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Electromagnetics:
Electromagnetic Field Theory

Example: Field Via Gauss' Law

1

Recall Gauss' Law

$$Q = \iiint_{\mathrm{v}} \rho_{\mathrm{v}} dv = \oiint_{S} \vec{D} \bullet d\vec{s}$$

$$\nabla \bullet \vec{D} = \rho_{\mathrm{v}}$$

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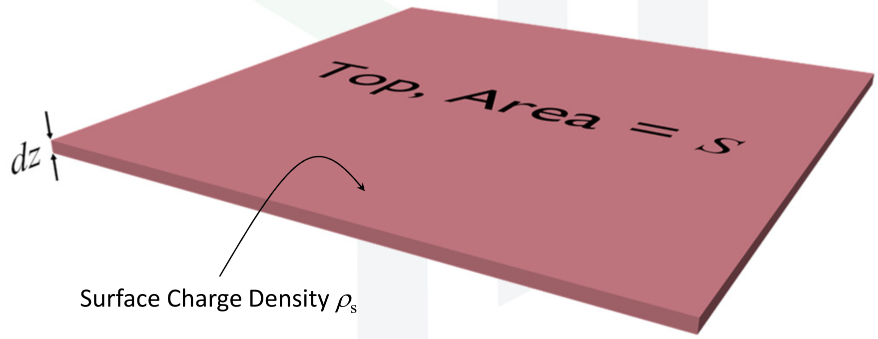
Problem Setup

Use Gauss' law to calculate the electric field around an infinite sheet charge.



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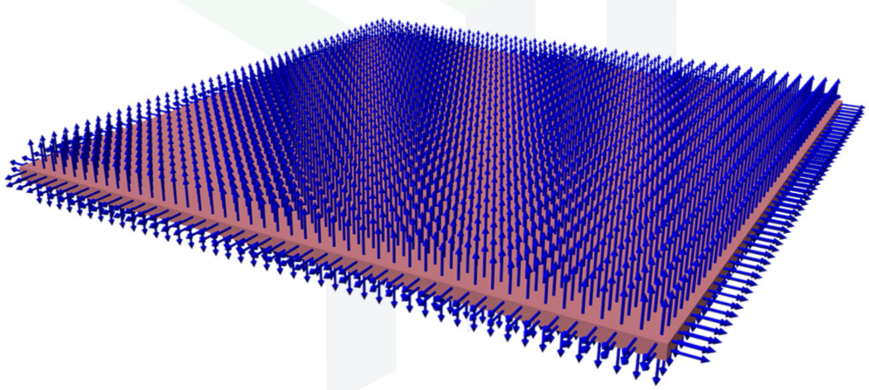
Integrate Flux

Method #1: $$Q = \oiint_S \vec{D} \bullet d\vec{s}$$ Integrate flux



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Integrate Flux

Method #1:

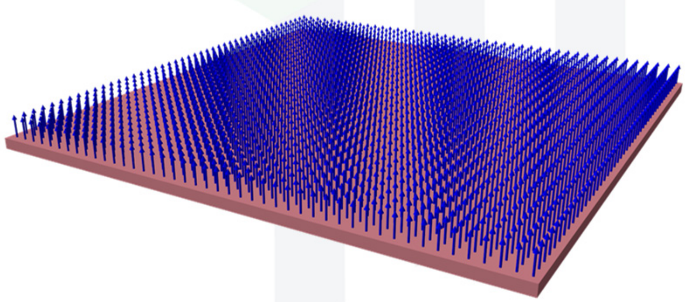
$$Q = \oiint_S \vec{D} \bullet d\vec{s} = \iint_{\text{top}} \vec{D} \bullet d\vec{s} + \iint_{\text{bottom}} \vec{D} \bullet d\vec{s} + \iint_{\text{front}} \vec{D} \bullet d\vec{s} + \iint_{\text{right}} \vec{D} \bullet d\vec{s} + \iint_{\text{back}} \vec{D} \bullet d\vec{s} + \iint_{\text{left}} \vec{D} \bullet d\vec{s}$$

Separate the closed-surface integral into six separate surface integrals, one for each side of our charged distribution.

Integrate Flux

Method #1:

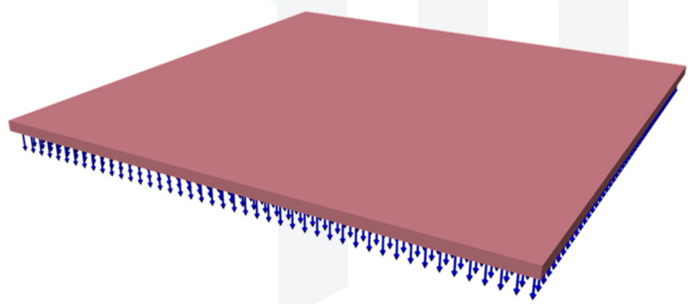
$$Q = \oiint_S \vec{D} \bullet d\vec{s} = \underbrace{\iint_{\text{top}} \vec{D} \bullet d\vec{s}}_{D_z S} + \iint_{\text{bottom}} \vec{D} \bullet d\vec{s} + \iint_{\text{front}} \vec{D} \bullet d\vec{s} + \iint_{\text{right}} \vec{D} \bullet d\vec{s} + \iint_{\text{back}} \vec{D} \bullet d\vec{s} + \iint_{\text{left}} \vec{D} \bullet d\vec{s}$$



Integrate Flux

Method #1:

$$Q = \oiint_S \vec{D} \bullet d\vec{s} = \iint_{\text{top}} \vec{D} \bullet d\vec{s} + \underbrace{\iint_{\text{bottom}} \vec{D} \bullet d\vec{s}}_{D_z S} + \iint_{\text{front}} \vec{D} \bullet d\vec{s} + \iint_{\text{right}} \vec{D} \bullet d\vec{s} + \iint_{\text{back}} \vec{D} \bullet d\vec{s} + \iint_{\text{left}} \vec{D} \bullet d\vec{s}$$



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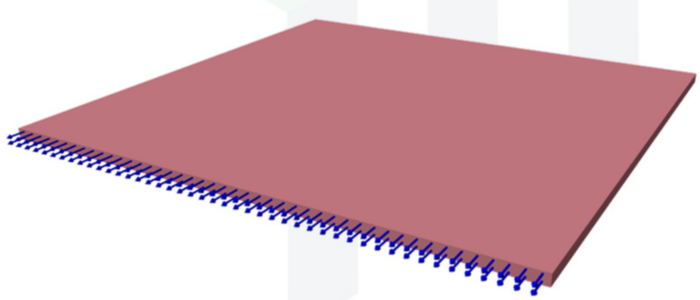
Integrate Flux

Method #1:

$$Q = \oiint_S \vec{D} \bullet d\vec{s} = \iint_{\text{top}} \vec{D} \bullet d\vec{s} + \iint_{\text{bottom}} \vec{D} \bullet d\vec{s} + \underbrace{\iint_{\text{front}} \vec{D} \bullet d\vec{s}}_{0} + \iint_{\text{right}} \vec{D} \bullet d\vec{s} + \iint_{\text{back}} \vec{D} \bullet d\vec{s} + \iint_{\text{left}} \vec{D} \bullet d\vec{s}$$

Area equals zero because $dz \rightarrow 0$.



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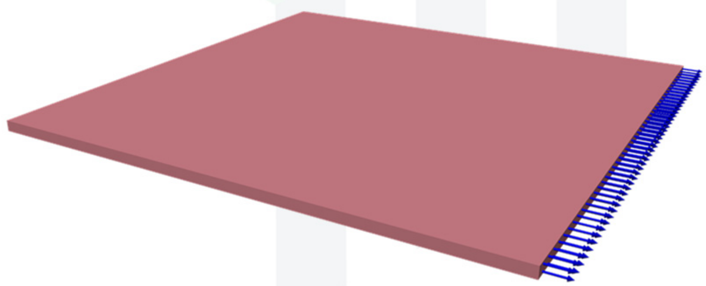
Integrate Flux

Method #1:

$$Q = \oiint_S \vec{D} \bullet d\vec{s} = \iint_{\text{top}} \vec{D} \bullet d\vec{s} + \iint_{\text{bottom}} \vec{D} \bullet d\vec{s} + \iint_{\text{front}} \vec{D} \bullet d\vec{s} + \underbrace{\iint_{\text{right}} \vec{D} \bullet d\vec{s}}_{0} + \iint_{\text{back}} \vec{D} \bullet d\vec{s} + \iint_{\text{left}} \vec{D} \bullet d\vec{s}$$

Area equals zero because $dz \rightarrow 0$.



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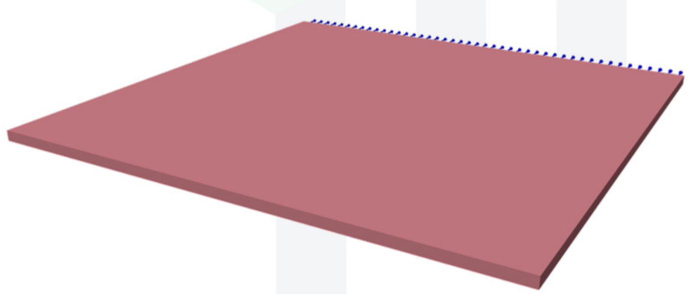
Integrate Flux

Method #1:

$$Q = \oiint_S \vec{D} \bullet d\vec{s} = \iint_{\text{top}} \vec{D} \bullet d\vec{s} + \iint_{\text{bottom}} \vec{D} \bullet d\vec{s} + \iint_{\text{front}} \vec{D} \bullet d\vec{s} + \iint_{\text{right}} \vec{D} \bullet d\vec{s} + \underbrace{\iint_{\text{back}} \vec{D} \bullet d\vec{s}}_{0} + \iint_{\text{left}} \vec{D} \bullet d\vec{s}$$

Area equals zero because $dz \rightarrow 0$.



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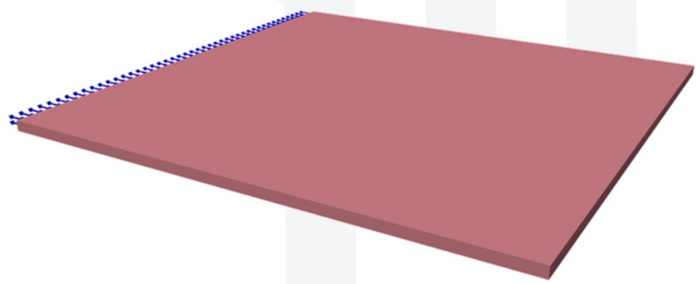
Integrate Flux

Method #1:

$$Q = \oiint_S \vec{D} \bullet d\vec{s} = \iint_{\text{top}} \vec{D} \bullet d\vec{s} + \iint_{\text{bottom}} \vec{D} \bullet d\vec{s} + \iint_{\text{front}} \vec{D} \bullet d\vec{s} + \iint_{\text{right}} \vec{D} \bullet d\vec{s} + \iint_{\text{back}} \vec{D} \bullet d\vec{s} + \underbrace{\iint_{\text{left}} \vec{D} \bullet d\vec{s}}_{0}$$

Area equals zero because $dz \rightarrow 0$.



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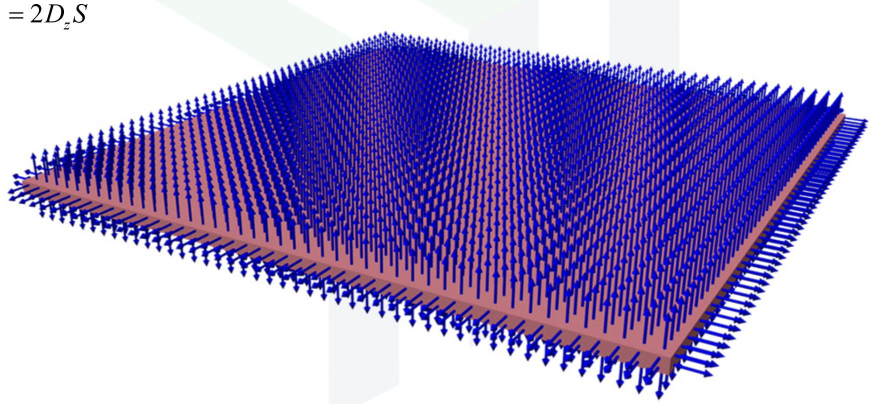
Integrate Flux

Method #1:

$$Q = \oiint_S \vec{D} \bullet d\vec{s} = D_z S + D_z S + 0 + 0 + 0 + 0$$

$$= 2D_z S$$



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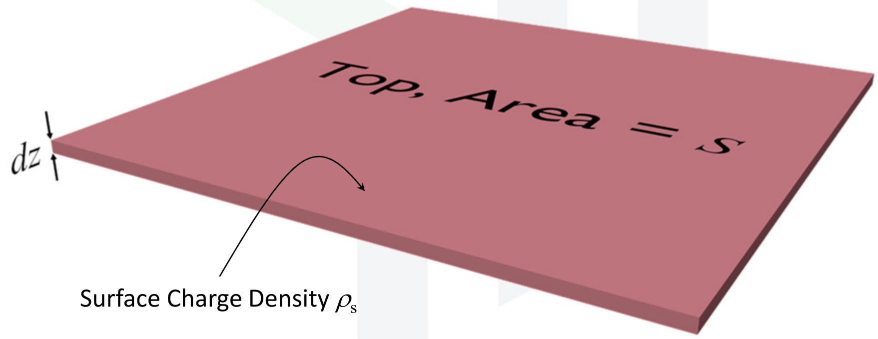
Integrate Charge Density

Method #2: $$Q = \iint_S \rho_s \, ds$$ Integrate charge density



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Integrate Charge Density

Method #2: $$Q = \iint_S \rho_s \, ds$$ Integrate charge density

$$= \rho_s \iint_S ds$$

$$= \rho_s S$$

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Equate Two Results

Method #1 = Method #2

$$2D_z S = \rho_s S$$

$$D_z = \frac{\rho_s}{2}$$

$$\boxed{\vec{D} = \frac{\rho_s}{2}\hat{a}_z}$$

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