Pioneering 21st Century Electromagnetics and Photonics

Module 2 –

Spatially-Variant Planar Gratings

Course Website:
http://emlab.utep.edu/scSVL.htm

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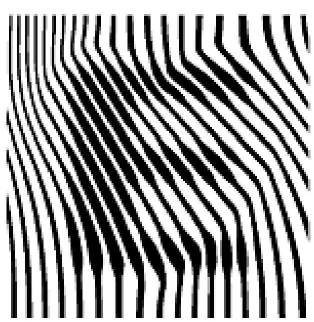
Lecture Outline

Short Course

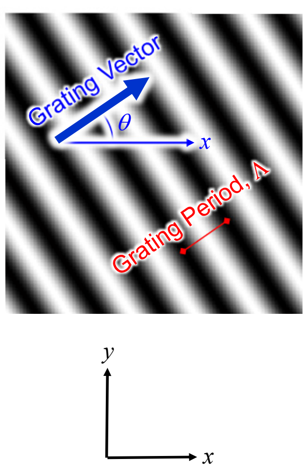
- The grating vector
- Grating phase
- Generating spatially-variant planar gratings



The Grating Vector $\vec{K}$

The Grating Vector in Two Dimensions

SVL Short Course



- The grating vector $\vec{K}$ is very similar to a wave vector $\vec{k}$.
- The direction of $\vec{K}$ is perpendicular to the grating planes.
- The magnitude of $\vec{K}$ is 2π divided by the period of the grating Λ.

$$\left|\vec{K}\right| = \frac{2\pi}{\Lambda}$$

- Given the slant angle θ, it is calculated as

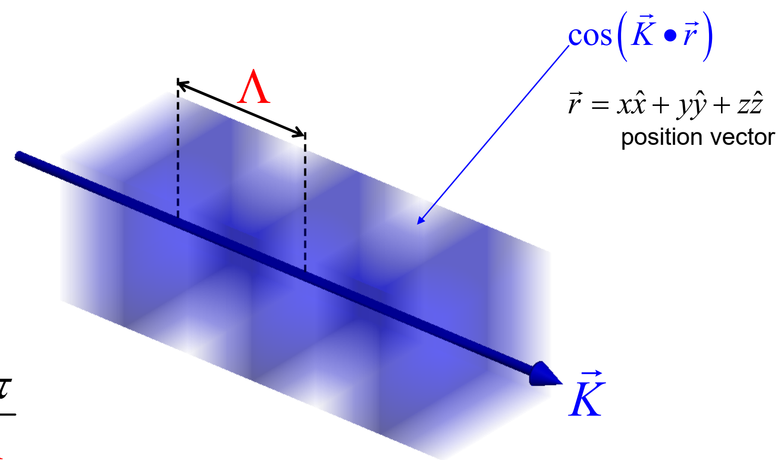
$$\vec{K} = \frac{2\pi}{\Lambda}\left(\hat{a}_x \cos\theta + \hat{a}_y \sin\theta\right)$$

- It allows a convenient calculation of the analog grating.

$$\cos\left(\vec{K} \bullet \vec{r}\right)$$

The Grating Vector in Three Dimensions

Short Course

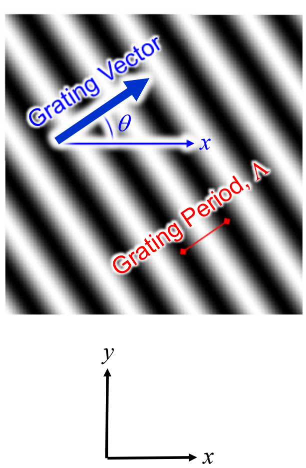


$$\left|\vec{K}\right| = \frac{2\pi}{\Lambda}$$

$$\vec{K} = K_x\hat{x} + K_y\hat{y} + K_z\hat{z}$$

The Analog Grating

Short Course



For simplicity, calculation of the analog grating will be written as

$$\varepsilon_a\left(\vec{r}\right) = \cos\left(\vec{K} \bullet \vec{r}\right)$$

or

$$\varepsilon_a\left(\vec{r}\right) = \mathrm{Re}\left[\exp\left(j\vec{K} \bullet \vec{r}\right)\right]$$

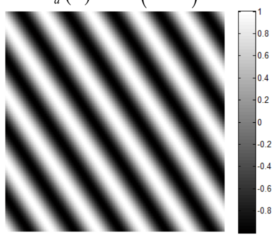
If instead you wish to generate an actual analog grating, it will need to be scaled to convey physical permittivity values.

$$\varepsilon\left(\vec{r}\right) = \varepsilon_{\mathrm{avg}} + \Delta\varepsilon\cos\left(\vec{K} \bullet \vec{r}\right)$$

The Binary Grating

Analog Grating

$$\varepsilon_a(\vec{r}) = \cos(\vec{K} \bullet \vec{r})$$



Generation of the grating using a cosine function gives us a smooth analog profile.

This requires functionally grading the material properties to realize this. That is hard.

Binary Grating

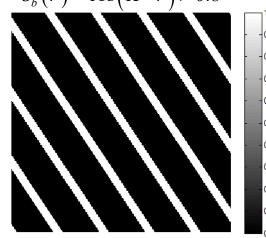
$$\varepsilon_b(\vec{r}) = \cos(\vec{K} \bullet \vec{r}) > 0$$

We can use a threshold to convert the analog profile to a binary grating.

This us much easier to physically realize.

Controlling the Fill Fraction

$$\varepsilon_b(\vec{r}) = \cos(\vec{K} \bullet \vec{r}) > 0.8$$



We can adjust the threshold value to control the duty cycle, or fill fraction, of the grating.

Grating Phase

$$\Phi(\vec{r})$$

Concept of Grating Phase

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Here we make an analogy with a standard wave. A wave propagates in the direction of the wave vector $\vec{k}$.

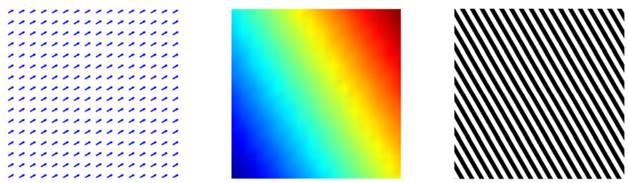
$$\vec{E}(\vec{r}) = \vec{E}_0 \cos\left(\vec{k} \bullet \vec{r} - \omega t\right)$$

As the wave propagates, it accumulates phase ϕ which is a function that increases in the direction of $\vec{k}$. The phase results from the wave propagating with wave vector $\vec{k}$ and we could alternatively reconstruct the wave just from the phase.

$$\vec{E}(\vec{r}) = \vec{E}_0 \cos\left[\phi(\vec{r})\right] \qquad \phi(\vec{r}) = \vec{k} \bullet \vec{r} - \omega t$$

For gratings, the grating vector $\vec{K}$ serves a similar purpose as the wave vector $\vec{k}$ does for waves.

$$\vec{k}(\vec{r}) \leftrightarrow \vec{K}(\vec{r})$$
$$\phi(\vec{r}) \leftrightarrow \Phi(\vec{r})$$
$$\vec{E}(\vec{r}) \leftrightarrow \varepsilon_a(\vec{r})$$



$\vec{k}(\vec{r})$ or $\vec{K}(\vec{r})$ $\qquad$ $\phi(\vec{r})$ or $\Phi(\vec{r})$ $\qquad$ $\vec{E}(\vec{r})$ or $\varepsilon_a(\vec{r})$

Definition of Grating Phase

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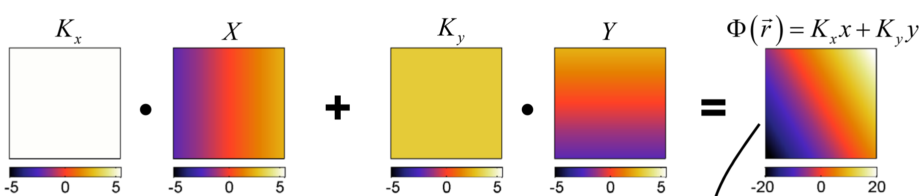
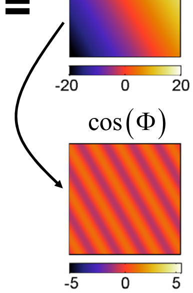
We can think of a grating as a wave because it has the same mathematical form. As a wave propagates it accumulates phase. The wave can be calculated from just the phase. Think of the grating phase Φ this way…

$$\varepsilon_a(\vec{r}) = \cos\left(\vec{K} \bullet \vec{r}\right) = \cos\left[\Phi(\vec{r})\right] \qquad \Phi(\vec{r}) = \vec{K} \bullet \vec{r} = K_x x + K_y y$$





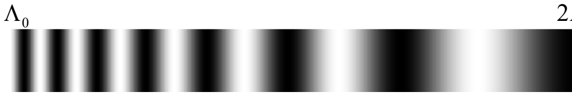
Note: Unfortunately, this intuitive definition of grating phase only holds when $\vec{K}$ is a constant and not a function of position.

Problem of a Chirped Grating

Short Course

Suppose we wish to generate the following chirped grating with period $\Lambda(z)$.

$$\Lambda(z) = \Lambda_0 z \qquad 1 \le z \le 2$$



This 1D grating is described by the following grating vector function.

$$K(z) = \frac{2\pi}{\Lambda(z)} = \frac{2\pi}{\Lambda_0 z}$$

What happens when we try to calculate the grating using $\cos(\vec{K} \bullet \vec{r})$.

$$\varepsilon_r(z) = \cos\left[K(z)\cdot z\right] = \cos\left[\frac{2\pi}{\Lambda_0 z}\cdot z\right] = \cos\left[\frac{2\pi}{\Lambda_0}\right]$$

The argument in the cosine is a constant so we do not even generate a grating here.

Conclusion From the Chirped Grating

Short Course

When the grating vector is a function of position, we can no longer calculate the grating directly from it.

$$\varepsilon_a(\vec{r}) \ne \cos\left[\vec{K}(\vec{r}) \bullet \vec{r}\right] \qquad \Phi(\vec{r}) \ne \vec{K}(\vec{r}) \bullet \vec{r}$$

Instead, we must generate the grating through the intermediate parameter of the grating phase.

$$\varepsilon_a(\vec{r}) = \cos\left[\Phi(\vec{r})\right]$$

However, we must adopt a more rigorous definition of grating phase.

$$\boxed{\nabla\Phi(\vec{r}) = \vec{K}(\vec{r})}$$

⬅ Key equation

Revised Solution for Chirped Gratings

Short Course

Now let's construct the chirped grating using the grating phase.

$$\nabla\Phi = \vec{K}(\vec{r})$$

$$\frac{d\Phi}{dz} = \frac{2\pi}{\Lambda_0 z}$$

$$\Phi(z) = \int_{-\infty}^{z} \frac{2\pi}{\Lambda_0 z'}dz' = \cancel{\int_{-\infty}^{1} \frac{2\pi}{\Lambda_0 z'}dz'} + \int_{1}^{z} \frac{2\pi}{\Lambda_0 z'}dz'$$

This term is just a constant. Since it is phase, we are free to choose whatever is convenient. Here we choose zero.

$$\Phi(z) = \frac{2\pi}{\Lambda_0}\ln z'\Big|_1^z = \frac{2\pi}{\Lambda_0}\left(\ln z - \cancel{\ln 1}\right)$$

The analog grating is then

$$\varepsilon_a(z) = \cos\left[\Phi(z)\right] = \cos\left[\frac{2\pi}{\Lambda_0}\ln z\right]$$

Generating Spatially-Variant Planar Gratings

Procedure for Generating Spatially-Variant Planar Gratings

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1. Define the grating vector function $\vec{K}(\vec{r})$, or *K*-function.

$$\vec{K}(\vec{r}) = \frac{2\pi}{\Lambda(\vec{r})}\left\{\hat{a}_x \cos\left[\theta(\vec{r})\right] + \hat{a}_y \sin\left[\theta(\vec{r})\right]\right\}$$

2. Calculate the grating phase $\Phi(\vec{r})$ from $\vec{K}(\vec{r})$.

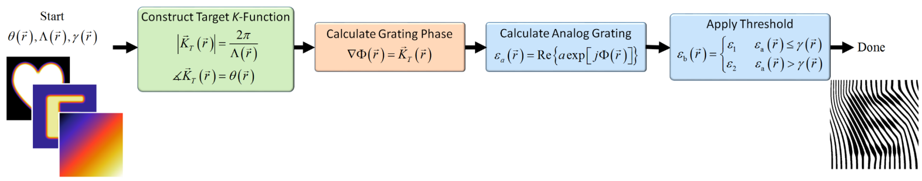
$$\nabla\Phi(\vec{r}) = \vec{K}(\vec{r})$$

3. Calculate the analog grating from the grating phase.

$$\varepsilon_a(\vec{r}) = \cos\left[\Phi(\vec{r})\right]$$

4. Calculate the binary grating from the analog grating.

$$\varepsilon_b(\vec{r}) = \begin{cases} \varepsilon_1 & \varepsilon_a(\vec{r}) \le \gamma(\vec{r}) \\ \varepsilon_2 & \varepsilon_a(\vec{r}) > \gamma(\vec{r}) \end{cases}$$



Spatially-Variant Planar Gratings

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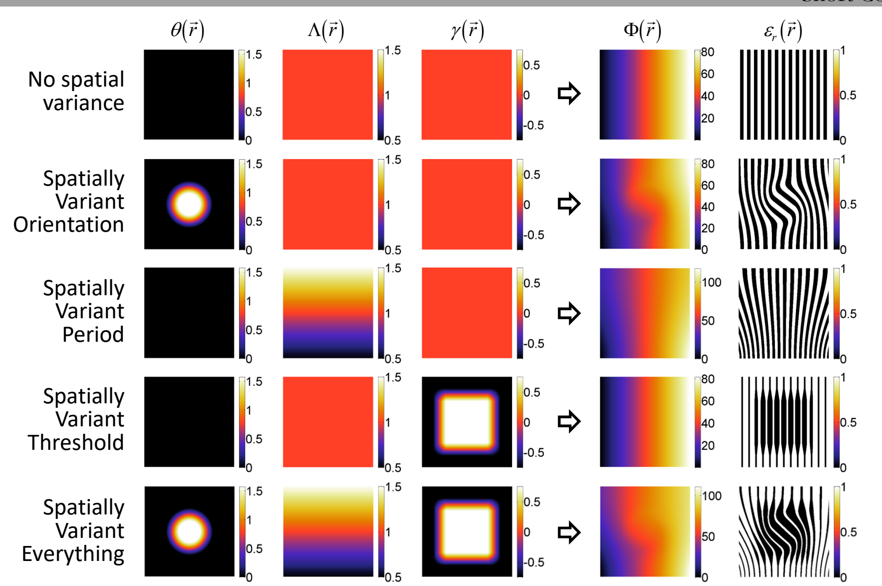

Build Input Functions

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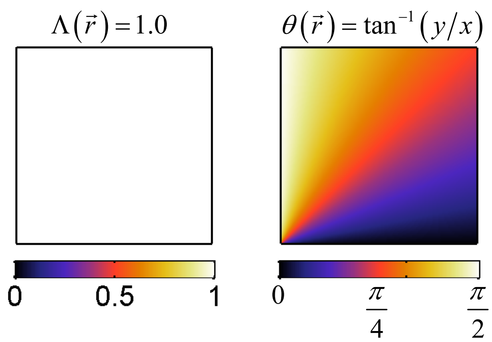
We need two things to build the *K*-function:

1. Orientation of the grating as a function of position, $\theta(\vec{r})$.
2. Period of the grating as a function of position, $\Lambda(\vec{r})$.




```
% GRATING PARAMETERS
a   = 1;
er1 = 2.5;
er2 = 1.0;
gth = 0;

% GRID PARAMETERS
Sx = 5*a;
Sy = Sx;
Nx = 100;
Ny = round(Nx*Sy/Sx);

% CALCULATE GRID
dx = Sx/Nx;
dy = Sy/Ny;
xa = [1:Nx]*dx;
ya = [1:Ny]*dy;
[Y,X] = meshgrid(ya,xa);

% SPATIALLY-VARIANT PARAMETERS
PER   = a*ones(Nx,Ny);
THETA = atan2(Y,X);
```


Calculate the K-Function

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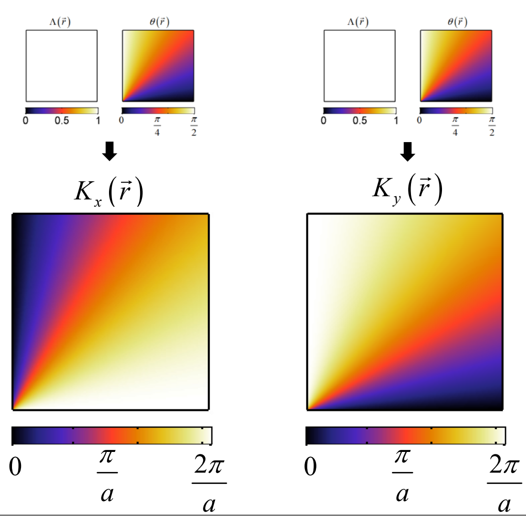
The K-function is calculated directly from the input functions.



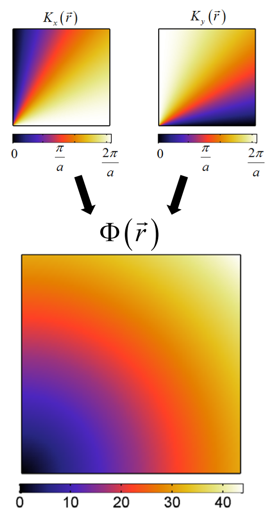
$$K_x(\vec{r}) = \frac{2\pi}{\Lambda(\vec{r})}\cos\left[\theta(\vec{r})\right]$$

$$K_y(\vec{r}) = \frac{2\pi}{\Lambda(\vec{r})}\sin\left[\theta(\vec{r})\right]$$

```
% CALCULATE K-FUNCTION
Kx = 2*pi./PER.*cos(THETA);
Ky = 2*pi./PER.*sin(THETA);
```

Calculate Grating Phase

Short Course



We calculate the grating phase from the *K*-function as a best fit.

$$\nabla\Phi(\vec{r}) = \vec{K}$$

$$\begin{bmatrix} \mathbf{D}_x \\ \mathbf{D}_y \end{bmatrix} \mathbf{\Phi} = \begin{bmatrix} \mathbf{k}_x \\ \mathbf{k}_y \end{bmatrix}$$

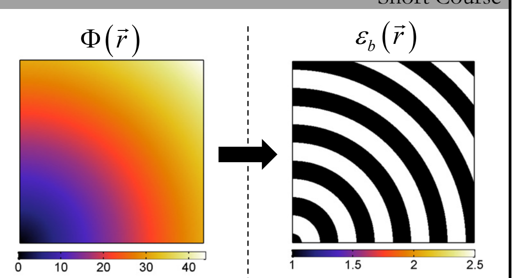
```
% COMPUTE GRATING PHASE
PHI = svlsolve(Kx,Ky,dx,dy);
```


A More Efficient Grid Strategy

Short Course

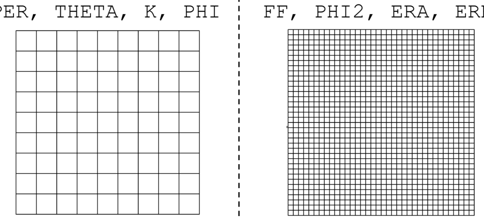
Observe how smooth the grating phase function is compared to the final grating.

We can get away with a much coarser grid up to the point where the planar grating is calculated.



Coarse Grid
`PER, THETA, K, PHI`

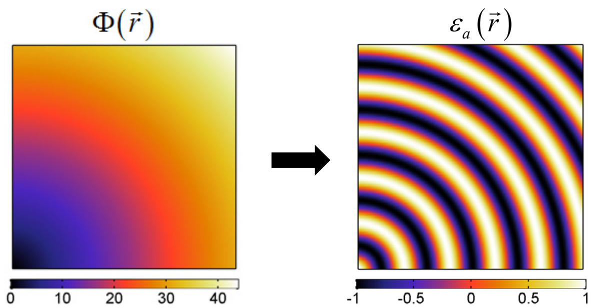
Fine Grid
`FF, PHI2, ERA, ERB`



Calculate Analog Grating

Short Course

The analog grating is calculated directly from the grating phase.



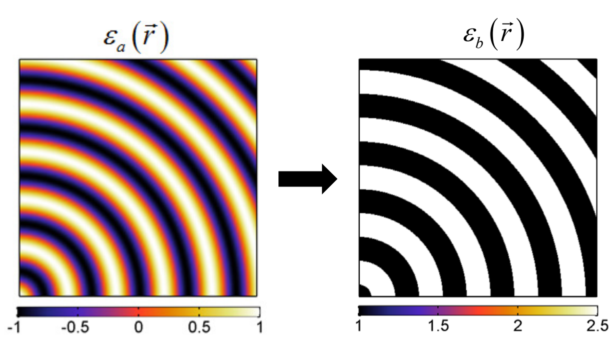
$$\varepsilon_a(\vec{r}) = \cos\left[\Phi(\vec{r})\right]$$

```
% COMPUTE ANALOG GRATING
ERA = cos(PHI);
```


Calculate Binary Grating

Short Course

The binary grating is calculated directly from the analog grating using the threshold technique.



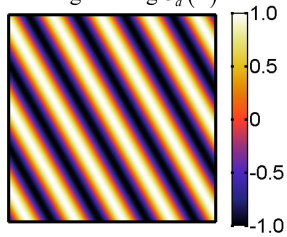
$$\varepsilon_b(\vec{r}) = \begin{cases} \varepsilon_1 & \varepsilon_a(\vec{r}) \le \gamma(\vec{r}) \\ \varepsilon_2 & \varepsilon_a(\vec{r}) > \gamma(\vec{r}) \end{cases}$$

```
% COMPUTE BINARY GRATING
ERB = er1*(ERA <= gth) + er2*(ERA > gth);
```

Controlling Fill Fraction Through the Threshold Function

Short Course

Analog Grating $\varepsilon_a(\vec{r})$



$$\varepsilon_b(\vec{r}) = \begin{cases} \varepsilon_1 & \varepsilon_a(\vec{r}) \le \gamma(\vec{r}) \\ \varepsilon_2 & \varepsilon_a(\vec{r}) > \gamma(\vec{r}) \end{cases}$$

We can estimate the threshold value in order to realize a given fill fraction f of ε_1.

$$\gamma(\vec{r}) \cong \cos\left[\pi f(\vec{r})\right]$$