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Electromagnetics:
Electromagnetic Field Theory

Example:
Uniform Spherical Charge

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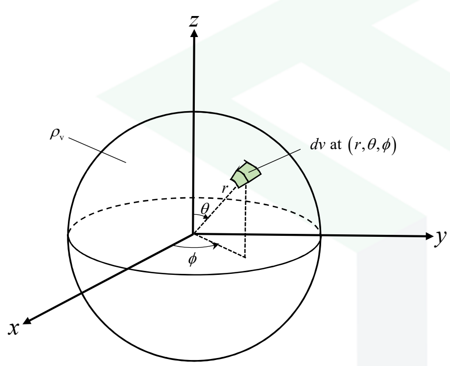
Example #5 – Spherical Volume Charge



What is the total charge Q_{Total}?

1. Draw the problem.
2. Choose a coordinate system.
   Spherical
3. Write general equation.
   $$Q_{\text{Total}} = \iiint_{v} \rho_v dv$$
4. Write expressions for each term.
   $$\rho_v = \rho_v \qquad dv = r^2 \sin\theta dr d\theta d\phi$$
5. Choose limits of integration.
   $$Q_{\text{Total}} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \int_{r=0}^{R} \rho_v r^2 \sin\theta dr d\theta d\phi$$

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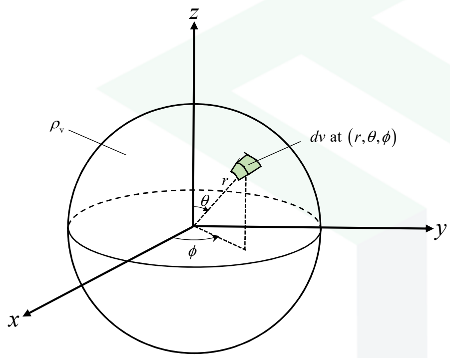
Example #5 – Spherical Volume Charge



What is the total charge Q_{Total}?

6. Solve the integral.

$$Q_{\text{Total}} = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\int_{r=0}^{R} \rho_v r^2 \sin\theta\, dr\, d\theta\, d\phi$$

$$= \rho_v \int_{\theta=0}^{\pi}\left[\int_{\phi=0}^{2\pi}\left(\int_{r=0}^{R} r^2 dr\right) d\phi\right]\sin\theta\, d\theta$$

$$= \rho_v \int_{\theta=0}^{\pi}\left[\int_{\phi=0}^{2\pi}\left(\left.\frac{r^3}{3}\right|_{r=0}^{R}\right) d\phi\right]\sin\theta\, d\theta$$

$$= \frac{\rho_v R^3}{3}\int_{\theta=0}^{\pi}\left[\int_{\phi=0}^{2\pi} d\phi\right]\sin\theta\, d\theta$$

$$= \frac{\rho_v R^3}{3}\int_{\theta=0}^{\pi}\left[\phi\Big|_{\phi=0}^{2\pi}\right]\sin\theta\, d\theta$$

$$= \frac{\rho_v R^3}{3}\int_{\theta=0}^{\pi}[2\pi]\sin\theta\, d\theta$$

$$= \frac{2\pi\rho_v R^3}{3}\int_{\theta=0}^{\pi}\sin\theta\, d\theta$$

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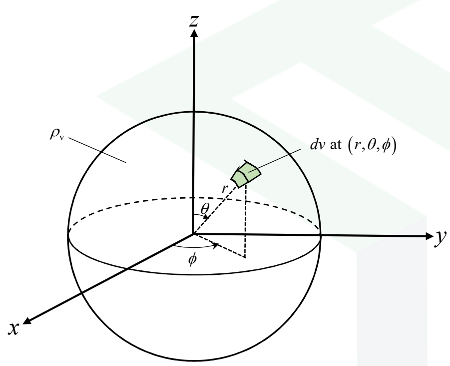
Example #5 – Spherical Volume Charge



What is the total charge Q_{Total}?

6. Solve the integral.

$$Q_{\text{Total}} = \frac{2\pi\rho_v R^3}{3}\int_{\theta=0}^{\pi}\sin\theta\, d\theta$$

$$= \frac{2\pi\rho_v R^3}{3}\left(-\cos\theta\right)\Big|_{\theta=0}^{\pi}$$

$$= \frac{2\pi\rho_v R^3}{3}\left[\left(-\cos\pi\right)-\left(-\cos 0\right)\right]$$

$$= \frac{2\pi\rho_v R^3}{3}[1+1]$$

$$\boxed{Q_{\text{Total}} = \rho_v \frac{4\pi R^3}{3}}$$

7. Interpret the result.

$Q_{\text{Total}} = \rho_v V$ *for uniform charge density*

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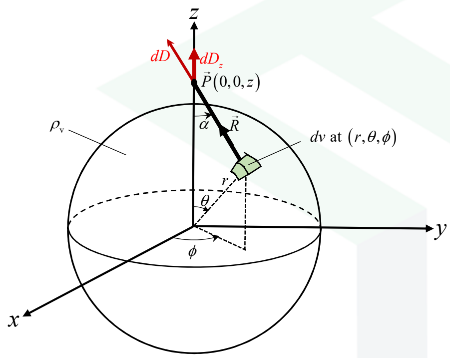
Example #5 – Spherical Volume Charge



What is the total field $\vec{D}$

1. Draw the problem.
2. Choose a coordinate system.

   Spherical
3. Write general equation.

   $$\vec{D}_{\text{Total}} = \iiint_{v} \frac{\rho_v dv}{4\pi R^2}\hat{a}_R$$
4. Write expressions for each term.

   Due to symmetry, the field will only point outward in the $\hat{a}_r$ direction.

   For this reason, we can solve for the field at any convenient point and we will know the field at any other point.

   We will choose to calculate the D field along the z-axis.

   $$\vec{D}_{\text{Total}} = \frac{\rho_v}{4\pi}\iiint_{v} \frac{1}{R^2}\hat{a}_R dv$$

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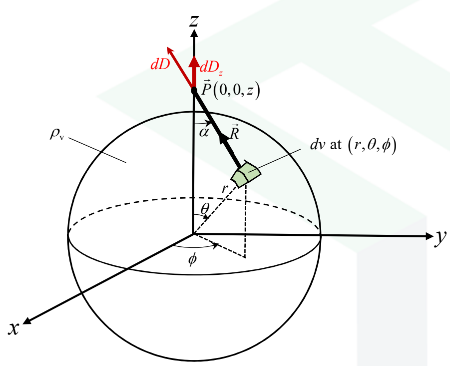
Example #5 – Spherical Volume Charge



What is the total field $\vec{D}$

5. Solve the integral.

   $$\vec{D}_{\text{Total}} = \frac{\rho_v}{4\pi}\iiint_{v} \frac{1}{R^2}\hat{a}_R dv$$

   Due to symmetry, all field components will cancel after the integration except the z-component.

   $$\vec{D}_{\text{Total}}(0,0,z) = \frac{\rho_v}{4\pi}\iiint_{v} \frac{1}{R^2}\left(\cos\alpha\,\hat{a}_z\right)dv$$

   $$\vdots$$

   $$\vec{D}_{\text{Total}}(0,0,z) = \frac{\rho_v}{4\pi}\frac{4\pi R^3}{3r^2}\hat{a}_z = \frac{1}{4\pi}\left(\rho_v \frac{4\pi R^3}{3}\right)\frac{1}{r^2}\hat{a}_z$$

   We generalize this to any point in spherical coordinates.

   $$\boxed{\vec{D}_{\text{Total}} = \frac{Q}{4\pi r^2}\hat{a}_r}$$

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