Course Instructor
Dr. Raymond C. Rumpf
Office: A-337
Phone: (915) 747-6958
E-Mail: rcrumpf@utep.edu

Maxwell's Equations:

Ampere's Circuit Law

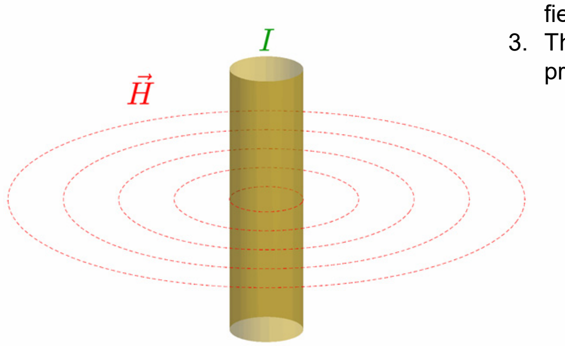
EE-3321

Electromagnetic Field Theory

Outline

- Ampere's Experiment
- Three Types of Current
- Ampere's Law in Integral Form
- Ampere's Law in Differential Form

Ampere's Experiment



Observations:

1. There exists a circulating magnetic field H around a conductor carrying a current I.
2. The current I can induce the magnetic field H, or the magnetic field H can induce the current I.
3. The measured current I is in proportion to the circulation of H.

$$I = \oint_L \vec{H} \bullet d\vec{\ell}$$

Three Types of Current

The total current I can be calculated by integrating the flux of the electric current density J through a cross section of the conductor.

$$I = \iint_S \vec{J}_{\text{total}} \bullet d\vec{s}$$

However, recall that there are three types of electric current.

$$\vec{J}_{\text{total}} = \underbrace{\vec{J}_\sigma + \vec{J}_\varepsilon}_{\vec{J}} + \vec{J}_D$$

$$\vec{J}_D = \frac{\partial \vec{D}}{\partial t} \equiv \text{displacement current}$$

$$\vec{J} \equiv \text{current due to free charges}$$

Putting this together gives

$$I = \iint_S \left[\vec{J} + \frac{\partial \vec{D}}{\partial t} \right] \bullet d\vec{s}$$

Ampere's Circuit Law in Integral Form

We have two ways of calculating the total current I in a conductor. Setting these equal gives…

Ampere's Experiment → $\oint_L \vec{H} \bullet d\vec{\ell}$

Simple integration of current density → $\iint_S \left[\vec{J} + \frac{\partial \vec{D}}{\partial t} \right] \bullet d\vec{s}$

$$I = \oint_L \vec{H} \bullet d\vec{\ell} = \iint_S \left[\vec{J} + \frac{\partial \vec{D}}{\partial t} \right] \bullet d\vec{s}$$

Apply Stoke's Theorem

Stoke's theorem allows us to write a closed-contour line integral as a surface integral.

$$\oint_L \vec{A} \bullet d\vec{\ell} = \iint_S \left(\nabla \times \vec{A} \right) \bullet d\vec{s}$$

Applying this to Ampere's law in integral form gives us

$$I = \oint_L \vec{H} \bullet d\vec{\ell} = \iint_S \left[\vec{J} + \frac{\partial \vec{D}}{\partial t} \right] \bullet d\vec{s}$$

$$\downarrow$$

$$I = \iint_S \left(\nabla \times \vec{H} \right) \bullet d\vec{s} = \iint_S \left[\vec{J} + \frac{\partial \vec{D}}{\partial t} \right] \bullet d\vec{s}$$

Ampere's Circuit Law in Differential Form

If the line L and surface S describe the same space, the argument of both integrals must be equal. Setting these arguments equal gives Ampere's circuit law in differential form.

$$I = \iint_S \left(\nabla \times \vec{H}\right) \bullet d\vec{s} = \iint_S \left[\vec{J} + \frac{\partial \vec{D}}{\partial t}\right] \bullet d\vec{s}$$

$$\boxed{\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

Visualization of Ampere's Circuit Law in Differential Form

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$