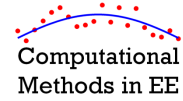




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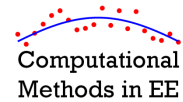


Topic 7f – Time-Domain FDM

EE 4386/5301 Computational Methods in EE

Outline

- Introduction
- Example #1



Introduction

Basic Algorithm

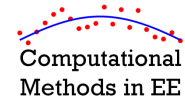
Formulation

1. Identify governing equation
2. Approximate equation using finite-differences
3. Solve finite-difference equation for function at the future time-value
4. Collect constants into update coefficients.
5. Write final update equation.

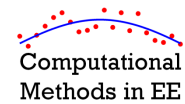
Implementation

1. Dashboard
2. Calculate grid
3. Build device on grid
4. Calculate update coefficients
5. Initial unknown function(s)
6. Main loop – iterate over time
7. Post process data
8. Visualize data

Notes

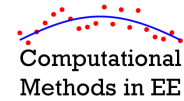


- Time-domain FDM does not require linear algebra.
- Scales nearly linearly
- Excellent technique for very large scale simulations
- Excellent for simulating transient response
- Excellent technique for learning about devices

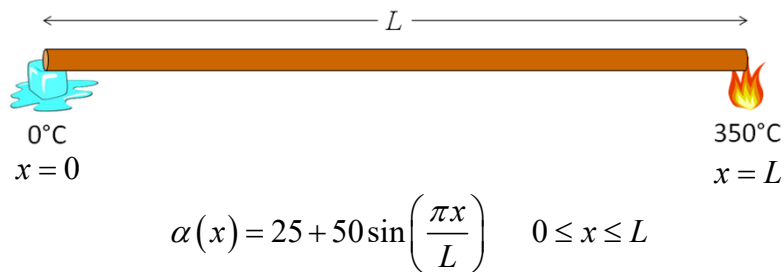


Example #1

Define Problem



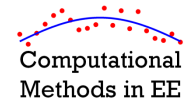
A long bar of length L is at a temperature 0°C . At time $t = 0$, a 350°C heat source is applied to the far end of the bar. Calculate and show how the temperature evolves with time over the length of the bar if the thermal diffusivity $\alpha(x)$ is



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Governing Equation



The governing equation is the time-domain form of the heat equation.

$$\frac{\partial T}{\partial t} - \alpha \nabla^2 T = 0$$

$T(x) \equiv$ temperature

$\alpha(x) \equiv$ thermal diffusivity

$t \equiv$ time

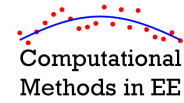
$\nabla \equiv$ del operator

Material	α
Metal	10^{-4}
Water	10^{-7}
Air	10^{-5}
Nylon	10^{-7}
Wood	10^{-7}

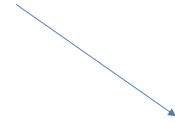
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Reduce to 1D



$$\frac{\partial T}{\partial t} - \alpha \nabla^2 T = 0$$

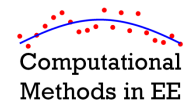


$$\frac{\partial T(x,t)}{\partial t} - \alpha(x) \frac{\partial^2 T(x,t)}{\partial x^2} = 0$$

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Approximate with Finite-Differences

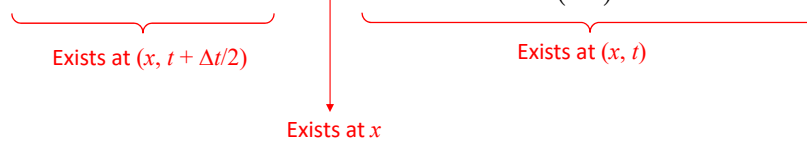


Over governing equation is

$$\frac{\partial T(x,t)}{\partial t} - \alpha(x) \frac{\partial^2 T(x,t)}{\partial x^2} = 0$$

Let's make a guess at the finite-difference approximation

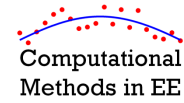
$$\frac{T(x, t + \Delta t) - T(x, t)}{\Delta t} - \alpha(x) \frac{T(x + \Delta x, t) - 2T(x, t) + T(x - \Delta x, t)}{(\Delta x)^2} = 0$$



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Fixing the Finite-Difference Equation (1 of 2)



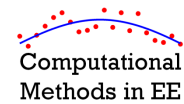
Solution 1 – Interpret the time-difference as a backward finite-difference.

$$\underbrace{\frac{T(x, t + \Delta t) - T(x, t)}{\Delta t}}_{\text{Exists at } (x, t)} - \alpha(x) \frac{T(x + \Delta x, t) - 2T(x, t) + T(x - \Delta x, t)}{(\Delta x)^2} = 0$$

Solution 2 – Interpret the time-difference as a forward finite-difference.

$$\underbrace{\frac{T(x, t) - T(x, t - \Delta t)}{\Delta t}}_{\text{Exists at } (x, t)} - \alpha(x) \frac{T(x + \Delta x, t) - 2T(x, t) + T(x - \Delta x, t)}{(\Delta x)^2} = 0$$

Fixing the Finite-Difference Equation (2 of 2)

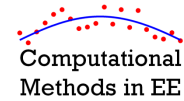


Solution 3 – We interpolate the second term in the equation to exist at $t + \Delta t/2$.

$$\underbrace{\frac{T(x, t + \Delta t) - T(x, t)}{\Delta t}}_{\text{Exists at } (x, t + \Delta t/2)} - \underbrace{\alpha(x) \frac{T(x + \Delta x, t + \Delta t) - 2T(x, t + \Delta t) + T(x - \Delta x, t + \Delta t)}{(\Delta x)^2} + \alpha(x) \frac{T(x + \Delta x, t) - 2T(x, t) + T(x - \Delta x, t)}{(\Delta x)^2}}_{\text{Exists at } (x, t + \Delta t/2)} = 0$$

This is called the Crank-Nicholson scheme.

Solve for Future Value



We will choose Solution 1 because it is the simplest, but not necessarily the most accurate.

$$\underbrace{\frac{T(x, t + \Delta t) - T(x, t)}{\Delta t}}_{\text{Exists at } (x, t)} - \alpha(x) \frac{T(x + \Delta x, t) - 2T(x, t) + T(x - \Delta x, t)}{(\Delta x)^2} = 0$$

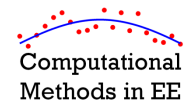
We now write this equation in terms of array indices.

$$\frac{T_i^{k+1} - T_i^k}{\Delta t} - \alpha_i \frac{T_{i+1}^k - 2T_i^k + T_{i-1}^k}{(\Delta x)^2} = 0$$

Solve for future value of T .

$$T_i^{k+1} = T_i^k + \frac{\alpha_i \Delta t}{(\Delta x)^2} (T_{i+1}^k - 2T_i^k + T_{i-1}^k)$$

Write Update Equation



The update equation is

$$T_i^{k+1} = T_i^k + c_i (T_{i+1}^k - 2T_i^k + T_{i-1}^k)$$

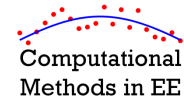
When calculating the update, calculate the new values based only on old values. Do not mix the two!

The update coefficient is

$$c_i = \frac{\alpha_i \Delta t}{(\Delta x)^2}$$

The update coefficient does not change with time so it should be calculated before the main time loop.

Stability Condition



Our finite-difference equations couples only adjacent points on the grid.

Thus, it is numerically impossible for a number to travel farther than one grid cell in one time step.

We need to make sure that the speed of the heat front is slower than what is numerically possible.

The stability condition that ensures this is

$$\frac{\alpha_i \Delta t}{(\Delta x)^2} < \frac{1}{2}$$

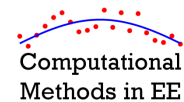
$$\Delta t < \frac{(\Delta x)^2}{2\alpha_i}$$

Typically we calculate the time step based on this stability condition.

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Revised Algorithm



1. Initialize MATLAB
2. Dashboard – define all simulation parameters
3. Calculate grid: N_x and Δx .
4. Build device on grid: α
5. Calculate a stable time step (and number of iterations): Δt and N_T
6. Calculate update coefficients: $[c_1 \ c_2 \ c_3 \ \dots \ c_N]$
7. Initialize arrays: T , $DT2$, etc.
8. Main loop – iterate over time t
 - a. Update T_i all the way across grid (loop over x)
 - b. Enforce boundary conditions
 - c. Record intermediate results (if needed)
 - d. Visualize intermediate results (if needed)
9. Post process results
10. Visualize results
11. Finished!

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