



# Electrostatics:

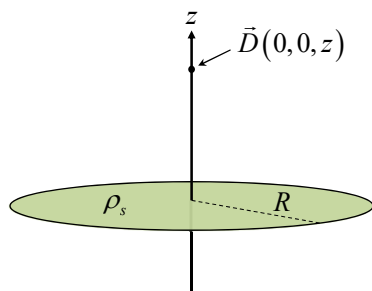
## Example – Uniform Circular Plate Charge

EE3321

Electromagnetic Field Theory



## Total Charge

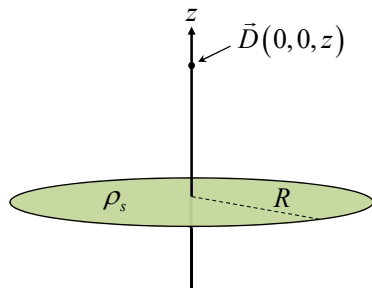


What is the total charge  $Q_{\text{Total}}$ ?

1. Draw the problem.
2. Choose a coordinate system.  
*Cylindrical*
3. Write general equation.
4. Write expressions for each term.  
 $\rho_s = \rho_s \quad ds = \rho d\rho d\phi$
5. Choose limits of integration.

$$Q_{\text{Total}} = \int_{\phi=0}^{2\pi} \int_{\rho=0}^R \rho_s \rho d\rho d\phi$$

## Total Charge



What is the total charge  $Q_{\text{Total}}$ ?

6. Solve the integral.

$$\begin{aligned} Q_{\text{Total}} &= \int_{\phi=0}^{2\pi} \int_{\rho=0}^R \rho_s \rho d\rho d\phi \\ &= \rho_s \int_{\phi=0}^{2\pi} \left( \int_{\rho=0}^R \rho d\rho \right) d\phi \\ &= \rho_s \int_{\phi=0}^{2\pi} \left( \frac{\rho^2}{2} \Big|_{\rho=0}^R \right) d\phi \\ &= \frac{\rho_s R^2}{2} \int_{\phi=0}^{2\pi} d\phi \\ &= \frac{\rho_s R^2}{2} \cdot 2\pi \end{aligned}$$

$$\boxed{Q_{\text{Total}} = \pi R^2 \rho_s}$$

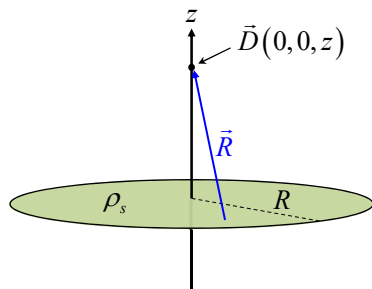
7. Interpret the result.

$$Q_{\text{Total}} = \rho_s S \quad \text{for uniform charge density}$$

Electrostatics -- Uniform Circular Plate

Slide 3

## Total Field



What is the total field  $\vec{D}$ ?

1. Draw the problem.
2. Choose a coordinate system.

*Cylindrical*

3. Write general equation.

$$\vec{D}_{\text{Total}} = \iint_s \frac{\rho_s ds}{4\pi R^2} \hat{a}_R = \frac{\rho_s}{4\pi} \iint_s \frac{\vec{R}}{|\vec{R}|^3} ds$$

4. Write expressions for each term.

$$\rho_s = \rho_s \quad \vec{R} = (0, \phi, z) - (\rho, \phi, 0) = (-\rho, 0, z)$$

$$ds = \rho d\rho d\phi \quad |\vec{R}| = \sqrt{\rho^2 + z^2}$$

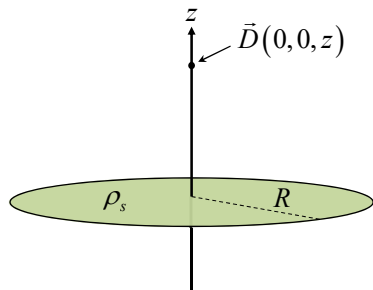
5. Choose limits of integration.

$$\vec{D}_{\text{Total}} = \frac{\rho_s}{4\pi} \int_{\phi=0}^{2\pi} \int_{\rho=0}^R \frac{-\rho \hat{a}_\rho + z \hat{a}_z}{(\rho^2 + z^2)^{3/2}} \rho d\rho d\phi$$

Electrostatics -- Uniform Circular Plate

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# Total Field



What is the total field  $\vec{D}$ ?

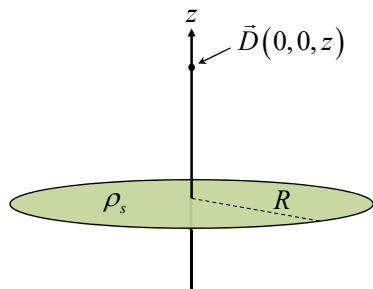
6. Solve the integral.

$$\vec{D}_{\text{Total}} = \frac{\rho_s}{4\pi} \int_{\phi=0}^{2\pi} \int_{\rho=0}^R \frac{-\rho \hat{a}_\rho + z \hat{a}_z}{(\rho^2 + z^2)^{3/2}} \rho d\rho d\phi$$

Due to symmetry, we can ignore the  $\rho$  component.

$$\begin{aligned} \vec{D}_{\text{Total}} &= \frac{\rho_s}{4\pi} \int_{\phi=0}^{2\pi} \int_{\rho=0}^R \frac{-\cancel{\rho \hat{a}_\rho} + z \hat{a}_z}{(\rho^2 + z^2)^{3/2}} \rho d\rho d\phi \\ &= \frac{\rho_s z}{4\pi} \hat{a}_z \int_{\phi=0}^{2\pi} \int_{\rho=0}^R (\rho^2 + z^2)^{-3/2} \rho d\rho d\phi \\ &= \frac{\rho_s z}{4\pi} \hat{a}_z \int_{\rho=0}^R \left( \int_{\phi=0}^{2\pi} d\phi \right) (\rho^2 + z^2)^{-3/2} \rho d\rho \\ &= \frac{\rho_s z}{4\pi} \hat{a}_z \int_{\rho=0}^R (2\pi) (\rho^2 + z^2)^{-3/2} \rho d\rho \\ &= \frac{\rho_s z}{2} \hat{a}_z \int_{\rho=0}^R (\rho^2 + z^2)^{-3/2} \rho d\rho \end{aligned}$$

# Total Field



What is the total field  $\vec{D}$ ?

6. Solve the integral.

$$\vec{D}_{\text{Total}} = \frac{\rho_s z}{2} \hat{a}_z \int_{\rho=0}^R (\rho^2 + z^2)^{-3/2} \rho d\rho$$

Let  $v = \rho^2 + z^2$

Then  $dv = 2\rho d\rho + \cancel{0}$

$$d\rho = \frac{dv}{2\rho}$$

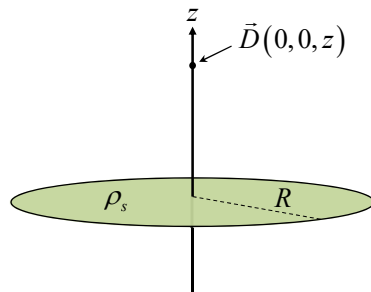
$$\rho_1 = 0 \rightarrow v_1 = 0^2 + z^2 = z^2$$

$$\rho_2 = R \rightarrow v_2 = R^2 + z^2$$

Our integral in terms of  $v$  is then

$$\begin{aligned} &= \frac{\rho_s z}{2} \hat{a}_z \int_{v=z^2}^{R^2+z^2} v^{-3/2} \rho \frac{dv}{2\rho} \\ &= \frac{\rho_s z}{4} \hat{a}_z \int_{v=z^2}^{R^2+z^2} v^{-3/2} dv \end{aligned}$$

# Total Field



What is the total field  $\vec{D}$ ?

6. Solve the integral.

$$\begin{aligned}
 \vec{D}_{\text{Total}} &= \frac{\rho_s z}{4} \hat{a}_z \int_{v=z^2}^{R^2+z^2} v^{-3/2} dv \\
 &= \frac{\rho_s z}{4} \hat{a}_z \left. \frac{v^{-1/2}}{-1/2} \right|_{z^2}^{R^2+z^2} \\
 &= -\frac{\rho_s z}{2} \hat{a}_z \left. \frac{1}{\sqrt{v}} \right|_{z^2}^{R^2+z^2} \\
 &= -\frac{\rho_s z}{2} \hat{a}_z \left( \frac{1}{\sqrt{R^2+z^2}} - \frac{1}{\sqrt{z^2}} \right) \\
 &= \frac{\rho_s}{2} \left( \frac{z}{\sqrt{z^2}} - \frac{z}{\sqrt{R^2+z^2}} \right) \hat{a}_z
 \end{aligned}$$

$$\vec{D}_{\text{Total}} = \frac{\rho_s}{2} \left( 1 - \frac{z}{\sqrt{R^2+z^2}} \right) \hat{a}_z$$