



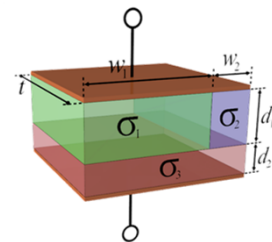
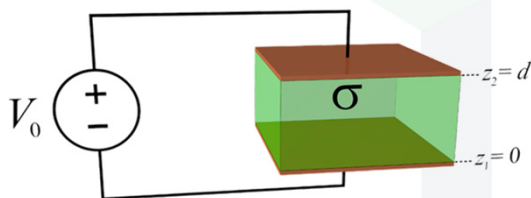
Electromagnetics:
Electromagnetic Field Theory

Resistor Examples

1

Outline

- Parallel plate resistor
- Inhomogeneous resistor



2

Example #1: Parallel Plate Resistor

Slide 3

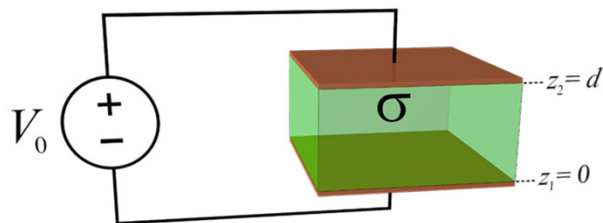
3

Example #1

Step 1 – Choose a coordinate system.

Cartesian

Step 2 – Assume applied voltage is V_0



 EMPossible

Slide 4

4

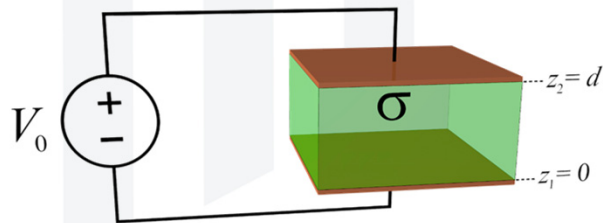
Example #1

Step 3 – Solve Laplace's equation in the area between the plates.

$$\nabla^2 V = 0$$

$$\frac{d^2 V}{dz^2} = 0 \quad V(0) = 0 \text{ and } V(d) = V_0$$

$$V(z) = \frac{V_0 z}{d} \quad 0 \leq z \leq d$$



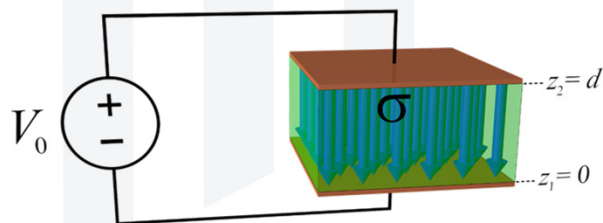
Example #1

Step 4 – Calculate the electric field intensity \vec{E}

$$\vec{E} = -\nabla V = -\frac{dV}{dz} \hat{a}_z$$

$$\vec{E} = -\frac{d}{dz} \left(\frac{V_0 z}{d} \right) \hat{a}_z \quad 0 \leq z \leq d$$

$$\vec{E} = -\frac{V_0}{d} \hat{a}_z \quad 0 \leq z \leq d$$



Example #1

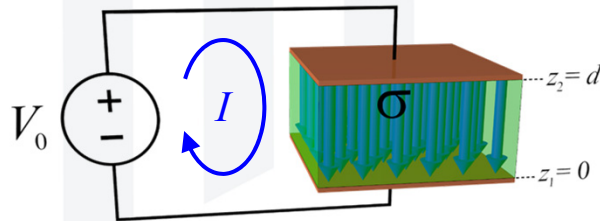
Step 5 – Calculate total current I

$$I = \iint_S \sigma \vec{E} \cdot d\vec{s} = \iint_S \sigma \left(-\frac{V_0}{d} \hat{a}_z \right) \cdot d\vec{s}$$

$$I = -\sigma \frac{V_0}{d} \underbrace{\iint_S \hat{a}_z \cdot d\vec{s}}_S = -\frac{\sigma V_0}{d} S$$

The sign can be ignored since the direction of the current is known.

$$I = \frac{\sigma V_0}{d} S$$

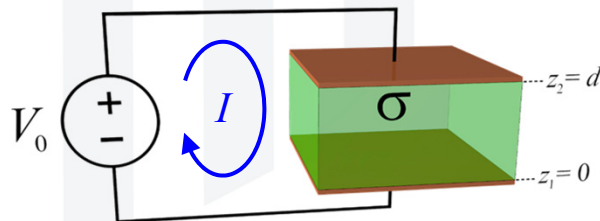


Example #1

Step 6 – Calculate equivalent resistance R using $R = V_0/I$.

$$R = \frac{V_0}{I} = \frac{V_0}{\frac{\sigma V_0}{d} S} = \frac{d}{\sigma S}$$

$$R = \frac{d}{\sigma S}$$



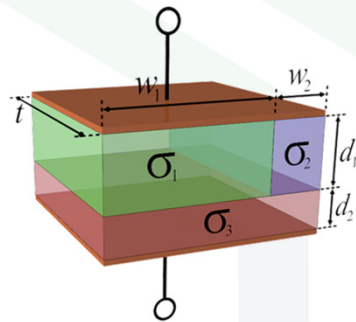
Example #2: Inhomogeneous Resistor

Slide 9

9

Example #2

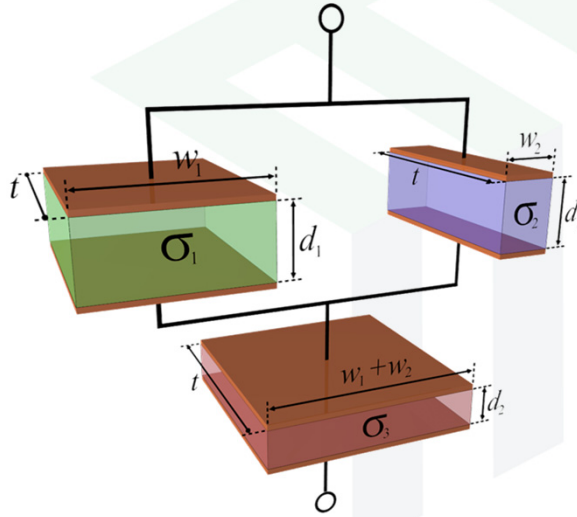
Suppose there exists an inhomogeneous resistor as shown here.



Slide 10

10

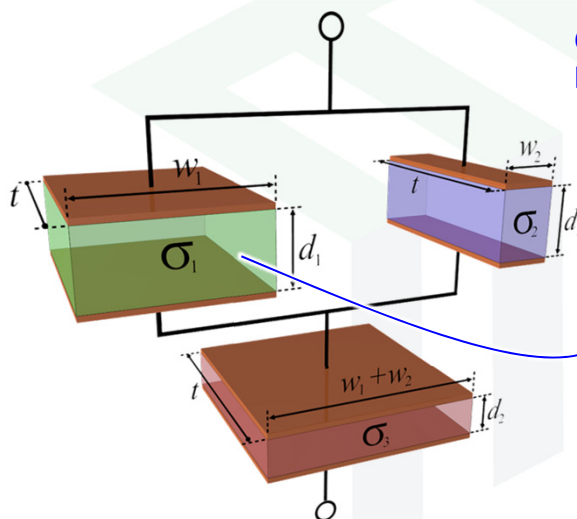
Example #2



The inhomogeneous resistor is separated into a network of homogeneous resistors.

11

Example #2

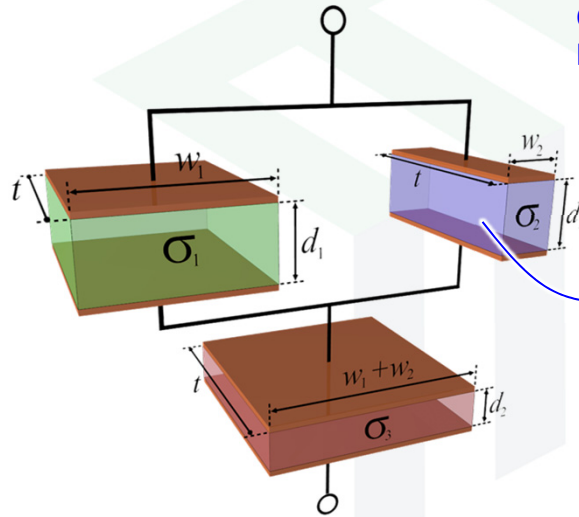


Calculate the resistance of each homogeneous resistor independently.

$$R_1 = \frac{d_1}{\sigma_1 S_1} = \frac{d_1}{\sigma_1 t w_1}$$

12

Example #2

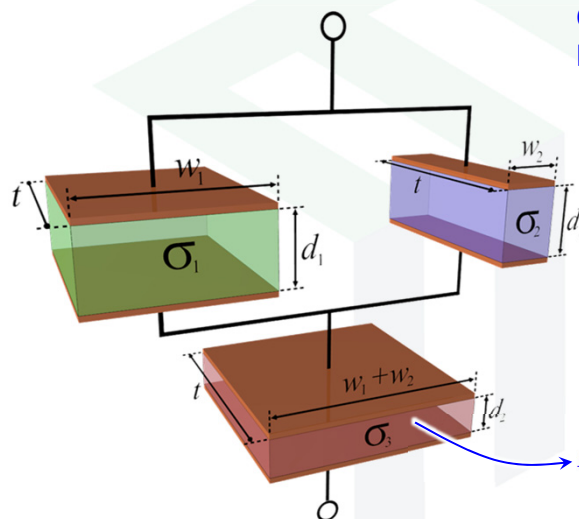


Calculate the resistance of each homogeneous resistor independently.

$$R_1 = \frac{d_1}{\sigma_1 S_1} = \frac{d_1}{\sigma_1 t w_1}$$

$$R_2 = \frac{d_1}{\sigma_2 S_2} = \frac{d_1}{\sigma_2 t w_2}$$

Example #2



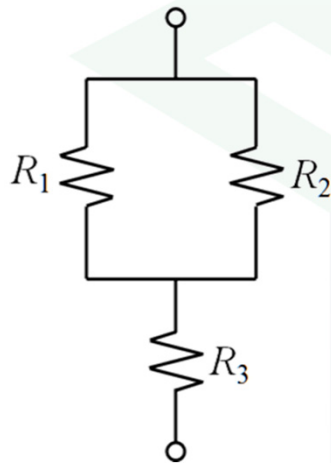
Calculate the resistance of each homogeneous resistor independently.

$$R_1 = \frac{d_1}{\sigma_1 S_1} = \frac{d_1}{\sigma_1 t w_1}$$

$$R_2 = \frac{d_1}{\sigma_2 S_2} = \frac{d_1}{\sigma_2 t w_2}$$

$$R_3 = \frac{d_2}{\sigma_3 S_3} = \frac{d_2}{\sigma_3 t (w_1 + w_2)}$$

Example #2



View the inhomogeneous resistor as a series/parallel combination of homogeneous resistors.

The equivalent resistance is

$$R_{\text{eq}} = R_1 \parallel R_2 + R_3$$

$$= \left(\frac{d_1}{\sigma_1 t w_1} \parallel \frac{d_1}{\sigma_2 t w_2} \right) + \frac{d_2}{\sigma_3 t (w_1 + w_2)}$$

$$R_{\text{eq}} = \frac{1}{t} \left[\frac{d_1 \sigma_1 \sigma_2 w_1 w_2}{\sigma_1 w_1 + \sigma_2 w_2} + \frac{d_2}{\sigma_3 (w_1 + w_2)} \right]$$