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Electromagnetics:
Electromagnetic Field Theory

Resistor Examples

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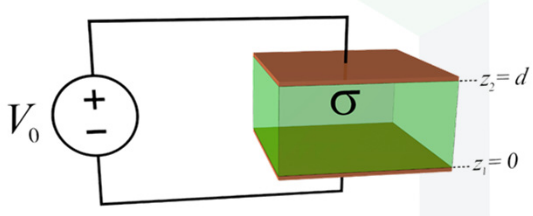
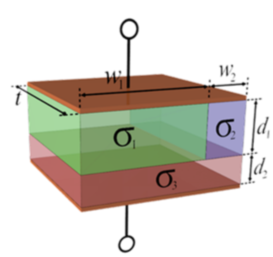
Outline

- Parallel plate resistor
- Inhomogeneous resistor





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Example #1:

Parallel Plate Resistor

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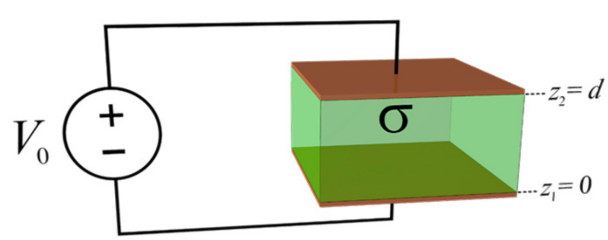
Example #1

Step 1 – Choose coordinate system.

Cartesian

Step 2 – Assume V_0



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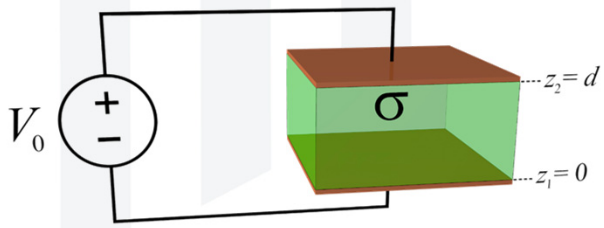
Example #1

Step 3 – Solve Laplace's equation

$$\nabla^2 V = 0$$

$$\frac{d^2V}{dz^2} = 0 \qquad V(0) = 0 \text{ and } V(d) = V_0$$

$$V(z) = \frac{V_0 z}{d} \qquad 0 \le z \le d$$



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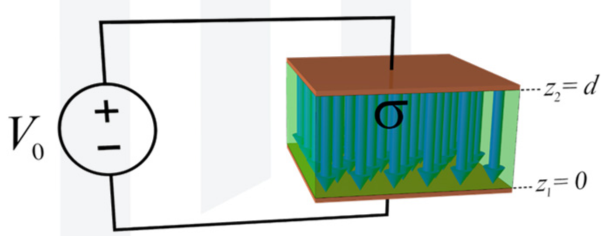
Example #1

Step 4 – Calculate $\vec{E}$

$$\vec{E} = -\nabla V = -\frac{dV}{dz}\hat{a}_z$$

$$\vec{E} = -\frac{d}{dz}\left(\frac{V_0 z}{d}\right)\hat{a}_z \qquad 0 \le z \le d$$

$$\vec{E} = -\frac{V_0}{d}\hat{a}_z \qquad 0 \le z \le d$$



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Example #1

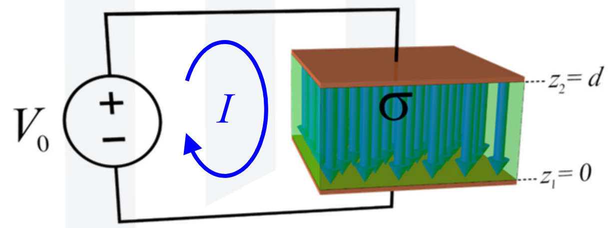
Step 5 – Calculate Current I

$$I = \iint_S \sigma \vec{E} \bullet d\vec{s} = \iint_S \sigma \left(-\frac{V_0}{d} \hat{a}_z \right) \bullet d\vec{s}$$

$$I = -\sigma \frac{V_0}{d} \underbrace{\iint_S \hat{a}_z \bullet d\vec{s}}_{S} = -\frac{\sigma V_0}{d} S$$

The sign can be ignored since the direction of the current is known.

$$I = \frac{\sigma V_0}{d} S$$



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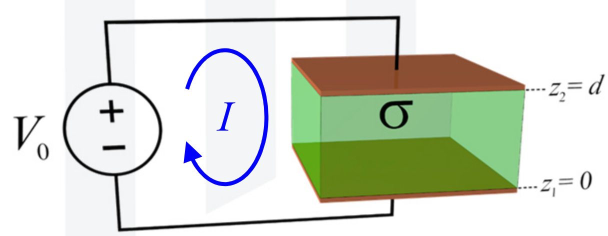
Example #1

Step 6 – Calculate R using $R = V_0/I$.

$$R = \frac{V_0}{I} = \frac{V_0}{\frac{\sigma V_0}{d} S} = \frac{d}{\sigma S}$$

$$\boxed{R = \frac{d}{\sigma S}}$$



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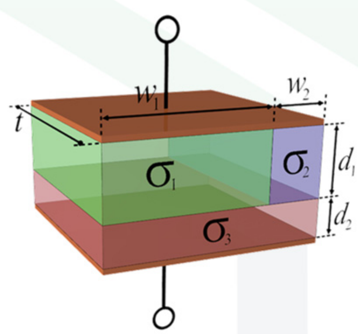
Example #2:

Inhomogeneous Resistor

Example #2

Suppose there exists an inhomogeneous resistor.



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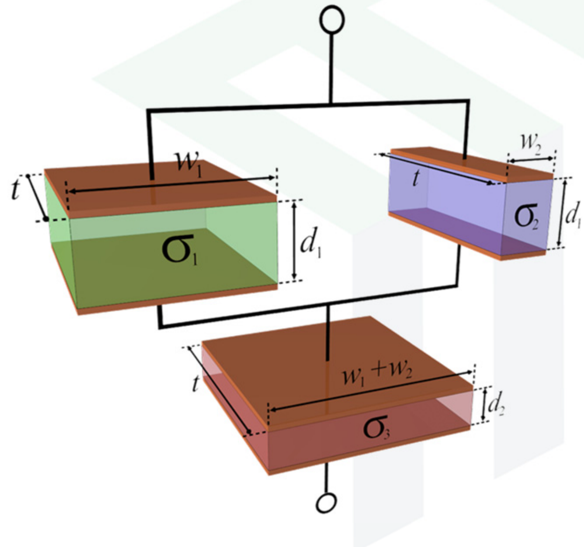
Example #2



The inhomogeneous resistor is separated into a combination of homogeneous resistors.

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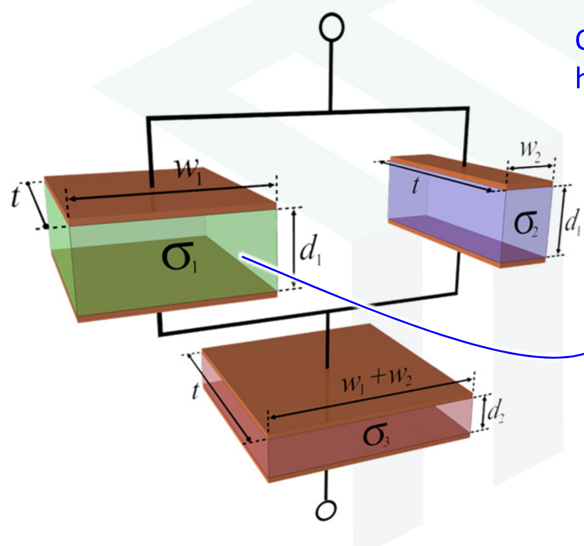
Example #2



Calculate the capacitance of each homogeneous resistor independently.

$$R_1 = \frac{d_1}{\sigma_1 S_1} = \frac{d_1}{\sigma_1 t w_1}$$

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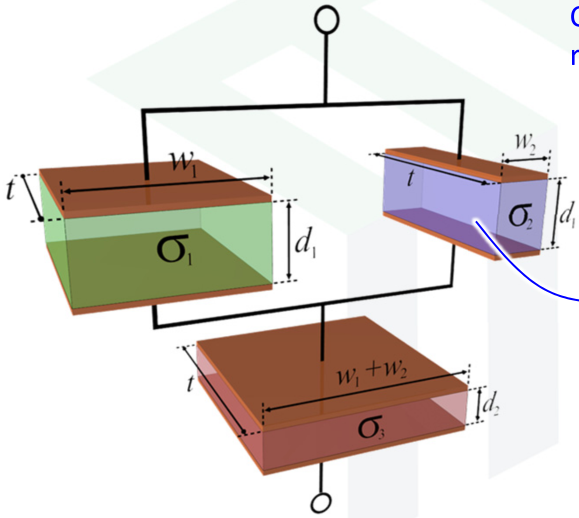
Example #2



Calculate each homogeneous resistor independently.

$$R_1 = \frac{d_1}{\sigma_1 S_1} = \frac{d_1}{\sigma_1 t w_1}$$

$$R_2 = \frac{d_1}{\sigma_2 S_2} = \frac{d_1}{\sigma_2 t w_2}$$

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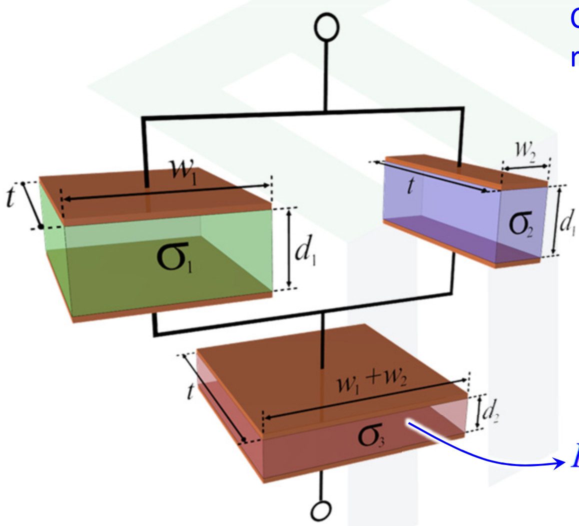
Example #2



Calculate each homogeneous resistor independently.

$$R_1 = \frac{d_1}{\sigma_1 S_1} = \frac{d_1}{\sigma_1 t w_1}$$

$$R_2 = \frac{d_1}{\sigma_2 S_2} = \frac{d_1}{\sigma_2 t w_2}$$

$$R_3 = \frac{d_2}{\sigma_3 S_3} = \frac{d_2}{\sigma_3 t \left(w_1 + w_2 \right)}$$

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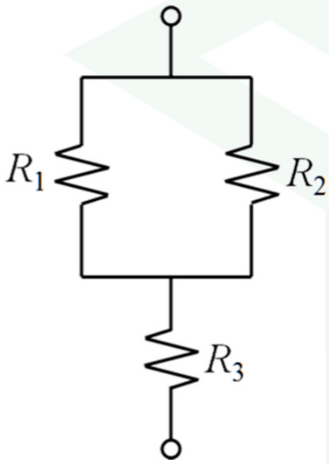
Example #2



We view the resistor as a series/parallel combination of resistors.

The equivalent resistance is

$$R_{\text{eq}} = R_1 \| R_2 + R_3$$

$$= \left(\frac{d_1}{\sigma_1 t w_1} \middle\| \frac{d_1}{\sigma_2 t w_2} \right) + \frac{d_2}{\sigma_3 t \left(w_1 + w_2 \right)}$$

$$\boxed{R_{\text{eq}} = \frac{1}{t}\left[\frac{d_1 \sigma_1 \sigma_2 w_1 w_2}{\sigma_1 w_1 + \sigma_2 w_2} + \frac{d_2}{\sigma_3 \left(w_1 + w_2 \right)} \right]}$$