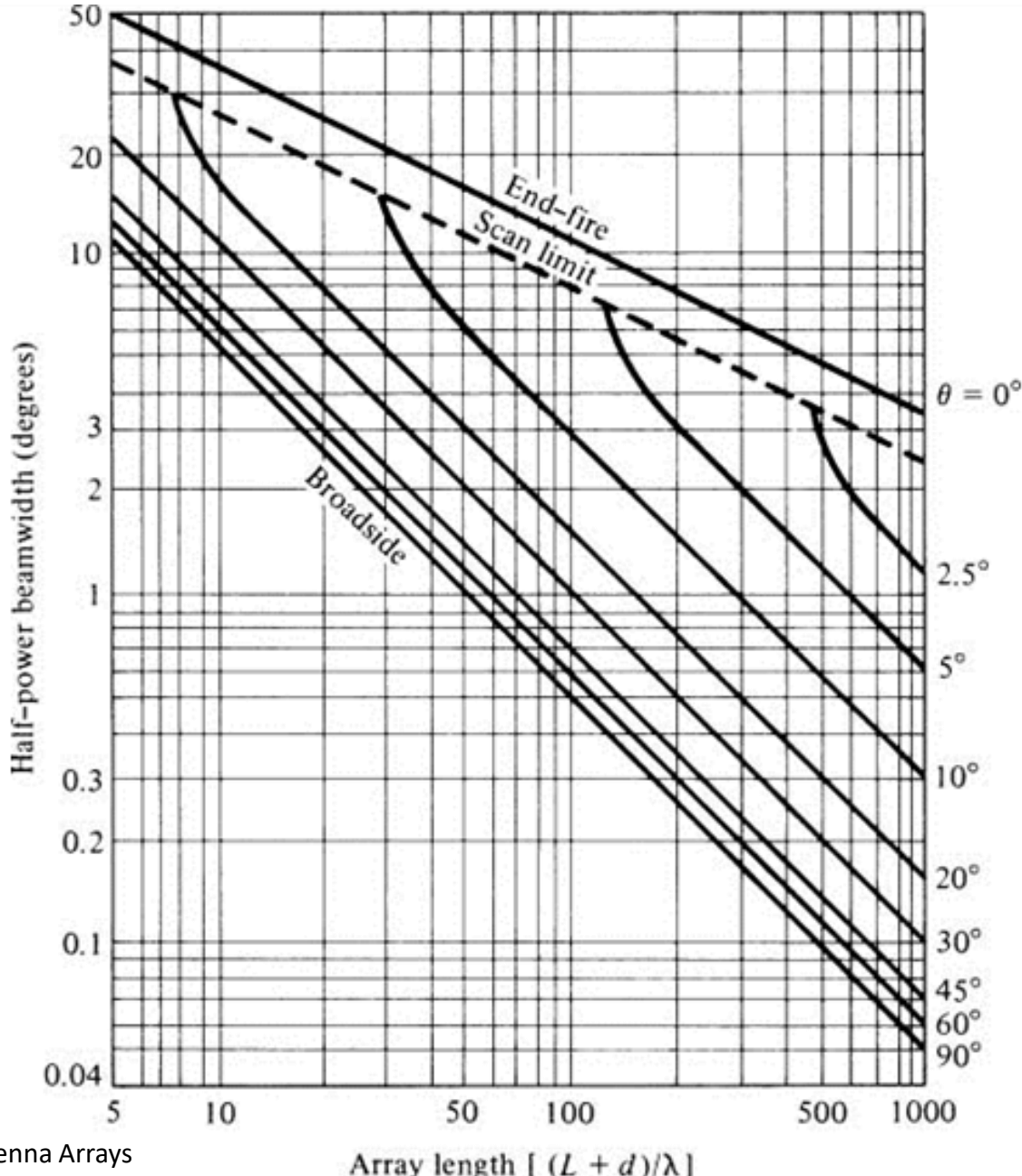
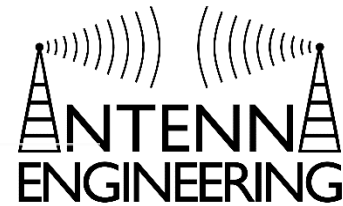
$$D_0 \cong 4N \left( \frac{d}{\lambda} \right) = 4 \left( 1 + \frac{L}{d} \right) \left( \frac{d}{\lambda} \right) \cong 4 \left( \frac{L}{\lambda} \right)$$

For a Hansen-Woodyard end-fire array ( $L \gg d$ ), directivity reduces to

$$D_0 \cong 1.805 \left[ 4N \left( \frac{d}{\lambda} \right) \right] = 1.805 \left[ 4 \left( 1 + \frac{L}{d} \right) \left( \frac{d}{\lambda} \right) \right] \cong 1.805 \left[ 4 \left( \frac{L}{\lambda} \right) \right]$$

# Linear Arrays: Design Procedure

# Linear Arrays: Design Procedure



$$N = \frac{L + d}{d}$$

$$L = (N - 1)d$$

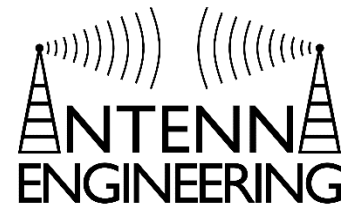
**Fig. 6.12**

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Chapter 6  
*Arrays: Linear, Planar, & Circular*

# Linear Arrays: Design Procedure

## Example



Design an uniform linear scanning array whose maximum array factor is  $30^\circ$  from the axis of the array ( $\theta = 30^\circ$ ). The desired half-power beamwidth is  $2^\circ$  while the spacing of the elements is  $\lambda/4$ . Determine the phase excitation of the elements, length of the array (in wavelengths), number of the elements, and directivity (in dB).

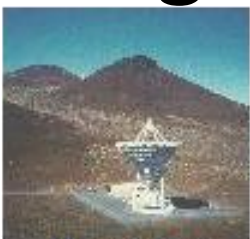
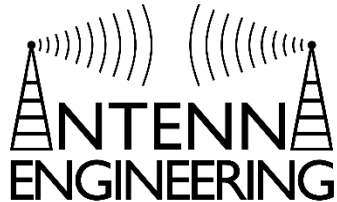
# Radio Observatory Antenna Arrays

# Karl G. Jansky Very Large Array (VLA)

- It's a cm-wavelength radio astronomy observatory located 50 miles west of Socorro, NM
- The radio telescope comprises 27 independent antennae, each of which has a dish diameter of 25 meters and weighs 209 metric tons.
- The antennae are distributed along the three arms of a track, shaped in a wye-configuration, (each of which measures 21 km).
- The frequency coverage is 74 MHz to 50 GHz (400 to 0.7 cm)



# Very Long Baseline Array



Mauna Kea  
Hawaii



Owens Valley  
California



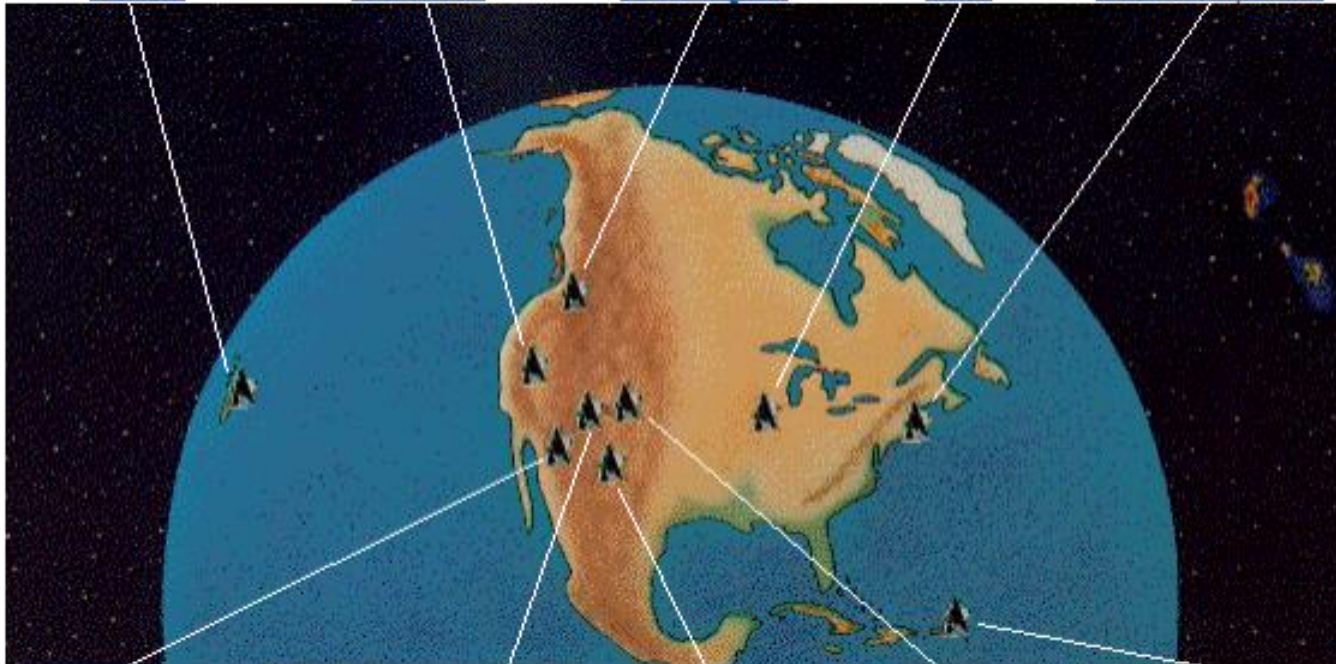
Brewster  
Washington



North Liberty  
Iowa



Hancock  
New Hampshire



Kitt Peak  
Arizona



Pie Town  
New Mexico



Fort Davis  
Texas



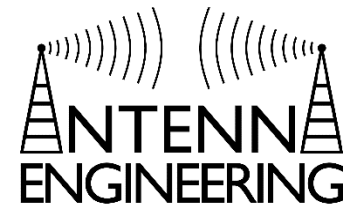
Los Alamos  
New Mexico



St. Croix  
Virgin Islands

<http://www.vlba.nrao.edu/sites/>

# Very Long Baseline Array (VLBA) and High Sensitivity Array (HSA)



- VLBA is an interferometer consisting of 10 identical antennas on transcontinental baselines up to 8000 km (Mauna Kea, Hawaii to St. Croix, Virgin Islands).
- The VLBA is controlled remotely from the Science Operations Center in Socorro, New Mexico.
- The VLBA observes at wavelengths of 28 cm to 3 mm (1.2 GHz to 96 GHz)
- It is part of the High Sensitivity Array (HSA), which comprises the VLBA, phased Very Large Array (VLA), Green Bank Telescope (GBT), Effelsberg, and Arecibo telescopes, and subsets thereof. This array spans around 12,000 km in length.

