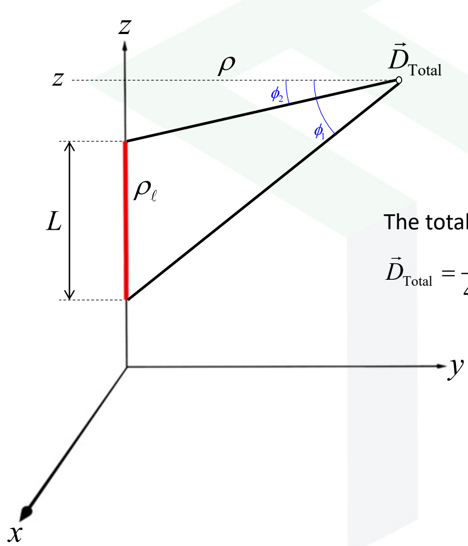
Electromagnetics:
Electromagnetic Field Theory

Example: Uniform Infinite Line Charge

Result from Uniform Finite Line Charge Example



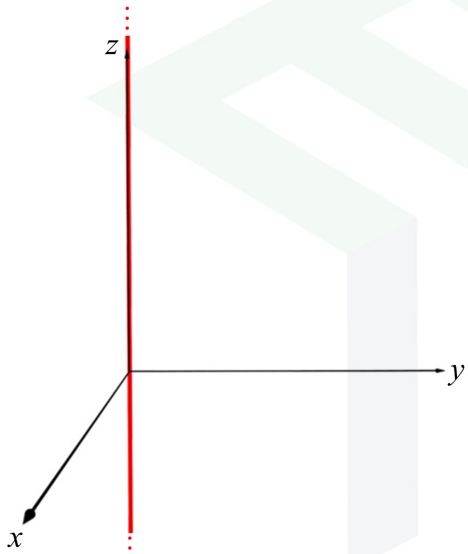
The total charge was

$$Q_{\text{Total}} = \rho_\ell L$$

The total field was

$$\vec{D}_{\text{Total}} = \frac{\rho_\ell}{4\pi\rho}\left[\left(\sin\phi_1 - \sin\phi_2\right)\hat{a}_\rho + \left(\cos\phi_2 - \cos\phi_1\right)\hat{a}_z\right]$$

Total Charge Q_{Total}



What is the total charge Q_{Total}?

Use the result from the finite line charge to get this answer simply by letting L go to infinity.

$$Q_{\text{Total}} = \rho_\ell L$$
$$= \rho_\ell \cdot \infty$$

$$\boxed{Q_{\text{Total}} = \infty}$$

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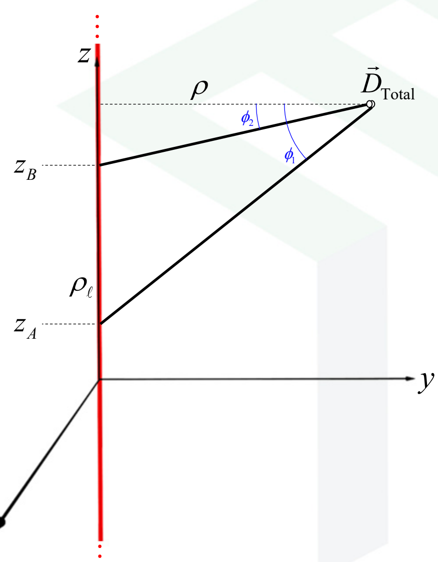
Total Field $\vec{D}_{\text{Total}}$



What is the total field $\vec{D}_{\text{Total}}$?

When the length of the line is infinity, the angles ϕ_1 and ϕ_2 become

$$\phi_1 = 90^\circ \qquad \phi_2 = -90^\circ$$

The total field is then derived from the expression for the finite length charge.

$$\vec{D}_{\text{Total}} = \frac{\rho_\ell}{4\pi\rho}\begin{bmatrix} (\sin\phi_1 - \sin\phi_2)\hat{a}_\rho \\ +(\cos\phi_2 - \cos\phi_1)\hat{a}_z \end{bmatrix}$$
$$= \frac{\rho_\ell}{4\pi\rho}\begin{Bmatrix} [\sin(90^\circ) - \sin(-90^\circ)]\hat{a}_\rho \\ +[\cos(-90^\circ) - \cos(90^\circ)]\hat{a}_z \end{Bmatrix}$$
$$= \frac{\rho_\ell}{4\pi\rho}\{[1-(-1)]\hat{a}_\rho + [0-0]\hat{a}_z\}$$

$$\boxed{\vec{D}_{\text{Total}} = \frac{\rho_\ell}{2\pi\rho}\hat{a}_\rho}$$

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