Advanced Computation:
Computational Electromagnetics

Preliminary Topics

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Outline

- Review of Linear Algebra
- Complex Wave Vector
- TE & TM Polarization
- Index Ellipsoids
- Electromagnetic Behavior at an Interface
  - Phase matching at an interface
  - The Fresnel equations
  - Visualization
- Image Theory

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Review of Linear Algebra

Matrices Represent Sets of Equations

A set of linear algebraic equations can be written in "matrix" form.

$$a_{11}w + a_{12}x + a_{13}y + a_{14}z = b_1$$
$$a_{21}w + a_{22}x + a_{23}y + a_{24}z = b_2$$
$$a_{31}w + a_{32}x + a_{33}y + a_{34}z = b_3$$
$$a_{41}w + a_{42}x + a_{43}y + a_{44}z = b_4$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

Interpretation of Matrices

$$a_{11}x + a_{12}y + a_{13}z = b_1$$
$$a_{21}x + a_{22}y + a_{23}z = b_2$$
$$a_{31}x + a_{32}y + a_{33}z = b_3$$

$$\Rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

EQUATION FOR…

Equation for x → $a_{11} \quad a_{12} \quad a_{13}$

Equation for y → $a_{21} \quad a_{22} \quad a_{23}$

Equation for z → $a_{31} \quad a_{32} \quad a_{33}$

RELATION TO…

Relation to x ↓   Relation to y ↓   Relation to z ↓

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

From a purely mathematical perspective, this interpretation does not make sense. This interpretation will be highly useful and insightful because of how we derive the equations.

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Compact Matrix Notation

Matrices and vectors can be represented and treated as single variables.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \Rightarrow \mathbf{Ax} = \mathbf{b}$$

or

$$[A][x] = [b]$$

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

square matrix

$$\mathbf{x} = \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}$$

column vector

$$\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

column vector

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Matrices Require Special Algebra

$$\mathbf{AB} \neq \mathbf{BA}$$

Commutative Laws

$$\mathbf{AB} \neq \mathbf{BA}$$
$$\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}$$

Associative Laws

$$(\mathbf{AB})\mathbf{C}=\mathbf{A}(\mathbf{BC})$$
$$(\mathbf{A}+\mathbf{B})+\mathbf{C}=\mathbf{A}+(\mathbf{B}+\mathbf{C})$$

Distributive Laws

$$\mathbf{A}(\mathbf{B}+\mathbf{C})=\mathbf{AB}+\mathbf{AC}$$
$$(\mathbf{A}+\mathbf{B})\mathbf{C}=\mathbf{AC}+\mathbf{BC}$$

Multiplication with a Scalar

$$\alpha(\mathbf{A}+\mathbf{B})=\alpha\mathbf{A}+\alpha\mathbf{B}$$
$$\alpha(\mathbf{AB})=(\alpha\mathbf{A})\mathbf{B}=\mathbf{A}(\alpha\mathbf{B})$$

Addition with a Scalar

$$\alpha+\mathbf{A}=?$$
$$\alpha\mathbf{I}+\mathbf{A}=\begin{bmatrix}\alpha+a_{11} & \cdots & a_{1n}\\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & \alpha+a_{mn}\end{bmatrix}$$

Matrix Inverses and Transposes

$$\left(\mathbf{A}^{-1}\right)^{-1}=\mathbf{A} \qquad (\mathbf{AB})^{-1}=\mathbf{B}^{-1}\mathbf{A}^{-1}$$
$$\left(\mathbf{A}^{T}\right)^{T}=\mathbf{A} \qquad (\mathbf{A}+\mathbf{B})^{T}=\mathbf{A}^{T}+\mathbf{B}^{T} \qquad (\mathbf{AB})^{T}=\mathbf{B}^{T}\mathbf{A}^{T}$$
$$\left(\mathbf{A}^{T}\right)^{-1}=\left(\mathbf{A}^{-1}\right)^{T}$$

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Manipulating Matrix Equations

Example #1: Dividing both sides by a matrix on the right

$(\mathbf{A}+\mathbf{B})\mathbf{C}=\mathbf{D}+\mathbf{E}$	Starting equation
$(\mathbf{A}+\mathbf{B})\mathbf{CC}^{-1}=(\mathbf{D}+\mathbf{E})\mathbf{C}^{-1}$	Post-multiply both sides by $\mathbf{C}^{-1}$
$\mathbf{A}+\mathbf{B}=(\mathbf{D}+\mathbf{E})\mathbf{C}^{-1}$	Recall that $\mathbf{CC}^{-1}=\mathbf{I}$

Example #2: Dividing both sides by a matrix on the left

$\mathbf{C}(\mathbf{A}+\mathbf{B})=\mathbf{D}+\mathbf{E}$	Starting equation
$\mathbf{C}^{-1}\mathbf{C}(\mathbf{A}+\mathbf{B})=\mathbf{C}^{-1}(\mathbf{D}+\mathbf{E})$	Pre-multiply both sides by $\mathbf{C}^{-1}$
$\mathbf{A}+\mathbf{B}=\mathbf{C}^{-1}(\mathbf{D}+\mathbf{E})$	Recall that $\mathbf{C}^{-1}\mathbf{C}=\mathbf{I}$

Example #3: Simplify an expression

$\left(\mathbf{C}^{-1}\mathbf{A}\right)^{-1}+\mathbf{D}=\mathbf{BC}+\mathbf{D}$	Starting equation
$\left(\mathbf{C}^{-1}\mathbf{A}\right)^{-1}=\mathbf{BC}$	Subtract $\mathbf{D}$ from both sides
$\mathbf{A}^{-1}\mathbf{C}=\mathbf{BC}$	Recall inverse of a product rule
$\mathbf{A}^{-1}\mathbf{CC}^{-1}=\mathbf{BCC}^{-1}$	Post-multiply both sides by $\mathbf{C}^{-1}$
$\mathbf{A}^{-1}=\mathbf{B}$	Recall that $\mathbf{C}^{-1}\mathbf{C}=\mathbf{I}$

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The Zero and the Identity Matrices

Zero Matrix

$$\mathbf{0} = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}$$

$$\mathbf{0} \cdot \mathbf{A} = \mathbf{A} \cdot \mathbf{0} = \mathbf{0}$$
$$\mathbf{0} + \mathbf{A} = \mathbf{A} + \mathbf{0} = \mathbf{A}$$
$$\mathbf{A} - \mathbf{A} = \mathbf{0}$$

Identity Matrix

$$\mathbf{I} = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$

$$\mathbf{I} \cdot \mathbf{A} = \mathbf{A} \cdot \mathbf{I} = \mathbf{A}$$
$$\mathbf{A}^{-1}\mathbf{A} = \mathbf{A}\mathbf{A}^{-1} = \mathbf{I}$$

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Matrix Division

It is very rare to calculate the inverse of matrix because it is very computationally intensive to do this.

At first glance, matrix division appears to require calculating the inverse of a matrix, but highly efficient algorithms exist that do not need to do this.

Pre-Division: $\mathbf{A} = \mathbf{B}^{-1}\mathbf{C}$ ~~`A = inv(B)*C;`~~ `A = B\C;`

Post-Division: $\mathbf{A} = \mathbf{C}\mathbf{B}^{-1}$ ~~`A = C*inv(B);`~~ `A = C/B;`

Despite that both of these equations are dividing by $\mathbf{B}$, pre- and post-division do NOT lead to the same result.

$$\mathbf{B}^{-1}\mathbf{C} \neq \mathbf{C}\mathbf{B}^{-1}$$

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Matrix Division With More than Two Terms

Suppose the following matrix expression is to be evaluated.

$$\mathbf{A} = \mathbf{B}\mathbf{C}^{-1}\mathbf{D}$$

Would the following MATLAB code work?

```
A = B*C\D;
```

No! Remember the order of operations.
MATLAB will first multiply $\mathbf{B}$ and $\mathbf{C}$.

$$\mathbf{A} = \mathbf{B}\mathbf{C}$$

It will then backward divide on $\mathbf{D}$.

$$\mathbf{A} = (\mathbf{B}\mathbf{C})^{-1}\mathbf{D}$$

So what is the correct code?

```
A = B/C*D;
A = B*(C\D);
```

It might be a good idea to time these two calculations to see which is faster.

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Proper Notation for Matrix Division

It is almost never correct to write matrix division as a fraction.

$$\frac{\mathbf{A}}{\mathbf{B}}$$

WRONG

Why?

Does this represent predivision or postdivision?

$$\mathbf{B}^{-1}\mathbf{A} \quad \text{or} \quad \mathbf{A}\mathbf{B}^{-1} \quad ???$$

It is impossible to say, thus fraction notation is incorrect for matrices.

One exception → When both $\mathbf{A}$ and $\mathbf{B}$ are diagonal matrices, both pre- and post-division will give the same answer.

$$\begin{bmatrix} b_{11} & & & \\ & b_{22} & & \\ & & \ddots & \\ & & & b_{MM} \end{bmatrix}^{-1} \begin{bmatrix} a_{11} & & & \\ & a_{22} & & \\ & & \ddots & \\ & & & a_{MM} \end{bmatrix} = \begin{bmatrix} a_{11} & & & \\ & a_{22} & & \\ & & \ddots & \\ & & & a_{MM} \end{bmatrix} \begin{bmatrix} b_{11} & & & \\ & b_{22} & & \\ & & \ddots & \\ & & & b_{MM} \end{bmatrix}^{-1} = \begin{bmatrix} a_{11}/b_{11} & & & \\ & a_{22}/b_{22} & & \\ & & \ddots & \\ & & & a_{MM}/b_{MM} \end{bmatrix}$$

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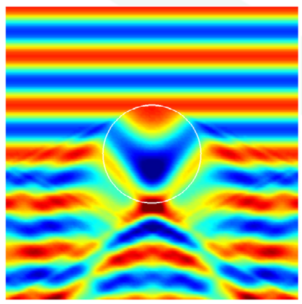
$\mathbf{Ax} = \mathbf{b}$ Vs. $\mathbf{Ax} = \lambda\mathbf{x}$

Standard Linear Problem

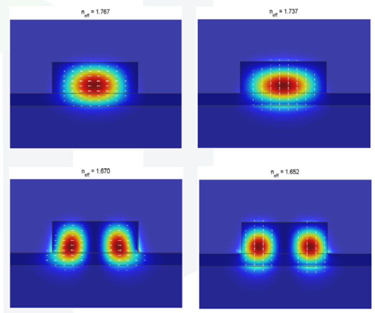
$$\mathbf{Ax} = \mathbf{b}$$



- Has only one answer
- Requires a source

Eigen-Value Problem

$$\mathbf{Ax} = \lambda\mathbf{x}$$



- Has an infinite number of answers
- Modes
- No source

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Complex Wave Vector

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The Complex Wave Number k

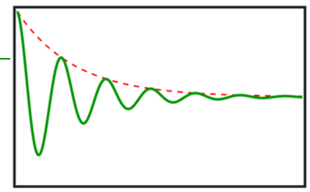
A wave travelling the $+z$ direction can be written in terms of the complex wave number k as

$$\vec{E}(z) = \vec{P}e^{-jkz} \qquad\qquad k = k' - jk''$$

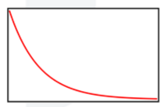
Substituting this into the wave solution yields

$$\vec{E}(z) = \vec{P}e^{-j(k'-jk'')z} = \underbrace{\vec{P}e^{-k''z}}_{\text{attenuation}}\underbrace{e^{-jk'z}}_{\text{oscillation}}$$

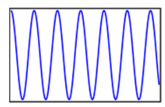
attenuation & oscillation



attenuation



oscillation



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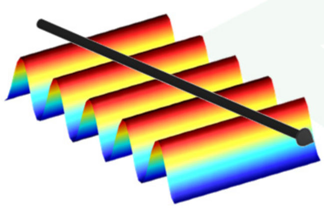
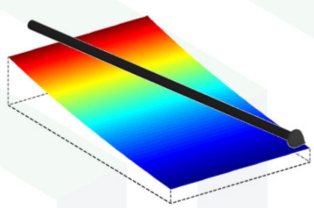
Waves with Complex k

Purely Real k



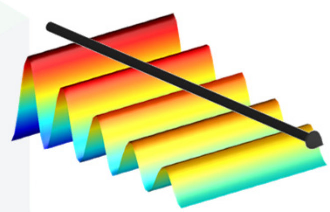
- Uniform amplitude
- Oscillations move power
- Considered to be a propagating wave

Purely Imaginary k



- Decaying amplitude
- No oscillations, no flow of power
- Considered to be evanescent

Complex k



- Decaying amplitude
- Oscillations move power
- Considered to be a propagating wave (not evanescent)

This implies that these are the only 2.5 configurations that electromagnetic fields can take on.

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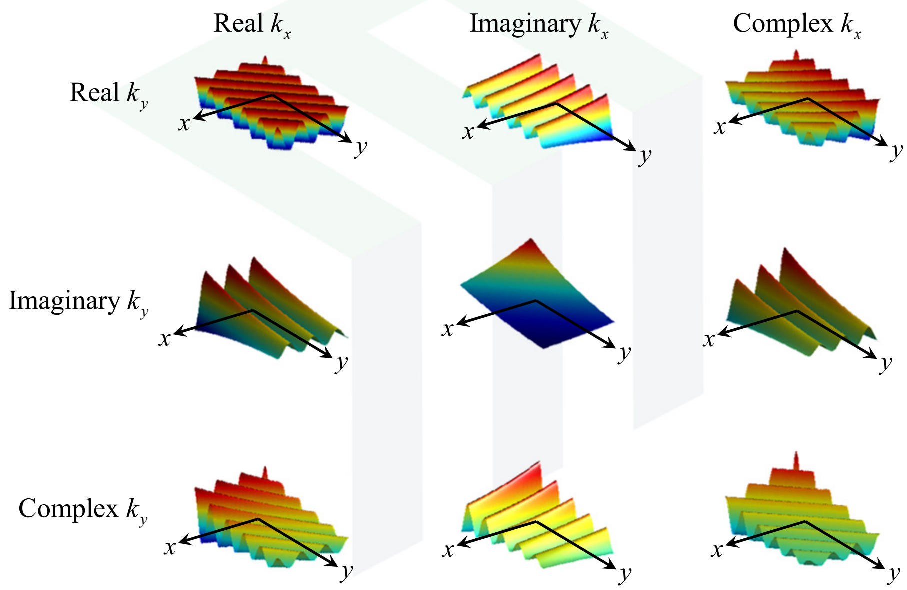
2D Waves with Doubly Complex $\vec{k}$



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TE & TM Polarization

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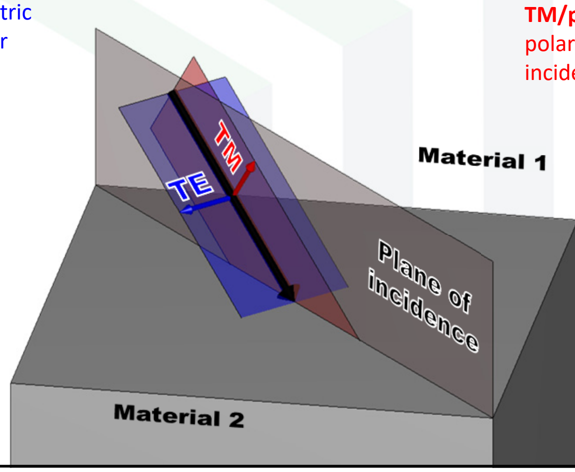
TE and TM

We use the labels "TE" and "TM" when we are describing the orientation of a linearly polarized wave relative to a device.

TE/perpendicular/s – the electric field is polarized perpendicular to the plane of incidence.

TM/parallel/p – the electric field is polarized parallel to the plane of incidence.



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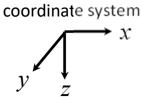
Calculating the Polarization Vectors

Incident Wave Vector

$$\vec{k}_{\text{inc}} = k_0 n_{\text{inc}} \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix}$$

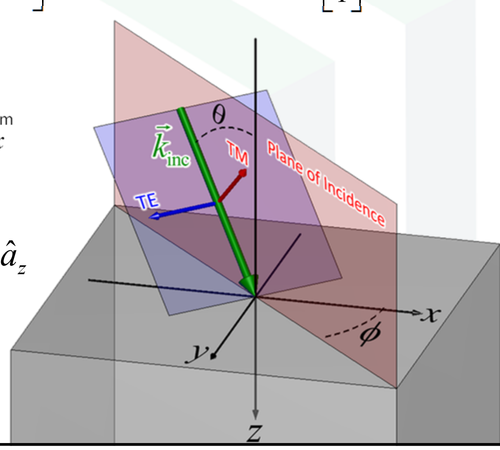
Right-handed coordinate system



$$\hat{a}_x \times \hat{a}_y = \hat{a}_z$$

Surface Normal

$$\hat{n} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$



Unit Vectors of Polarization Directions

$$\hat{a}_{\text{TE}} = \begin{cases} \hat{a}_y & \theta = 0^\circ \\ \dfrac{\hat{n}\times\vec{k}_{\text{inc}}}{\left|\hat{n}\times\vec{k}_{\text{inc}}\right|} & \theta \neq 0^\circ \end{cases}$$

$$\hat{a}_{\text{TM}} = \frac{\vec{k}_{\text{inc}}\times\hat{a}_{\text{TE}}}{\left|\vec{k}_{\text{inc}}\times\hat{a}_{\text{TE}}\right|}$$

Composite Polarization Vector

$$\vec{P} = p_{\text{TE}}\hat{a}_{\text{TE}} + p_{\text{TM}}\hat{a}_{\text{TM}}$$

In CEM, we usually make

$$\left|\vec{P}\right| = 1$$

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Phase Matching at an Interface

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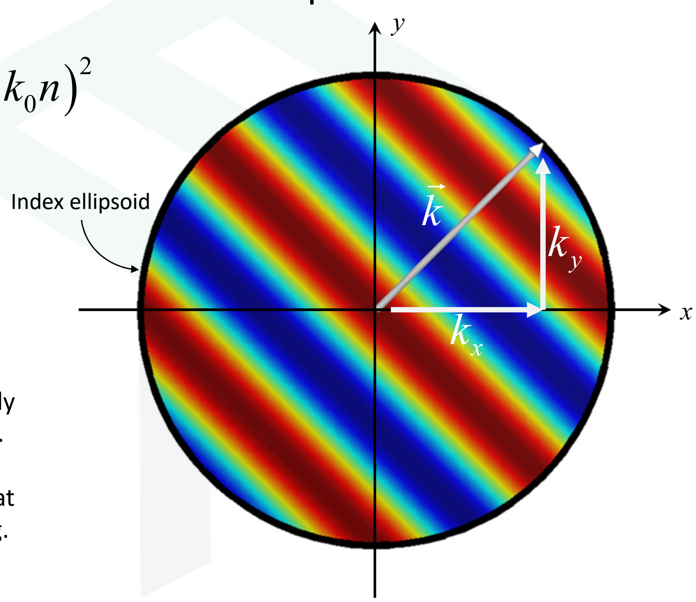
Illustration of the Dispersion Relation

$$k_x^2 + k_y^2 = \left|\vec{k}\right|^2 = \left(k_0 n\right)^2$$



The dispersion relation for isotropic materials is essentially just the Pythagorean theorem. It says a wave sees the same refractive index no matter what direction the wave is travelling.

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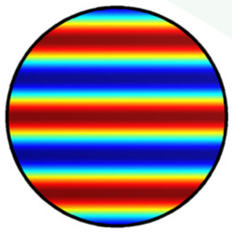
Index Ellipsoid in Two Different Materials

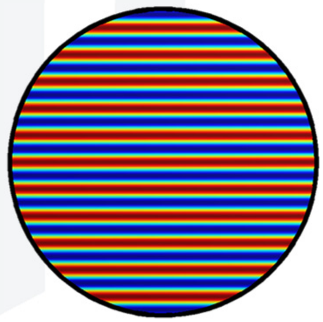
Material 1 (Low n)

$$k_{x,1}^2 + k_{y,1}^2 = \left|\vec{k}_1\right|^2 = \left(k_0 n_1\right)^2$$



Material 2 (High n)

$$k_{x,2}^2 + k_{y,2}^2 = \left|\vec{k}_2\right|^2 = \left(k_0 n_2\right)^2$$



$n_1 < n_2$

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Phase Matching at the Interface Between Two Materials Where $n_1 < n_2$

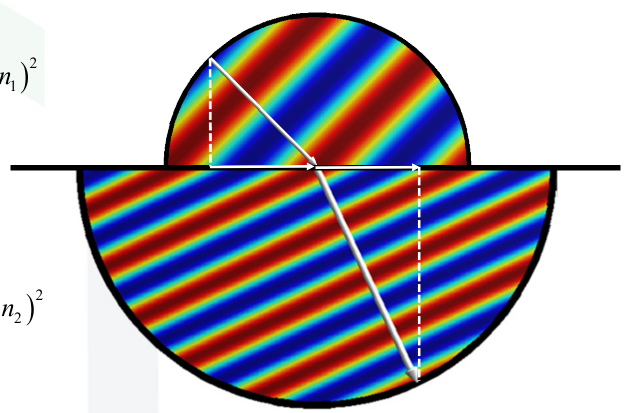
$n_1 < n_2$

Material 1

$$k_{x,1}^2 + k_{y,1}^2 = \left|\vec{k}_1\right|^2 = \left(k_0 n_1\right)^2$$

Material 2

$$k_{x,2}^2 + k_{y,2}^2 = \left|\vec{k}_2\right|^2 = \left(k_0 n_2\right)^2$$



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Summary of the Phase Matching Trend for $n_1 < n_2$

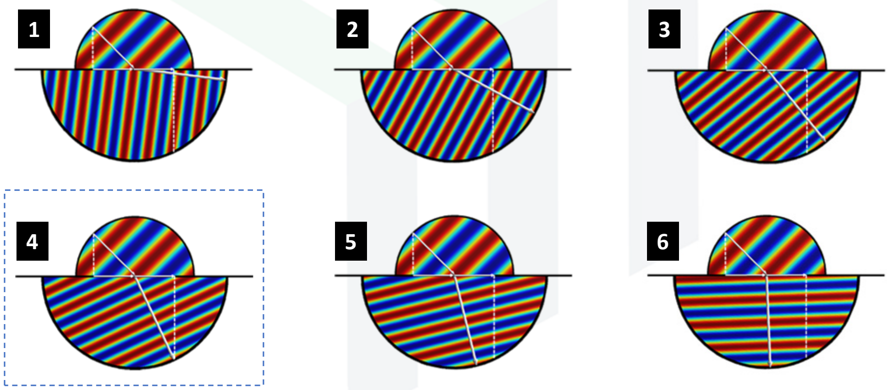
$n_1 < n_2$

Material 1

$k_{x,1}^2 + k_{y,1}^2 = \left|\vec{k}_1\right|^2 = \left(k_0 n_1\right)^2$

Material 2

$k_{x,2}^2 + k_{y,2}^2 = \left|\vec{k}_2\right|^2 = \left(k_0 n_2\right)^2$



Properly phased matched at the interface.

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Phase Matching at the Interface Between Two Materials Where $n_1 > n_2$

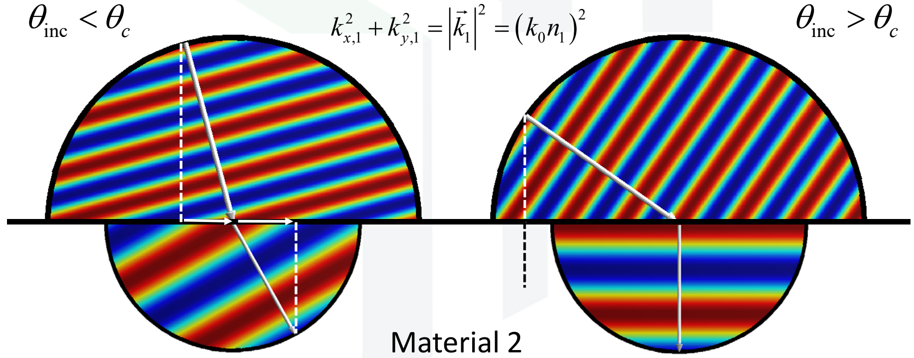
$n_1 > n_2$

Material 1

$k_{x,1}^2 + k_{y,1}^2 = \left|\vec{k}_1\right|^2 = \left(k_0 n_1\right)^2$

$\theta_{\text{inc}} < \theta_c$

$\theta_{\text{inc}} > \theta_c$



Material 2

$k_{x,2}^2 + k_{y,2}^2 = \left|\vec{k}_2\right|^2 = \left(k_0 n_2\right)^2$

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Summary of the Phase Matching Trend for $n_1 > n_2$

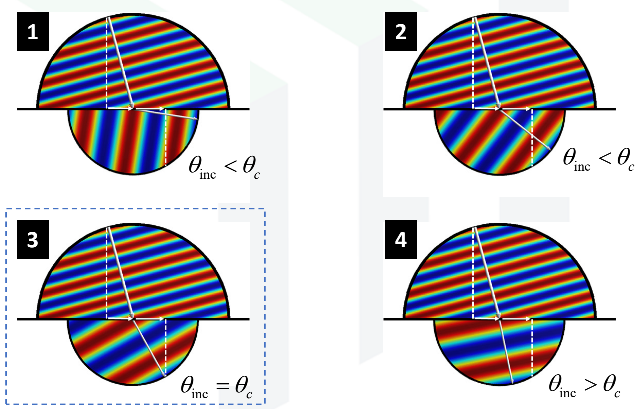
$$n_1 > n_2$$

Material 1

$$k_{x,1}^2 + k_{y,1}^2 = \left|\vec{k}_1\right|^2 = \left(k_0 n_1\right)^2$$

Material 2

$$k_{x,2}^2 + k_{y,2}^2 = \left|\vec{k}_2\right|^2 = \left(k_0 n_2\right)^2$$



Properly phased matched at the interface.

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Reflection and Transmission: The Fresnel Equations

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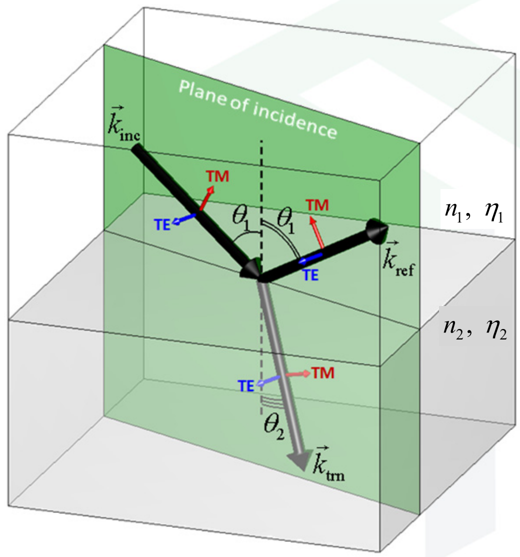
Reflection, Transmission, and Refraction at an Interface



Angles

$$\theta_{\mathrm{inc}} = \theta_{\mathrm{ref}} = \theta_1$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad \text{Snell's Law}$$

TE Polarization

$$r_{\mathrm{TE}} = \frac{\eta_2\cos\theta_1 - \eta_1\cos\theta_2}{\eta_2\cos\theta_1 + \eta_1\cos\theta_2}$$

$$t_{\mathrm{TE}} = \frac{2\eta_2\cos\theta_1}{\eta_2\cos\theta_1 + \eta_1\cos\theta_2}$$

$$1 + r_{\mathrm{TE}} = t_{\mathrm{TE}}$$

TM Polarization

$$r_{\mathrm{TM}} = \frac{\eta_2\cos\theta_2 - \eta_1\cos\theta_1}{\eta_1\cos\theta_1 + \eta_2\cos\theta_2}$$

$$t_{\mathrm{TM}} = \frac{2\eta_2\cos\theta_1}{\eta_1\cos\theta_1 + \eta_2\cos\theta_2}$$

$$1 + r_{\mathrm{TM}} = \frac{\cos\theta_2}{\cos\theta_1} t_{\mathrm{TM}}$$

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Reflectance and Transmittance

Reflectance

The fraction of power R reflected from an interface is called *reflectance*. It is related to the reflection coefficient r through

$$R_{\mathrm{TE}} = \left|r_{\mathrm{TE}}\right|^2 \qquad R_{\mathrm{TM}} = \left|r_{\mathrm{TM}}\right|^2 \qquad \frac{R_{\mathrm{TE}}}{R_{\mathrm{TM}}} = \frac{\left|r_{\mathrm{TE}}\right|^2}{\left|r_{\mathrm{TM}}\right|^2}$$

Transmittance

The fraction of power T transmitted through an interface is called *transmittance*. It is related to the transmission coefficient t through

$$T_{\mathrm{TE}} = \left|t_{\mathrm{TE}}\right|^2 \frac{\eta_1\cos\theta_2}{\eta_2\cos\theta_1} \qquad T_{\mathrm{TM}} = \left|t_{\mathrm{TM}}\right|^2 \frac{\eta_1\cos\theta_2}{\eta_2\cos\theta_1} \qquad \frac{T_{\mathrm{TE}}}{T_{\mathrm{TM}}} = \frac{\left|t_{\mathrm{TE}}\right|^2}{\left|t_{\mathrm{TM}}\right|^2}$$

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Amplitude Vs. Power Terms

Wave Amplitudes

The reflection and transmission coefficients, r and t, relate the amplitudes of the reflected and transmitted waves relative to the applied wave. They are complex numbers because both the magnitude and phase of the wave can change at an interface.

$$E_{\text{ref}} = rE_{\text{inc}} \qquad E_{\text{trn}} = tE_{\text{inc}}$$

Wave Power

The reflectance and transmittance, R and T, relate the power of the reflected and transmitted waves relative to the applied wave. They are real numbers bound between zero and one.

$$\left|E_{\text{ref}}\right|^2 = R \cdot \left|E_{\text{inc}}\right|^2 \qquad \left|E_{\text{trn}}\right|^2 = T \cdot \left|E_{\text{inc}}\right|^2$$

Often, these quantities are expressed on the decibel scale.

$$R_{\text{dB}} = 10\log_{10}\left(R\right) \qquad T_{\text{dB}} = 10\log_{10}\left(T\right)$$

Conservation of Power

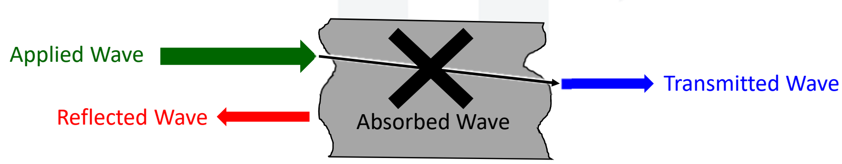
When an electromagnetic wave is incident onto a device, it can be absorbed (i.e. converted to another form of energy), reflected and/or transmitted. Without a nuclear reaction, nothing else can happen.

$$A + R + T = 1$$

Reflectance, R
Fraction of power from the applied wave that is reflected from the device.

Transmittance, T
Fraction of power from the applied wave that is transmitted through the device.

Absorptance, A
Fraction of power from the applied wave that is absorbed by the device.



Notes on a Single Interface

- It is a change in impedance that causes reflections
- Law of reflection says the angle of reflection is equal to the angle of incidence.
- Snell's Law quantifies the angle of transmission as a function of angle of incidence and the material properties.
- Angle of transmission and reflection do not depend on polarization.
- The Fresnel equations quantify the amount of reflection and transmission, but not the angles.
- Amount of reflection and transmission depends on the polarization and angle of incidence.

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Visualization of Wave Scattering at an Interface

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Longitudinal Component of the Wave Vector

1. Boundary conditions require that the tangential component of the wave vector is continuous across the interface.

   Assuming k_x is purely real in material 1, k_x will be purely real in material 2.

   → We have oscillations and energy flow in the x direction.

2. Knowing that the dispersion relation must be satisfied, the longitudinal component of the wave vector in material 2 is calculated from the dispersion relation in material 2.

$$k_{x,2}^2 + k_{y,2}^2 = \left(k_0 n_2\right)^2$$
$$\downarrow$$
$$k_{y,2} = \sqrt{\left(k_0 n_2\right)^2 - k_{x,2}^2}$$

We see that k_y will be purely real if $k_0 n_2 > \left|k_{x,2}\right|$.

We see that k_y will be purely imaginary if $k_0 n_2 < \left|k_{y,z}\right|$.

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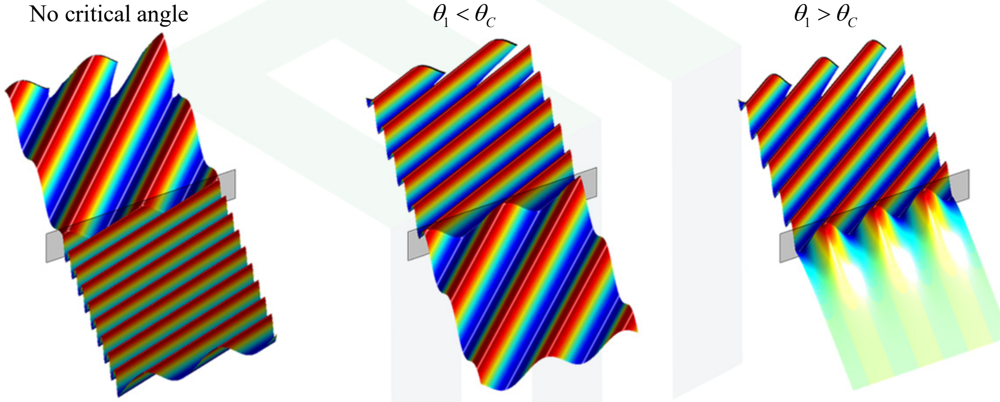
Field at an Interface Above and Below the Critical Angle (Ignoring Reflections)

$n_1 < n_2$ No critical angle | $n_1 > n_2$, $\theta_1 < \theta_C$ | $n_1 > n_2$, $\theta_1 > \theta_C$



1. The field always penetrates material 2, but it may not propagate.
2. Above the critical angle, penetration is greatest near the critical angle.
3. Very high spatial frequencies are supported in material 2 despite the dispersion relation.
4. In material 2, energy always flows along x, but not necessarily along y.

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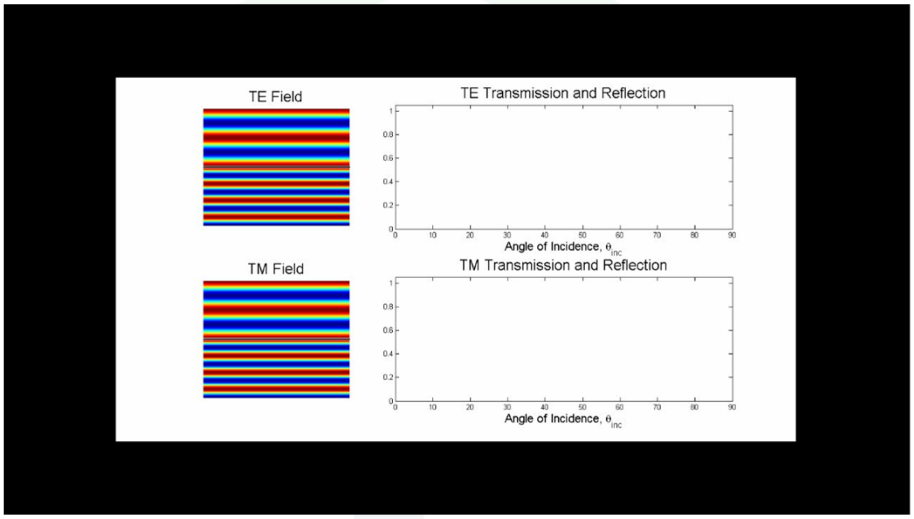
Simulation of Reflection and Transmission at a Single Interface ($n_1<n_2$)



n_1=1.0, n_2=1.73 → θ_B=60°

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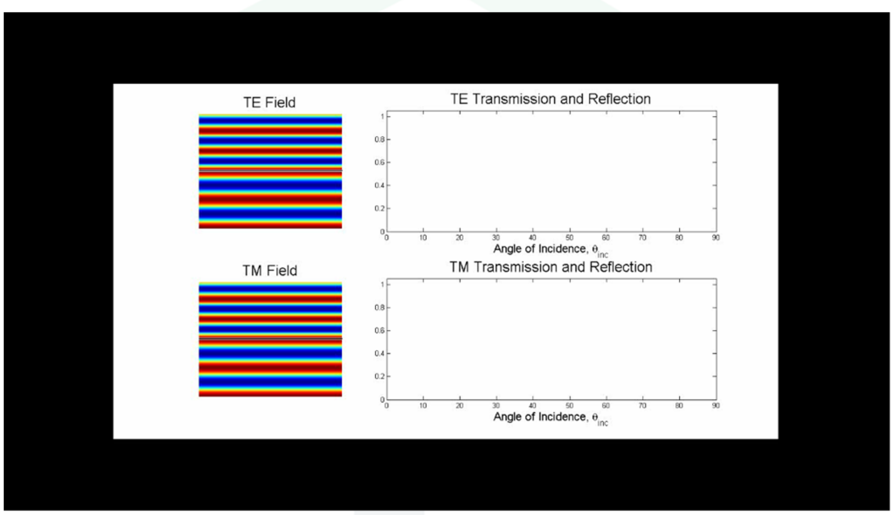
Simulation of Reflection and Transmission at a Single Interface ($n_1>n_2$)



n_1=1.41, n_2=1.0 → θ_C=45°

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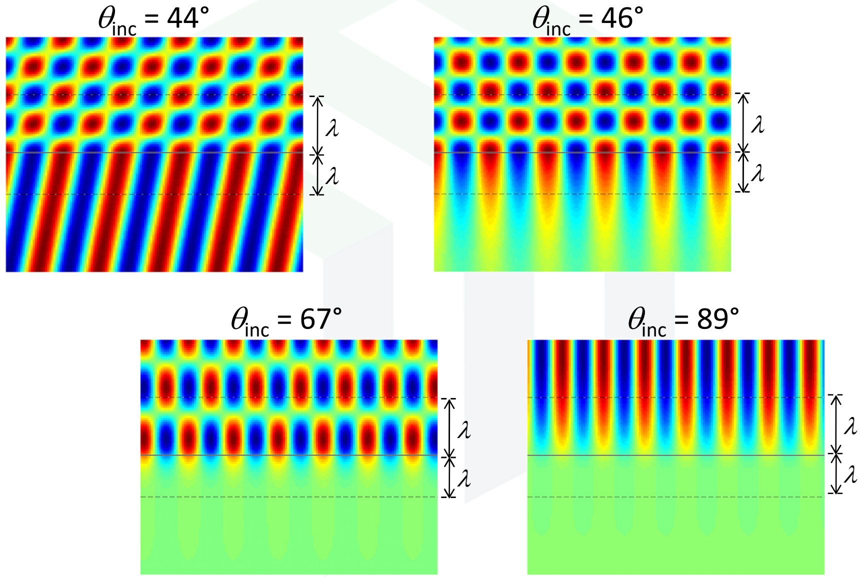
Field Visualization for θ_C=45°



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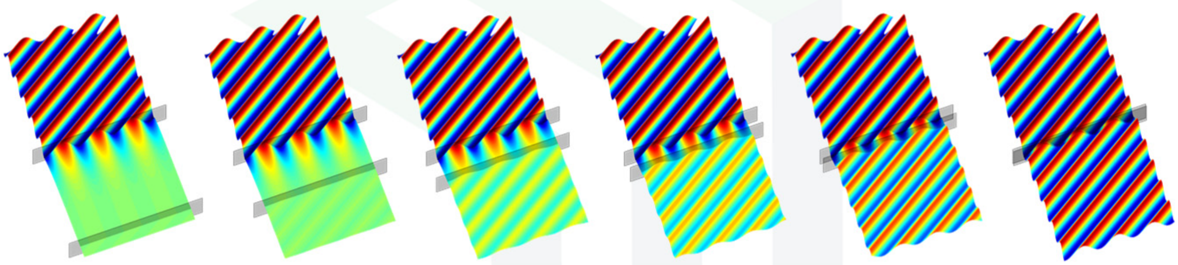
Electromagnetic Tunneling

If an evanescent field touches a medium with higher refractive index, the field may no longer be cutoff and become a propagating wave.



This is a very unusual phenomenon because the evanescent field is contributing to power flow.

This is called electromagnetic tunneling and is analogous to electron tunneling through thin insulators.

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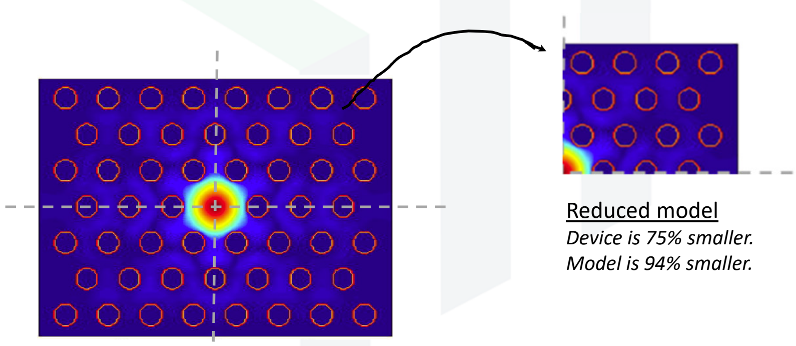
Image Theory

Image Theory Reduces Size of Models

When fields are symmetric in some manner about a plane, it is only necessary to calculate one half of the field because the other half contains only redundant information. Sometimes more than one plane of symmetry can be identified. Image theory can dramatically reduce the numerical size of the model being solved.



Reduced model
Device is 75% smaller.
Model is 94% smaller.

G. Bellanca, S. Trillo, "Full vectorial BPM modeling of Index-Guiding Photonic Crystal Fibers and Couplers," Optics Express 10(1), 54-59 (2002).

Summary of Image Theory

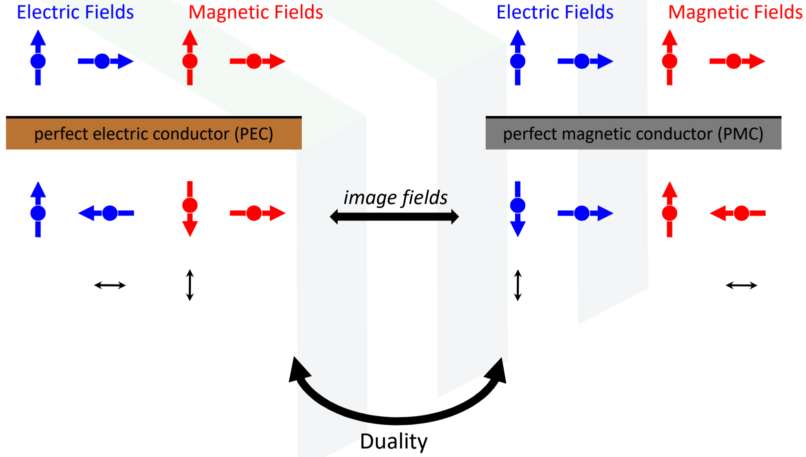


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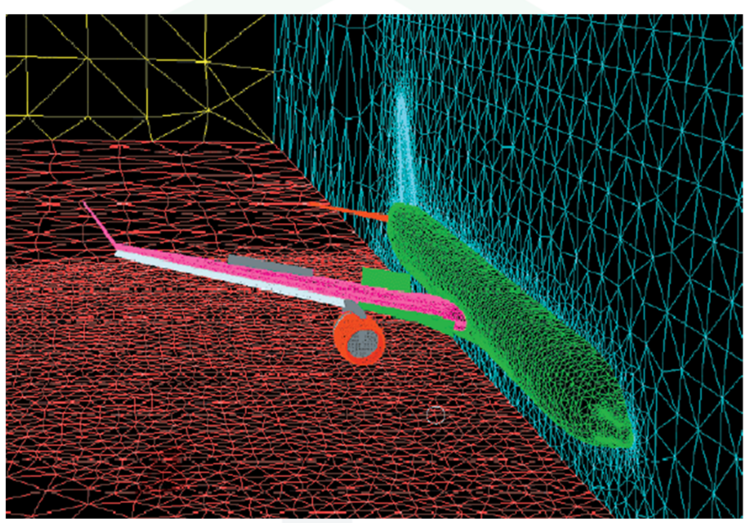
Image Theory Applied to Simulation of an Airplane



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