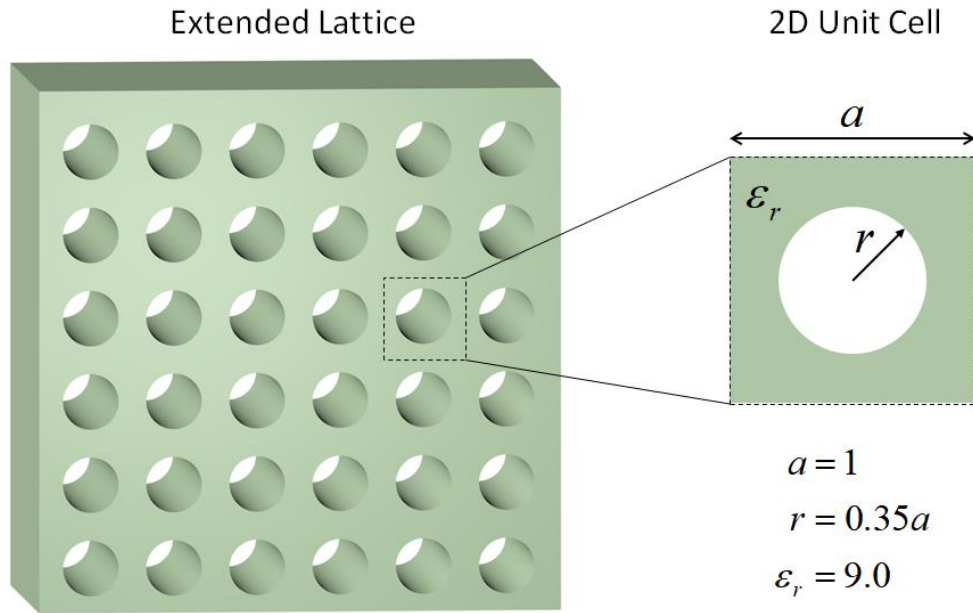
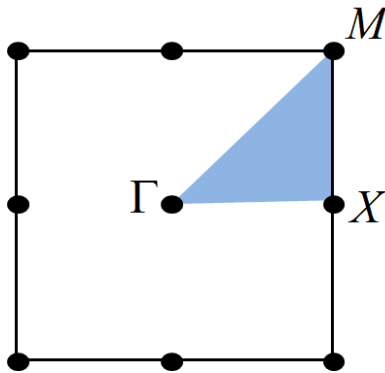


Problem Definition

This benchmarking document steps through the PWEM for the following unit cell:



For simplicity benchmarking your code, only 3×3 spatial harmonics were used. Data was generated at the three key points of symmetry Γ , X, and M.



MATLAB Code to Build Unit Cell

In order to replicate the results exactly, start with the following header and code:

```

% PWEM-2D (BENCHMARK)
% EE 5337 -- COMPUTATIONAL ELECTROMAGNETICS

% INITIALIZE MATLAB
close all;
clc
clear all;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% DASHBOARD
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% UNIT CELL
a = 1;
r = 0.35*a;
er = 9.0;

% GRID
Nx = 512;
Ny = 512;

% PWEM PARAMETERS
P = 3;
Q = P;
MODE = 'E';

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% BUILD UNIT CELL ONTO HIGH RESOLUTION REAL-SPACE GRID
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% COMPUTE GRID PARAMETERS
dx = a/Nx;
dy = a/Ny;
xa = [0:Nx-1]*dx;      xa = xa - mean(xa);
ya = [0:Ny-1]*dy;      ya = ya - mean(ya);
[Y,X] = meshgrid(ya,xa);

% BUILD UNIT CELL
UR = ones(Nx,Ny);
ER = (X.^2 + Y.^2) > r^2;
ER = 1 + (er - 1)*ER;

```

E Mode Benchmarking Data

```

=====
%% DASHBOARD
=====
a = 1
r = 0.35
er = 9

Nx = 512
Ny = 512

P = 3
Q = 3
E MODE

=====
%% CONVOLUTION MATRICES
=====

URC =
1 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 0 0 1

ERC =
5.9208 1.5571 - 0.00961 0.2834 - 0.00351 1.5571 - 0.00961 -0.5879 + 0.00721 -0.3972 + 0.00731 0.2834 - 0.00351 -0.3972 + 0.00731 0.2256 - 0.00551
1.5571 + 0.00961 5.9208 1.5571 - 0.00961 -0.5880 - 0.00001 1.5571 - 0.00961 -0.5879 + 0.00721 -0.3972 + 0.00241 0.2834 - 0.00351 -0.3972 + 0.00731
0.2834 + 0.00351 1.5571 + 0.00961 5.9208 -0.3972 + 0.00241 -0.3972 - 0.00241 -0.5880 - 0.00001 1.5571 - 0.00961 1.5571 - 0.00961 0.2256 - 0.00001 -0.3972 + 0.00731
1.5571 + 0.00961 -0.5880 + 0.00001 -0.3972 + 0.00241 5.9208 1.5571 - 0.00961 0.2834 - 0.00351 1.5571 - 0.00961 -0.5879 + 0.00721 -0.3972 + 0.00731
-0.5879 - 0.00721 1.5571 + 0.00961 -0.5880 + 0.00001 1.5571 + 0.00961 5.9208 1.5571 - 0.00961 -0.5880 - 0.00001 1.5571 - 0.00961 -0.5879 + 0.00721
-0.3972 - 0.00731 -0.5879 - 0.00721 1.5571 + 0.00961 0.2834 + 0.00351 1.5571 + 0.00961 5.9208 -0.3972 - 0.00241 -0.5880 - 0.00001 1.5571 - 0.00961
0.2834 + 0.00351 -0.3972 - 0.00241 0.2256 + 0.00001 1.5571 + 0.00961 -0.5880 + 0.00001 -0.3972 + 0.00241 5.9208 1.5571 - 0.00961 0.2834 - 0.00351
-0.3972 - 0.00731 0.2834 + 0.00351 -0.3972 - 0.00241 -0.5879 - 0.00721 1.5571 + 0.00961 -0.5880 + 0.00001 1.5571 + 0.00961 5.9208 1.5571 - 0.00961
0.2256 + 0.00551 -0.3972 - 0.00731 0.2834 + 0.00351 -0.3972 - 0.00731 -0.5879 - 0.00721 1.5571 + 0.00961 0.2834 + 0.00351 1.5571 + 0.00961 5.9208

=====
%% BLOCH WAVE VECTORS
=====

T1 =
6.2832
0

T2 =
0
6.2832

G =
0
0

X =
3.1416
0

M =
3.1416
3.1416

BETA =
0 3.1416 3.1416
0 0 3.1416

=====
%% PLANE WAVE EXPANSION METHOD
=====

P =
-1 0 1

q =
-1 0 1

-----
Beta 1
kx = 0
by = 0

KX =
6.2832 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 -6.2832 0 0 0 0 0 0 0
0 0 0 6.2832 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 -6.2832 0 0 0 0
0 0 0 0 0 0 6.2832 0 0 0
0 0 0 0 0 0 0 0 0 -6.2832

KY =
6.2832 0 0 0 0 0 0 0 0 0
0 6.2832 0 0 0 0 0 0 0 0
0 0 6.2832 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 -6.2832 0 0
0 0 0 0 0 0 0 0 -6.2832 0
0 0 0 0 0 0 0 0 0 -6.2832

```

Benchmarking Aid for 2D PWEM

A =												
78.9568	0	0	0	0	0	0	0	0	0	0	0	0
0	39.4784	0	0	0	0	0	0	0	0	0	0	0
0	0	78.9568	0	0	0	0	0	0	0	0	0	0
0	0	0	39.4784	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	39.4784	0	0	0	0	0	0	0
0	0	0	0	0	0	78.9568	0	0	0	0	0	0
0	0	0	0	0	0	0	0	39.4784	0	0	0	0
0	0	0	0	0	0	0	0	0	78.9568	0	0	0
B =												
5.9208	1.5571 - 0.00961	0.2834 - 0.00351	1.5571 - 0.00961	-0.5879 + 0.00721	-0.3972 + 0.00731	0.2834 - 0.00351	-0.3972 + 0.00731	0.2256 - 0.00551	0.2834 - 0.00351	-0.3972 + 0.00731	0.2256 - 0.00551	0.2834 - 0.00351
1.5571 + 0.00961	5.9208	1.5571 - 0.00961	-0.5880 - 0.00001	1.5571 - 0.00961	-0.5879 + 0.00721	-0.3972 + 0.00731	0.2834 - 0.00351	1.5571 + 0.00961	5.9208	1.5571 - 0.00961	-0.5880 - 0.00001	1.5571 - 0.00961
0.2834 + 0.00351	1.5571 + 0.00961	5.9208	-0.3972 + 0.00731	0.2834 - 0.00351	1.5571 - 0.00961	-0.5880 - 0.00001	1.5571 - 0.00961	0.2256 - 0.00551	0.2834 - 0.00351	-0.3972 + 0.00731	0.2256 - 0.00551	0.2834 - 0.00351
1.5571 + 0.00961	-0.5880 + 0.00001	-0.3972 + 0.00731	0.2834 - 0.00351	1.5571 - 0.00961	-0.5879 + 0.00721	-0.3972 + 0.00731	0.2834 - 0.00351	1.5571 + 0.00961	5.9208	1.5571 - 0.00961	-0.5880 - 0.00001	1.5571 - 0.00961
-0.5879 - 0.00721	1.5571 + 0.00961	-0.5880 + 0.00001	1.5571 - 0.00961	0.2834 + 0.00351	1.5571 + 0.00961	5.9208	1.5571 - 0.00961	-0.5880 - 0.00001	1.5571 - 0.00961	-0.5879 + 0.00721	-0.3972 + 0.00731	0.2834 - 0.00351
-0.3972 + 0.00731	-0.5879 + 0.00721	1.5571 + 0.00961	0.2834 + 0.00351	1.5571 + 0.00961	5.9208	1.5571 - 0.00961	-0.5880 - 0.00001	-0.3972 + 0.00731	0.2256 - 0.00551	0.2834 - 0.00351	-0.3972 + 0.00731	0.2256 - 0.00551
0.2834 + 0.00351	-0.3972 + 0.00731	0.2256 - 0.00551	0.2834 - 0.00351	-0.5879 + 0.00721	-0.3972 + 0.00731	0.2834 - 0.00351	-0.5879 + 0.00721	1.5571 + 0.00961	5.9208	1.5571 - 0.00961	-0.5880 - 0.00001	1.5571 - 0.00961
-0.3972 + 0.00731	0.2834 + 0.00351	-0.3972 + 0.00731	-0.5879 + 0.00721	1.5571 + 0.00961	-0.5880 - 0.00001	1.5571 + 0.00961	5.9208	1.5571 - 0.00961	0.2834 - 0.00351	1.5571 + 0.00961	5.9208	1.5571 - 0.00961
0.2256 + 0.00551	-0.3972 + 0.00731	0.2834 + 0.00351	0.2834 + 0.00351	-0.3972 + 0.00731	-0.5879 + 0.00721	-0.5879 + 0.00721	-0.5879 + 0.00721	1.5571 + 0.00961	0.2834 - 0.00351	1.5571 + 0.00961	5.9208	1.5571 - 0.00961
V =												
0.0000 - 0.00001	-0.0000 + 0.00001	-0.0496 + 0.00121	-0.0736 + 0.00181	-0.1056 + 0.00261	-0.2116 + 0.00521	-0.2968 + 0.00731	0.1385 - 0.00341	0.2288 - 0.00561	0.1385 - 0.00341	0.2288 - 0.00561	0.1385 - 0.00341	0.2288 - 0.00561
-0.0000 - 0.00001	-0.1840 + 0.00391	0.0467 - 0.00051	-0.2394 + 0.00421	-0.1549 + 0.00291	0.0000 - 0.00001	0.2240 - 0.00651	0.0813 - 0.00651	-0.3121 + 0.00571	0.0813 - 0.00651	-0.3121 + 0.00571	0.0813 - 0.00651	-0.3121 + 0.00571
0.0000 + 0.00001	0.0000 - 0.00001	0.0736 + 0.00071	-0.0496 - 0.00051	-0.1057 + 0.00131	0.2117 - 0.00261	-0.1384 + 0.00621	-0.2966 + 0.01341	0.2289 - 0.00281	-0.2966 + 0.01341	0.2289 - 0.00281	-0.2966 + 0.01341	0.2289 - 0.00281
-0.0000 + 0.00001	0.1840 - 0.00391	-0.2394 + 0.00411	-0.0467 + 0.00111	-0.1549 + 0.00291	0.0000 + 0.00001	0.0816 + 0.00081	-0.2239 + 0.00911	-0.3121 + 0.00571	-0.2239 + 0.00911	-0.3121 + 0.00571	-0.2239 + 0.00911	-0.3121 + 0.00571
-0.4109 + 0.00411	-0.0000 + 0.00001	0.0000 - 0.00001	0.0000 - 0.00001	0.1210 - 0.00151	0.0000 - 0.00001	0.0000 + 0.00001	0.0000 - 0.00001	0.4193 - 0.00511	0.0000 - 0.00001	0.0000 + 0.00001	0.0000 - 0.00001	0.4193 - 0.00511
0.0000 + 0.00001	0.1840 - 0.00171	0.2394 - 0.00111	0.0467 - 0.00051	-0.1550 + 0.00101	-0.0000 + 0.00001	-0.0816 - 0.00181	0.2240 - 0.00641	-0.3122 + 0.00191	-0.0816 - 0.00181	0.2240 - 0.00641	-0.3122 + 0.00191	-0.3122 + 0.00191
-0.0000 - 0.00001	0.0000 + 0.00001	-0.0736 + 0.00071	0.0496 - 0.00051	-0.1057 + 0.00131	0.2117 - 0.00261	0.1384 - 0.00621	0.2966 - 0.01341	0.2289 - 0.00281	0.2966 - 0.01341	0.2289 - 0.00281	0.2966 - 0.01341	0.2289 - 0.00281
-0.0000 - 0.00001	-0.1840 + 0.00171	-0.0467 - 0.00011	0.2394 - 0.00121	-0.1550 + 0.00101	-0.0000 + 0.00001	-0.0816 + 0.00181	-0.2240 + 0.00371	-0.3122 + 0.00191	-0.0816 + 0.00181	-0.2240 + 0.00371	-0.3122 + 0.00191	-0.3122 + 0.00191
0.0000	0.0000	0.0496	0.0737	-0.1057	-0.2117	0.2969	-0.1385	0.2289	-0.2969	-0.1385	0.2289	0.2289
D =												
-0.0000	0	0	0	0	0	0	0	0	0	0	0	0
0	5.3493	0	0	0	0	0	0	0	0	0	0	0
0	0	5.9424	0	0	0	0	0	0	0	0	0	0
0	0	0	5.9424	0	0	0	0	0	0	0	0	0
0	0	0	0	7.3189	0	0	0	0	0	0	0	0
0	0	0	0	0	14.1508	0	0	0	0	0	0	0
0	0	0	0	0	0	21.4393	0	0	0	0	0	0
0	0	0	0	0	0	0	21.4393	0	0	0	0	0
0	0	0	0	0	0	0	0	31.9392	0	0	0	0
Beta 2												
bx = 3.1416												
by = 0												
KX =												
9.4248	0	0	0	0	0	0	0	0	0	0	0	0
0	3.1416	0	0	0	0	0	0	0	0	0	0	0
0	0	-3.1416	0	0	0	0	0	0	0	0	0	0
0	0	0	9.4248	0	0	0	0	0	0	0	0	0
0	0	0	0	3.1416	0	0	0	0	0	0	0	0
0	0	0	0	0	-3.1416	0	0	0	0	0	0	0
0	0	0	0	0	0	9.4248	0	0	0	0	0	0
0	0	0	0	0	0	0	3.1416	0	0	0	0	0
0	0	0	0	0	0	0	0	-3.1416	0	0	0	0
KY =												
6.2832	0	0	0	0	0	0	0	0	0	0	0	0
0	6.2832	0	0	0	0	0	0	0	0	0	0	0
0	0	6.2832	0	0	0	0	0	0	0	0	0	0
0	0	0	6.2832	0	0	0	0	0	0	0	0	0
0	0	0	0	6.2832	0	0	0	0	0	0	0	0
0	0	0	0	0	6.2832	0	0	0	0	0	0	0
0	0	0	0	0	0	-6.2832	0	0	0	0	0	0
0	0	0	0	0	0	0	-6.2832	0	0	0	0	0
0	0	0	0	0	0	0	0	-6.2832	0	0	0	0
A =												
128.3049	0	0	0	0	0	0	0	0	0	0	0	0
0	49.3480	0	0	0	0	0	0	0	0	0	0	0
0	0	49.3480	0	0	0	0	0	0	0	0	0	0
0	0	0	88.8264	0	0	0	0	0	0	0	0	0
0	0	0	0	9.8696	0	0	0	0	0	0	0	0
0	0	0	0	0	9.8696	0	0	0	0	0	0	0
0	0	0	0	0	0	128.3049	0	0	0	0	0	0
0	0	0	0	0	0	0	49.3480	0	0	0	0	0
0	0	0	0	0	0	0	0	49.3480	0	0	0	0
B =												
5.9208	1.5571 - 0.00961	0.2834 - 0.00351	1.5571 - 0.00961	-0.5879 + 0.00721	-0.3972 + 0.00731	0.2834 - 0.00351	-0.3972 + 0.00731	0.2256 - 0.00551	0.2834 - 0.00351	-0.3972 + 0.00731	0.2256 - 0.00551	0.2834 - 0.00351
1.5571 + 0.00961	5.9208	1.5571 - 0.00961	-0.5880 - 0.00001	1.5571 - 0.00961	-0.5879 + 0.00721	-0.3972 + 0.00731	0.2834 - 0.00351	1.5571 + 0.00961	5.9208	1.5571 - 0.00961	-0.5880 - 0.00001	1.5571 - 0.00961
0.2834 + 0.00351	1.5571 + 0.00961	5.9208	-0.3972 + 0.00731	0.2834 - 0.00351	1.5571 - 0.00961	-0.5880 - 0.00001	1.5571 - 0.00961	0.2256 - 0.00551	0.2834 - 0.00351	-0.3972 + 0.00731	0.2256 - 0.00551	0.2834 - 0.00351
1.5571 + 0.00961	-0.5880 + 0.00001	-0.3972 + 0.00731	0.2834 - 0.00351	1.5571 - 0.00961	-0.5879 + 0.00721	-0.3972 + 0.00731	0.2834 - 0.00351	1.5571 + 0.00961	5.9208	1.5571 - 0.00961	-0.5880 - 0.00001	1.5571 - 0.00961
-0.5879 - 0.00721	1.5571 + 0.00961	-0.5880 + 0.00001	1.5571 - 0.00961	0.2834 + 0.00351	1.5571 + 0.00961	5.9208	1.5571 - 0.00961	-0.5880 - 0.00001	1.5571 - 0.00961	-0.5879 + 0.00721	-0.3972 + 0.00731	0.2834 - 0.00351
-0.3972 + 0.00731	-0.5879 + 0.00721	1.5571 + 0.00961	0.2834 + 0.00351	1.5571 + 0.00961	5.9208	1.5571 - 0.00961	-0.5880 - 0.00001	-0.3972 + 0.00731	0.2256 - 0.00551	0.2834 - 0.00351	-0.3972 + 0.00731	0.2256 - 0.00551
0.2834 + 0.00351	-0.3972 + 0.00731	0.2256 - 0.00551	0.2834 - 0.00351	-0.5879 + 0.00721	-0.3972 + 0.00731	0.2834 - 0.00351	-0.5879 + 0.00721	1.5571 + 0.00961	5.9208	1.5571 - 0.00961	-0.5880 - 0.00001	1.5571 - 0.00961
-0.3972 + 0.00731	0.2834 + 0.00351	-0.3972 + 0.00731	-0.5879 + 0.00721	1.5571 + 0.00961	-0.5880 - 0.00001	1.5571 + 0.00961	5.9208	1.5571 - 0.00961	0.2834 - 0.00351	1.5571 + 0.00961	5.9208	1.5571 - 0.00961
0.2256 + 0.00551	-0.3972 + 0.00731	0.2834 + 0.00351	0.2834 + 0.00351	-0.3972 + 0.00731	-0.5879 + 0.00721	-0.5879 + 0.00721	-0.5879 + 0.00721	1.5571 + 0.00961	0.2834 - 0.00351	1.5571 + 0.00961	5.9208	1.5571 - 0.00961
V =												
-0.0025 + 0.00011	0.0003 - 0.00001	0.0254 - 0.00061	0.0208 - 0.00051	0.0813 - 0.00201	0.1215 - 0.00301	-0.1139 + 0.00281	-0.3084 + 0.00761	0.3048 - 0.00751	-0.3084 + 0.00761	0.3048 - 0.00751	-0.3084 + 0.00761	0.3048 - 0.00751
0.0080 - 0.00011	-0.0331 + 0.00061	0.1832 - 0.00341	0.1859 - 0.00341	0.2075 - 0.00381	-0.0399 - 0.00071	-0.2235 + 0.00411	0.1993 - 0.00371	-0.2608 - 0.00481	0.1993 - 0.00371	-0.2608 - 0.00481	0.1993 - 0.00371	-0.2608 - 0.00481
0.0079 - 0.00011	0.0340 - 0.00041	0.1714 - 0.00211	0.1719 - 0.00211	-0.2533 + 0.00311	0.0831 - 0.00101	0.2781 - 0.00341	-0.0941 + 0.00121	0.0786 - 0.00101	-0.0941 + 0.00121	0.0786 - 0.00101	-0.0941 + 0.00121	0.0786 - 0.00101
0.0072 - 0.00011	-0.0097 + 0.00021	0.0000 - 0.00001	-0.0510 + 0.00091	0.0000 - 0.00001	0.2987 - 0.00551	0.0086 - 0.00021	0.0000 -					

```

KY =
9.4248 0 0 0 0 0 0 0 0 0
0 9.4248 0 0 0 0 0 0 0 0
0 0 9.4248 0 0 0 0 0 0 0
0 0 0 3.1416 0 0 0 0 0 0
0 0 0 0 3.1416 0 0 0 0 0
0 0 0 0 0 3.1416 0 0 0 0
0 0 0 0 0 0 -3.1416 0 0 0
0 0 0 0 0 0 0 -3.1416 0 0
0 0 0 0 0 0 0 0 -3.1416
    
```

```

A =
177.6529 0 0 0 0 0 0 0 0 0
0 98.6960 0 0 0 0 0 0 0 0
0 0 98.6960 0 0 0 0 0 0 0
0 0 0 98.6960 0 0 0 0 0 0
0 0 0 0 19.7392 0 0 0 0 0
0 0 0 0 0 19.7392 0 0 0 0
0 0 0 0 0 0 98.6960 0 0 0
0 0 0 0 0 0 0 19.7392 0 0
0 0 0 0 0 0 0 0 19.7392
    
```

```

B =
5.9208 1.5571 - 0.00961 0.2834 - 0.00351 1.5571 - 0.00961 -0.5879 + 0.00721 -0.3972 + 0.00731 0.2834 - 0.00351 -0.3972 + 0.00731 0.2256 - 0.00551
1.5571 + 0.00961 5.9208 1.5571 - 0.00961 -0.5880 - 0.00001 1.5571 - 0.00961 -0.5879 + 0.00721 -0.3972 + 0.00241 0.2834 - 0.00351 -0.3972 + 0.00731
0.2834 + 0.00351 1.5571 + 0.00961 5.9208 0.0186 + 0.00031 0.0549 - 0.00071 -0.1475 - 0.00151 -0.1861 + 0.00231 0.2146 - 0.00261 -0.3040 + 0.02631 0.1567 + 0.00191
1.5571 + 0.00961 -0.5880 + 0.00001 -0.3972 + 0.00241 5.9208 0.0186 + 0.00031 0.0549 - 0.00071 -0.1475 - 0.00151 -0.1861 + 0.00231 0.2146 - 0.00261 -0.3040 + 0.02631 0.1567 + 0.00191
-0.5879 - 0.00721 1.5571 + 0.00961 -0.5880 + 0.00001 1.5571 + 0.00961 5.9208 1.5571 - 0.00961 -0.5880 - 0.00001 1.5571 - 0.00961 -0.5879 + 0.00721
-0.3972 - 0.00731 -0.5879 - 0.00721 1.5571 + 0.00961 0.2834 + 0.00351 1.5571 + 0.00961 5.9208 -0.3972 - 0.00241 -0.5880 - 0.00001 1.5571 - 0.00961 0.2834 - 0.00351
0.2834 + 0.00351 -0.3972 - 0.00241 1.5571 - 0.00961 0.2256 + 0.00001 1.5571 - 0.00961 -0.5880 + 0.00001 -0.3972 + 0.00241 1.5571 - 0.00961 0.2834 - 0.00351
-0.3972 - 0.00731 0.2834 + 0.00351 -0.3972 - 0.00241 -0.5879 - 0.00721 1.5571 + 0.00961 -0.5880 + 0.00001 1.5571 - 0.00961 1.5571 - 0.00961 1.5571 - 0.00961
0.2256 + 0.00551 -0.3972 - 0.00731 0.2834 + 0.00351 -0.3972 - 0.00731 -0.5879 - 0.00721 1.5571 + 0.00961 0.2834 + 0.00351 1.5571 + 0.00961 5.9208
    
```

```

V =
-0.0026 + 0.00011 0.0030 - 0.00011 -0.0000 + 0.00001 -0.0163 + 0.00041 -0.0000 + 0.00001 -0.0918 + 0.00231 -0.2654 + 0.00651 -0.0000 + 0.00001 0.3799 - 0.00931
0.0041 - 0.00011 -0.0184 + 0.00031 -0.0074 - 0.00011 -0.0468 + 0.00091 -0.2007 - 0.00071 -0.1626 + 0.00301 -0.1142 + 0.00211 0.2265 - 0.02101 -0.3709 + 0.00681
0.0041 - 0.00011 0.0082 - 0.00011 0.0186 + 0.00031 0.0549 - 0.00071 -0.1475 - 0.00151 -0.1861 + 0.00231 0.2146 - 0.00261 -0.3040 + 0.02631 0.1567 + 0.00191
0.0041 - 0.00011 -0.0184 + 0.00031 0.0074 + 0.00011 -0.0468 + 0.00091 0.2007 + 0.00071 -0.1626 + 0.00301 -0.1142 + 0.00211 -0.2265 + 0.02101 -0.3709 + 0.00681
0.1743 - 0.00211 -0.2711 + 0.00331 0.0000 - 0.00001 -0.2630 + 0.00321 0.0000 - 0.00001 0.0313 - 0.00041 0.2182 - 0.00271 0.0000 - 0.00001 0.3710 - 0.00461
0.1715 - 0.00211 0.0073 - 0.00001 0.2701 + 0.00561 0.2932 - 0.00181 0.0232 + 0.00041 0.0271 - 0.00021 -0.1970 + 0.00121 0.1357 - 0.01091 -0.1572 + 0.00101
0.0041 - 0.00011 0.0082 - 0.00011 -0.0186 - 0.00031 0.0549 - 0.00071 0.1475 + 0.00151 -0.1861 + 0.00231 0.2146 - 0.00261 0.3040 - 0.02631 0.1567 + 0.00191
0.1715 - 0.00211 0.0073 - 0.00001 -0.2701 - 0.00561 0.2932 - 0.00181 -0.0232 - 0.00041 0.0271 - 0.00021 -0.1970 + 0.00121 -0.1357 + 0.01091 -0.1572 + 0.00101
0.1686 0.02685 -0.0000 -0.2288 0.0000 0.0000 -0.0154 0.00021 0.1136 0.0000 0.0000
    
```

```

D =
2.3299 0 0 0 0 0 0 0 0 0
0 2.9579 0 0 0 0 0 0 0 0
0 0 2.9607 0 0 0 0 0 0 0
0 0 0 7.9678 0 0 0 0 0 0
0 0 0 0 12.2661 0 0 0 0 0
0 0 0 0 0 13.6074 0 0 0 0
0 0 0 0 0 0 26.9175 0 0 0
0 0 0 0 0 0 0 29.3217 0 0
0 0 0 0 0 0 0 0 61.3951
    
```

```

*****
%% FINAL LIST OF NORMALIZED FREQUENCIES
*****
    
```

```

WN =
0 0.1817 0.2429
0.3681 0.2268 0.2737
0.3880 0.4020 0.2739
0.3880 0.4124 0.4493
0.4306 0.5578 0.5574
0.5987 0.5646 0.5871
0.7369 0.6642 0.8257
0.7369 0.8602 0.8618
0.8995 1.1107 1.2471
    
```




Benchmarking Aid for 2D PWEM

A =

17.6525	-0.00001	-3.7513	+0.02301	0	-3.7513	+0.02301	0	0.5979	-0.01101	0	0.5979	-0.01101	0.5397	-0.01321		
-3.7513	+0.02301	10.7279	+0.00001	-3.7513	+0.02301	0	0	0.5980	-0.00371	0	-1.5385	+0.01891	0.5979	-0.01101		
0	0	-3.7513	-0.02301	17.6525	+0.00001	0.5980	+0.00371	-3.7513	+0.02301	0.5399	-0.00001	0.5980	-0.00371	0		
-3.7513	-0.02301	0	0	0.5980	-0.00371	10.7279	-0.00001	0	0	-1.5385	+0.01891	-3.7513	+0.02301	0.5979	-0.01101	
0.5979	+0.01101	0	0	-3.7513	-0.02301	-1.5385	+0.01891	0	0	10.7279	+0.00001	0.5980	+0.00371	0	-3.7513	+0.02301
0	0	0.5980	+0.00371	0.5399	+0.00001	-3.7513	-0.02301	0	0	0.5980	-0.00371	17.6525	+0.00001	-3.7513	+0.02301	0
0.5979	+0.01101	-1.5385	+0.01891	0.5980	+0.00371	0	0	0	0	0	0	-3.7513	-0.02301	10.7279	-3.7513	+0.02301
0.5397	+0.01321	0.5979	+0.01101	0	0	0.5979	+0.01101	0	0	-3.7513	-0.02301	0	0	-3.7513	-0.02301	17.6525

B =

1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0

V =

0.5056	+0.00001	-0.9513	+0.00581	0.9763	+0.00001	0.9819	-0.01811	-0.9760	+0.02401	-0.2154	+0.00271	-0.0000	-0.00001	-0.0461	+0.00411	0
0.9821	+0.00601	-0.9880	-0.00001	0.0000	+0.00001	-0.4728	+0.00581	0.5026	-0.00931	0.5723	-0.01631	-0.9392	-0.05001	-0.9505	+0.04951	0
0.5056	+0.00621	-0.0000	-0.00001	-0.9763	-0.01201	-0.0000	+0.00001	-0.9763	+0.01201	-0.9734	+0.02661	-0.0000	-0.00001	-0.8693	+0.03851	0
0.9821	+0.00601	-0.9880	-0.00001	-0.0000	+0.00001	-0.4728	+0.00581	0.5026	-0.00931	-0.3648	+0.01501	0.9392	+0.05001	0.8546	-0.04151	1.0000
0.9819	+0.01811	0.9879	+0.01211	-0.0000	-0.00001	0.4728	+0.00001	0.5026	-0.00311	0.3650	-0.01051	0.9385	+0.06151	-0.8550	+0.03101	0
0.5056	+0.00621	-0.0000	-0.00001	-0.9763	-0.01201	-0.0000	+0.00001	-0.9763	+0.01201	0.9734	-0.02661	0.0000	+0.00001	0.8693	-0.03851	0
0.9819	+0.01811	0.9879	+0.01211	-0.0000	-0.00001	0.4728	+0.00001	0.5026	-0.00311	-0.5724	+0.00931	-0.9385	-0.06151	0.9510	-0.03791	0
0.5055	+0.01241	0.9511	+0.01751	0.9760	+0.02401	-0.9821	-0.00601	-0.9763	+0.00001	0.2154	+0.00261	-0.0000	-0.00001	0.0462	-0.00301	0

D =

5.9424	-0.00001	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	8.0785	-0.00001	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	18.1924	-0.00001	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	21.3007	-0.00001	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	21.4322	-0.00001	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	21.3007	-0.00001	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	9.1893	-0.00001	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	8.0785	+0.00001	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

 Beta 2
 bx = 3.1416
 by = 0

KX =

9.4248	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	3.1416	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-3.1416	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	9.4248	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	3.1416	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-3.1416	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	9.4248	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	3.1416	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	3.1416	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-3.1416	0	0	0	0	0	0

KY =

6.2832	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	6.2832	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	6.2832	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	6.2832	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	6.2832	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	6.2832	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	6.2832	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	6.2832	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	6.2832	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	6.2832	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	6.2832	0	0	0	0	0	0

A =

28.6853	-0.00001	-6.5649	+0.04031	0.1851	-0.00231	-8.4405	+0.05181	2.4631	-0.03021	0.4484	-0.00831	0.9255	-0.01141	0.1495	-0.00281	0.4722	-0.01161
-6.5649	-0.04031	13.4099	-0.00001	-2.8135	+0.01731	2.5939	+0.00001	-1.4772	+0.00911	-0.8646	+0.01061	0.1495	-0.00091	-1.1539	+0.01421	0.7474	-0.01381
0.1851	+0.00231	-2.8135	-0.01731	11.0328	+0.00001	0.4485	+0.00281	-0.8211	-0.00001	-0.9378	+0.00581	0.4724	-0.00001	0.7475	-0.00461	-0.5553	+0.00681
-8.4405	-0.05181	2.5939	-0.00001	0.4485	-0.00281	24.1378	-0.00001	-4.4316	+0.02721	-1.1539	+0.01421	-8.4405	+0.05181	2.5937	-0.03181	0.4484	-0.00831
2.4631	+0.03021	-1.4772	-0.00911	-0.8211	+0.00001	-4.4316	-0.02721	3.5471	+0.00001	1.4772	-0.00911	2.4633	+0.00001	-1.4772	+0.00911	-0.8210	-0.01011
0.4484	-0.00831	-0.8646	-0.01061	-0.9378	-0.00581	-1.1539	+0.01421	1.4772	+0.00911	2.6820	-0.00001	0.4485	+0.00281	-0.8646	-0.00001	-0.9378	+0.00581
0.9255	-0.01141	0.1495	+0.00091	0.4724	+0.00001	-8.4405	-0.05181	2.4633	-0.00001	0.4485	-0.00281	28.6853	+0.00001	-6.5649	+0.04031	0.1851	-0.00231
0.1495	+0.00281	-1.1539	-0.01421	0.7475	+0.00461	2.5937	+0.03181	-1.4772	-0.00911	-0.8646	+0.00001	-6.5649	-0.04031	13.4099	-0.00001	-2.8135	+0.01731
0.4722	-0.01161	0.7474	+0.01381	-0.5553	-0.00681	0.4484	+0.00831	-0.8210	-0.01011	-0.9378	-0.00581	0.1851	+0.00231	-2.8135	-0.01731	11.0328	0.00001

B =

1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0

V =

0.8873	-0.00001	0.9939	-0.00611	-0.0341	+0.00041	-0.0090	+0.00011	-0.4870	+0.00601	-0.4637	+0.00851	-0.5995	+0.00741	-0.3267	+0.00601	0.2031	-0.00501
-0.2943	-0.00181	-0.4295	-0.00001	0.0028	-0.00001	-0.2668	+0.00001	0.2701	-0.00171	-0.9219	+0.01131	-0.8995	+0.00551	-0.9071	+0.01111	0.9819	-0.01811
0.0150	+0.00021	0.0653	+0.00041	-0.0170	-0.00001	-0.2918	+0.00181	-0.3034	-0.00001	0.9939	-0.00611	0.9880	-0.00001	-0.9939	+0.00611	0.9302	-0.01141
-0.9939	-0.00611	-0.0000	+0.00001	0.1141	-0.00071	-0.1910	+0.00001	-0.9939	+0.00611	-0.0000	+0.00001	-0.5013	+0.00311	-0.0000	+0.00001	-0.3497	+0.00641
0.2533	+0.00311	0.0000	+0.00001	0.9745	-0.00001	-0.9939	-0.00611	0.1526	+0.00001	0.0000	-0.00001	0.0329	-0.00001	-0.0000	+0.00001	-0.4711	+0.00581
0.0713	+0.00131	0.0000	+0.00001	-0.9939	-0.00611	-0.9873	-0.01211	0.0809	+0.00051	0.0000	-0.						

Benchmarking Aid for 2D PWEM



```

KY =
9.4248      0      0      0      0      0      0      0      0
0      9.4248      0      0      0      0      0      0      0
0      0      9.4248      0      0      0      0      0      0
0      0      0      3.1416      0      0      0      0      0
0      0      0      0      3.1416      0      0      0      0
0      0      0      0      0      3.1416      0      0      0
0      0      0      0      0      0      -3.1416      0      0
0      0      0      0      0      0      0      -3.1416      0
0      0      0      0      0      0      0      0      -3.1416
    
```

```

A =
39.7181 - 0.00001 -11.2540 + 0.06911 1.1106 - 0.01361 -11.2540 + 0.06911 4.9263 - 0.06051 0 1.1106 - 0.01361 0 0.4048 - 0.00991
-11.2540 - 0.06911 26.8198 + 0.00001 -7.5027 + 0.04601 5.1879 + 0.00001 -5.9088 + 0.03631 1.7292 - 0.02121 -0.0000 + 0.00001 -0.7692 + 0.00941 0.5979 - 0.01101
1.1106 + 0.01361 -7.5027 - 0.04601 22.0656 - 0.00001 0.0000 - 0.00001 1.6422 + 0.00001 -3.7513 + 0.02301 0.4049 - 0.00001 0.5980 - 0.00371 -0.3702 + 0.00451
-11.2540 - 0.06911 5.1879 - 0.00001 -0.0000 - 0.00001 26.8198 - 0.00001 -5.9088 + 0.03631 -0.7692 + 0.00941 -7.5027 + 0.04601 1.7292 - 0.02121
4.9263 + 0.06051 -5.9088 - 0.03631 1.6422 - 0.00001 -5.9088 - 0.03631 7.0941 + 0.00001 0 1.6422 + 0.00001 0 -1.6421 + 0.02021
0 0 0 0 0 0 0 0 5.3640 + 0.00001 0.5980 + 0.00371 -1.7293 - 0.00001 0
1.1106 + 0.01361 -0.0000 + 0.00001 0.4049 + 0.00001 -7.5027 - 0.04601 1.6422 - 0.00001 0.5980 - 0.00371 22.0656 + 0.00001 -3.7513 + 0.02301 -0.3702 + 0.00451
0 0 -0.7692 - 0.00941 0.5980 + 0.00371 1.7292 + 0.02121 0 -1.7293 + 0.00001 -3.7513 - 0.02301 5.3640 0
0.4048 + 0.00991 0.5979 + 0.01101 -0.3702 - 0.00451 0.5979 + 0.01101 -1.6421 - 0.02021 0 -0.3702 - 0.00451 0 4.4131
    
```

```

B =
1 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 0 0 1
    
```

```

V =
1.0000 + 0.00001 0.9939 - 0.00611 0.9082 - 0.01111 0.0354 - 0.00021 0.0428 - 0.00051 -0.0412 + 0.00001 0.0000 - 0.00001 0.0000 + 0.00001 -0.0000 - 0.00001
-0.6142 - 0.00381 0.4710 + 0.00001 0.9618 - 0.00591 -0.1603 + 0.00001 -0.1209 + 0.00071 -0.2628 - 0.00161 0.9470 - 0.02311 0.9941 + 0.00591 -0.0345 + 0.00131
0.1901 + 0.00231 -0.9265 - 0.00571 1.0000 + 0.00001 0.1614 + 0.00101 -0.1421 + 0.00001 -0.0151 - 0.00021 -0.9821 + 0.01791 0.9007 + 0.01091 0.2526 + 0.00771
-0.6142 - 0.00381 0.4710 - 0.00001 0.9618 - 0.00591 -0.1603 + 0.00001 -0.1209 + 0.00071 -0.2628 - 0.00161 -0.9470 + 0.02311 -0.9941 - 0.00591 0.0345 - 0.00131
0.2665 + 0.00331 -0.2043 - 0.00131 -0.4243 - 0.00001 -0.9939 + 0.00611 -0.7238 + 0.00001 -0.9879 - 0.01211 0.0000 - 0.00001 -0.0000 - 0.00001 -0.0000 - 0.00001
-0.0230 - 0.00041 0.1508 + 0.00191 -0.1791 - 0.00111 0.9830 + 0.01211 -0.8249 - 0.00511 -0.0952 - 0.00181 0.2877 - 0.00351 -0.1979 - 0.00361 0.9760 - 0.02401
0.1901 + 0.00231 -0.9265 - 0.00571 1.0000 + 0.00001 0.1614 + 0.00101 -0.1421 + 0.00001 -0.0151 - 0.00021 0.9821 - 0.01791 -0.9007 - 0.01091 -0.2526 + 0.00771
-0.0230 - 0.00041 0.1508 + 0.00191 -0.1791 - 0.00111 0.9830 + 0.01211 -0.8249 - 0.00511 -0.0952 - 0.00181 -0.2877 + 0.00351 0.1979 + 0.00361 -0.9760 + 0.02401
-0.0179 - 0.00041 0.0920 + 0.00171 0.1263 + 0.00161 -0.9189 - 0.01691 -0.9879 - 0.01211 0.9494 + 0.02331 -0.00001 0.0000 - 0.00001 0.0000 - 0.00001 0.0000 + 0.00001
    
```

```

D =
55.2708 - 0.00001 0 0 0 0 0 0 0 0
0 26.0054 + 0.00001 16.0846 + 0.00001 0 0 0 0 0 0
0 0 0 2.9602 + 0.00001 0 0 0 0 0 0
0 0 0 0 3.2323 - 0.00001 0 0 0 0 0
0 0 0 0 0 5.7848 - 0.00001 30.1704 + 0.00001 0 0 0
0 0 0 0 0 0 0 0 14.3361 + 0.00001 0 0 0
0 0 0 0 0 0 0 0 0 5.8793 + 0.00001 0 0 0
    
```

```

*****
%% FINAL LIST OF NORMALIZED FREQUENCIES
*****
    
```

```

WN =
0.3880 1.0308 1.1832
0.4524 0.8810 0.8116
0.6798 0.1863 0.6393
0.7345 0.2725 0.738
0.7369 0.6267 0.2861
0.7345 0.6169 0.3828
0.4825 0.5450 0.8742
0.4524 0.4570 0.6026
0 0.4836 0.3859
    
```