Advanced Computation:
Computational Electromagnetics

Implementation of Finite-Difference Frequency-Domain

1

Outline

- Basic flow of FDFD
- 2× grid technique
- Calculating grid parameters
- Constructing your device on the grid
- Walkthrough of the FDFD algorithm
- Examples for benchmarking

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Basic Flow of FDFD

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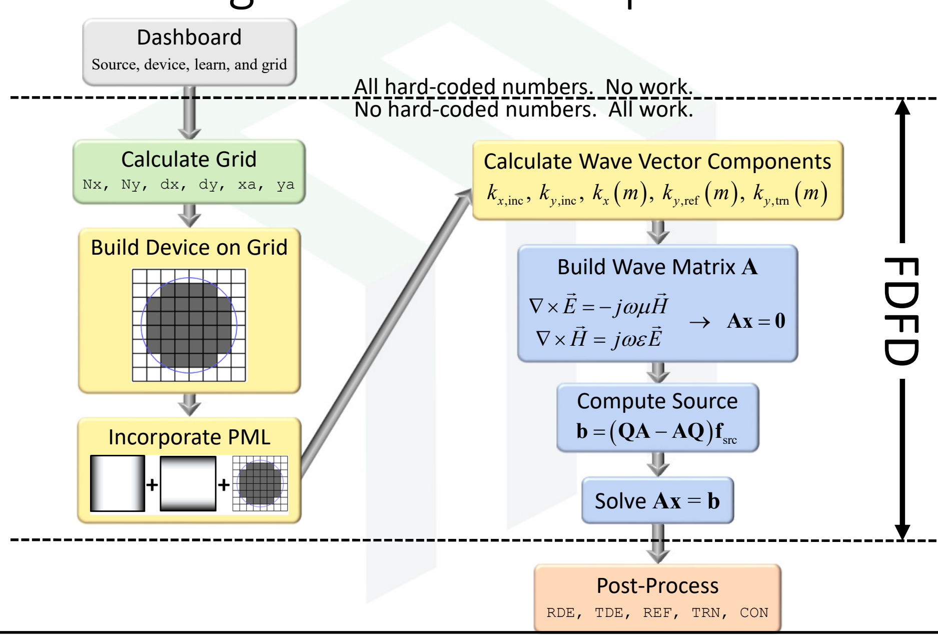
Block Diagram of FDFD Implementation



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Detailed FDFD Algorithm

1. Construct FDFD Problem
   a. Define your problem
   b. Calculate the grid parameters
   c. Assign materials to the grid to build `ER2` and `UR2` arrays

2. Handle PML and Materials
   a. Compute s_x and s_y
   b. Incorporate into ε_r and μ_r
   c. Overlay onto 1× grids

3. Compute Wave Vector Components
   a. Identify materials in reflected and transmitted regions: ε_{ref}, ε_{trn}, μ_{ref}, μ_{trn}, n_{ref}, n_{trn}
   b. Compute incident wave vector: $\mathbf{k}_{inc}$
   c. Compute transverse wave vector expansion: $k_{x,m}$
   d. Compute $k_{y,\text{ref}}$ and $k_{y,\text{trn}}$

4. Construct $\mathbf{A}$
   a. Construct diagonal materials matrices
   b. Compute derivative matrices
   c. Compute $\mathbf{A}$

5. Compute Source Vector, $\mathbf{b}$
   a. compute source field $\mathbf{f}_{src}$
   b. compute $\mathbf{Q}$
   c. compute source vector $\mathbf{b}$

6. Solve Matrix Problem: $\mathbf{e} = \mathbf{A}^{-1}\mathbf{b}$

7. Post Process Data
   a. Extract E_{ref} and E_{trn}
   b. Remove phase tilt
   c. FFT the fields
   d. Compute diffraction efficiencies
   e. Compute reflectance & transmittance
   f. Compute conservation of power

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2× Grid Technique

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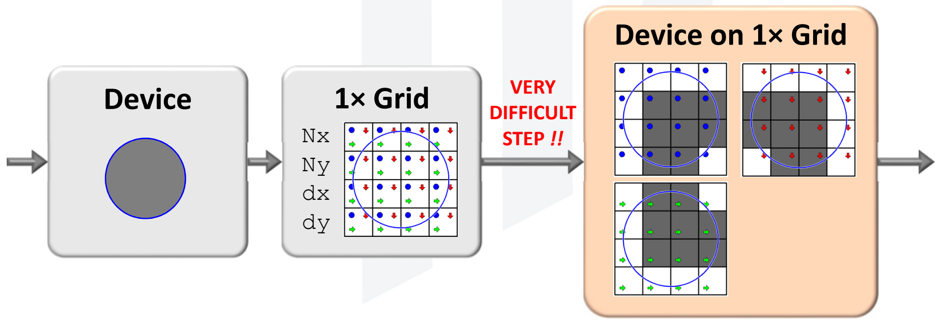
What is the 2× Grid Technique? (1 of 2)

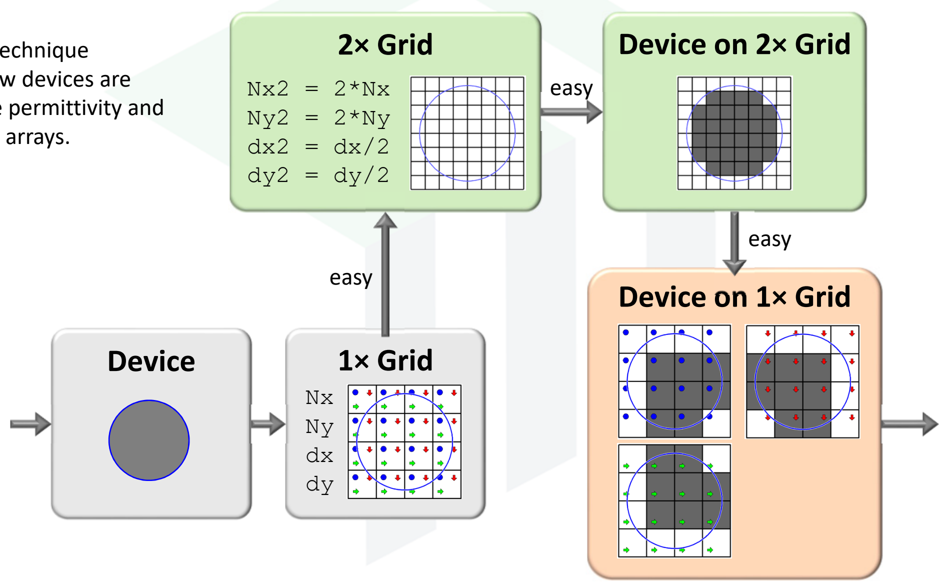
This is the traditional approach for building devices on a Yee grid.

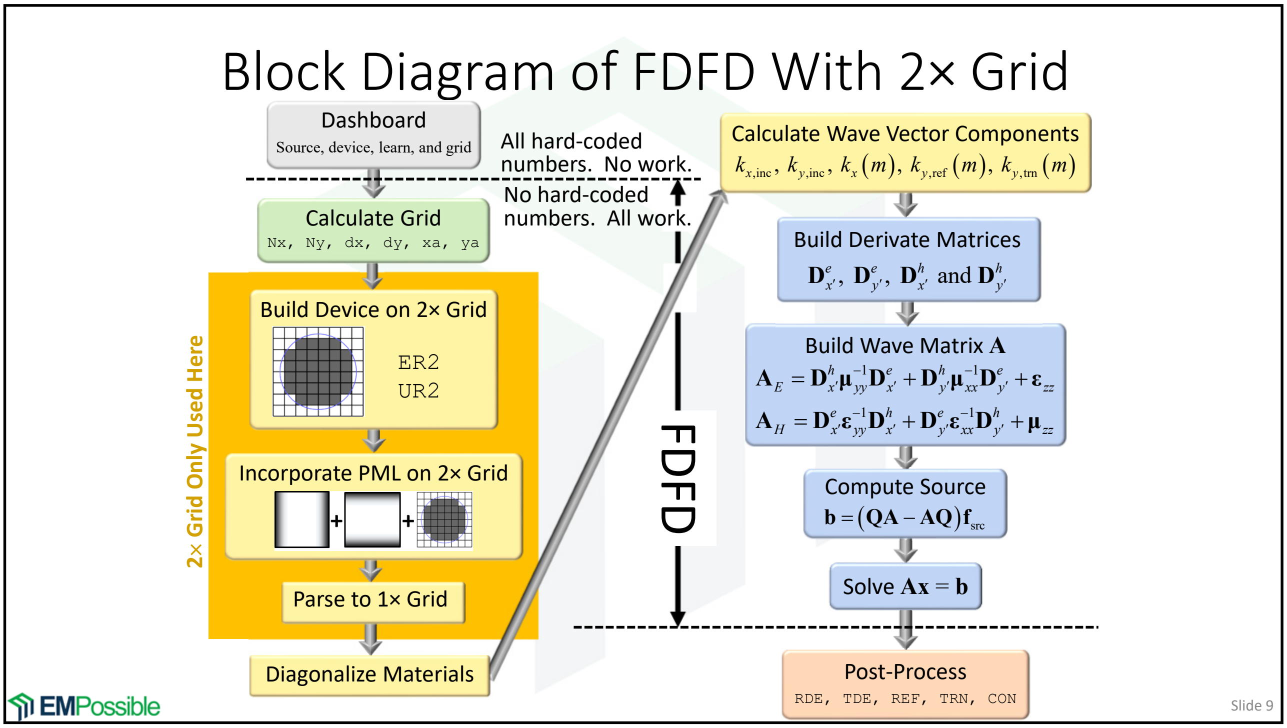
It is very tedious and cumbersome to determine which field components reside in which material.



What is the 2× Grid Technique? (2 of 2)

The 2× grid technique simplifies how devices are built into the permittivity and permeability arrays.




Block Diagram of FDFD With 2× Grid
Dashboard
Source, device, learn, and grid
All hard-coded numbers. No work.
No hard-coded numbers. All work.
Calculate Grid
Nx, Ny, dx, dy, xa, ya
2× Grid Only Used Here
Build Device on 2× Grid
ER2
UR2
Incorporate PML on 2× Grid
Parse to 1× Grid
Diagonalize Materials
FDFD
Calculate Wave Vector Components
$k_{x,\text{inc}},\ k_{y,\text{inc}},\ k_x(m),\ k_{y,\text{ref}}(m),\ k_{y,\text{trn}}(m)$
Build Derivate Matrices
$\mathbf{D}_{x'}^{e},\ \mathbf{D}_{y'}^{e},\ \mathbf{D}_{x'}^{h}$ and $\mathbf{D}_{y'}^{h}$
Build Wave Matrix A
$\mathbf{A}_E = \mathbf{D}_{x'}^{h}\boldsymbol{\mu}_{yy}^{-1}\mathbf{D}_{x'}^{e} + \mathbf{D}_{y'}^{h}\boldsymbol{\mu}_{xx}^{-1}\mathbf{D}_{y'}^{e} + \boldsymbol{\varepsilon}_{zz}$
$\mathbf{A}_H = \mathbf{D}_{x'}^{e}\boldsymbol{\varepsilon}_{yy}^{-1}\mathbf{D}_{x'}^{h} + \mathbf{D}_{y'}^{e}\boldsymbol{\varepsilon}_{xx}^{-1}\mathbf{D}_{y'}^{h} + \boldsymbol{\mu}_{zz}$
Compute Source
$\mathbf{b} = (\mathbf{QA} - \mathbf{AQ})\mathbf{f}_{\text{src}}$
Solve $\mathbf{Ax} = \mathbf{b}$
Post-Process
RDE, TDE, REF, TRN, CON
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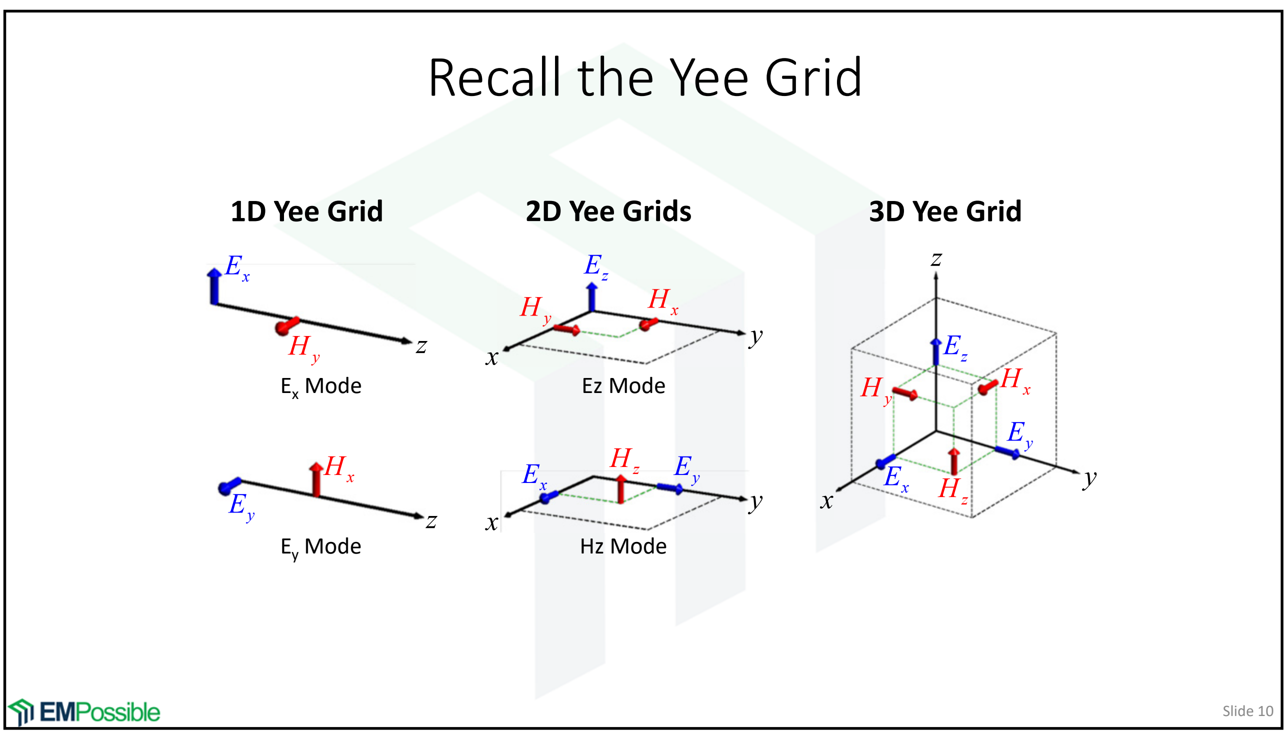
Recall the Yee Grid
1D Yee Grid
E_x
H_y
z
Ex Mode
H_x
E_y
z
Ey Mode
2D Yee Grids
E_z
H_y
H_x
x
y
Ez Mode
E_x
H_z
E_y
x
y
Hz Mode
3D Yee Grid
z
E_z
H_y
H_x
E_y
E_x
H_z
x
y
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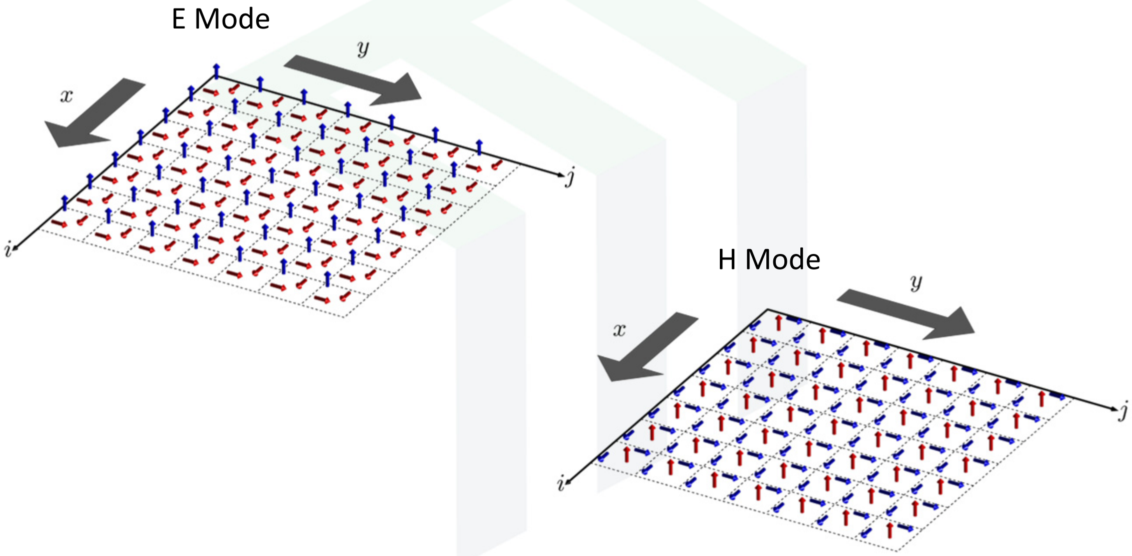
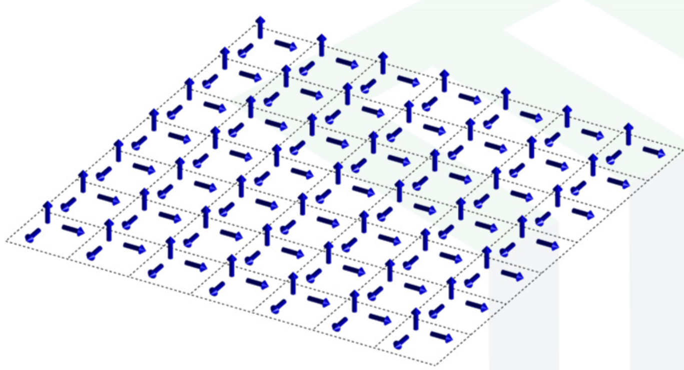
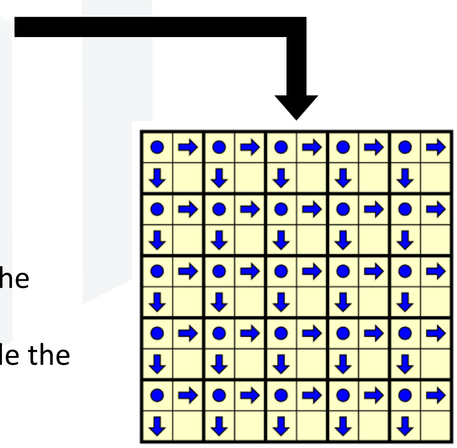
4×4 Yee Grids



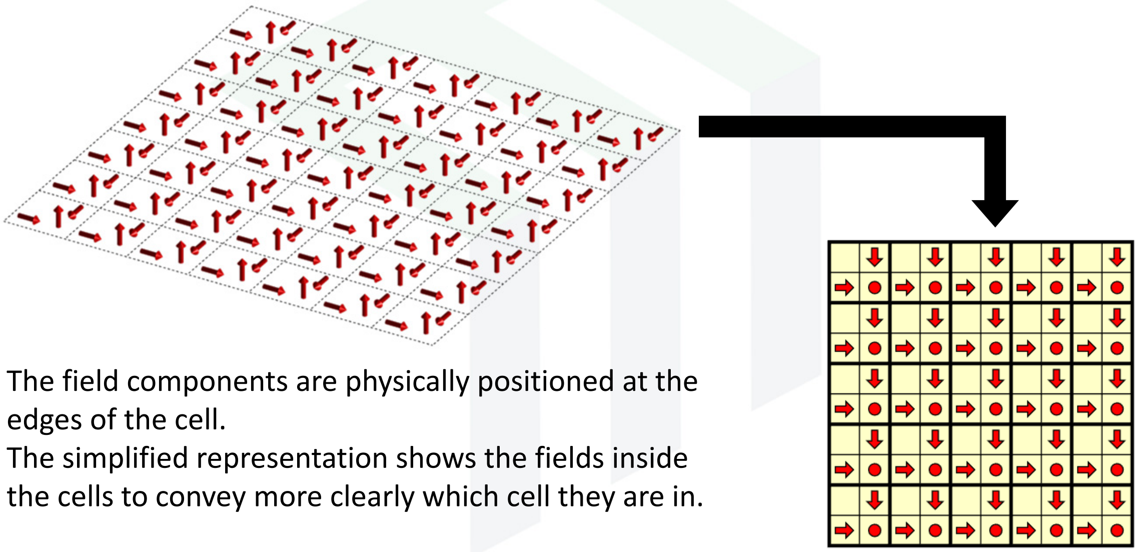
Simplified Representation of E-Field Components





- The field components are physically positioned at the edges of the cell.
- The simplified representation shows the fields inside the cells to convey more clearly which cell they are in.

Simplified Representation of H-Field Components



- The field components are physically positioned at the edges of the cell.
- The simplified representation shows the fields inside the cells to convey more clearly which cell they are in.

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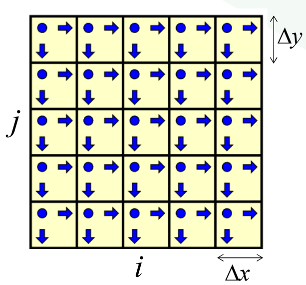
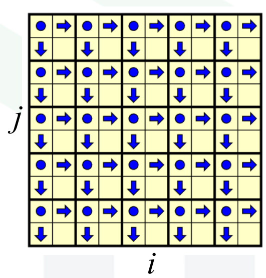
The 2× Grid

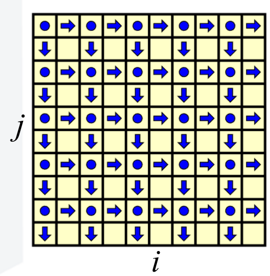
The Conventional 1× Grid



Due to the staggered nature of the Yee grid, we are effectively getting twice the resolution.



It now makes sense to talk about a grid that is at twice the resolution, the "2× grid."



The 2× grid concept is useful because we can create devices (or PMLs) on the 2× grid without having to think about where the different field components are located. In a second step, we can easily pull off the values from the 2× grid where they exist for a particular field component.

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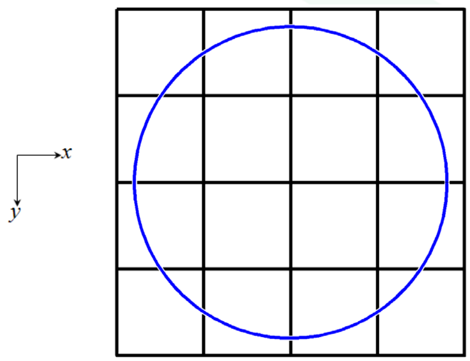
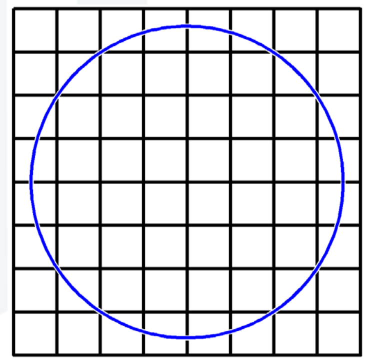
2×→1× (1 of 4): *Define Grids*

Suppose a circle is to be built onto the Yee grid. First, define the standard 1x grid.



Second, define a corresponding 2× grid.



Note: the 2× grid represents the same physical space, but with twice the number of points.

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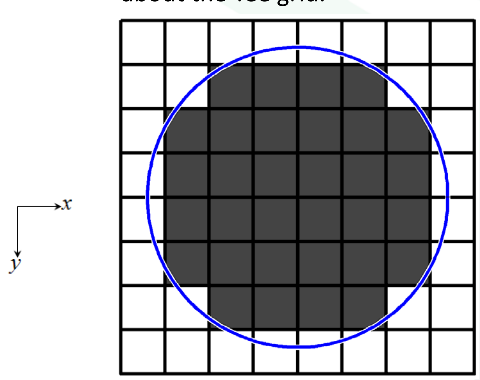
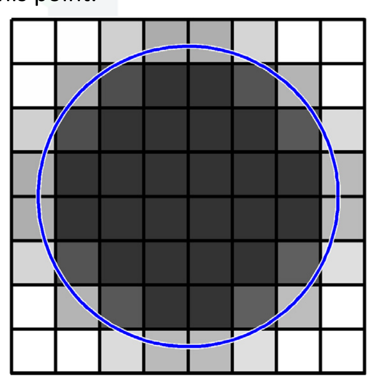
2×→1× (2 of 4): *Build Device*

Third, we construct a cylinder on the 2× grid without having to consider anything about the Yee grid.



Fourth, if desired we could perform dielectric averaging on the 2× grid at this point.



Note: This is discussed more in Lecture 14 FDFD Extras.

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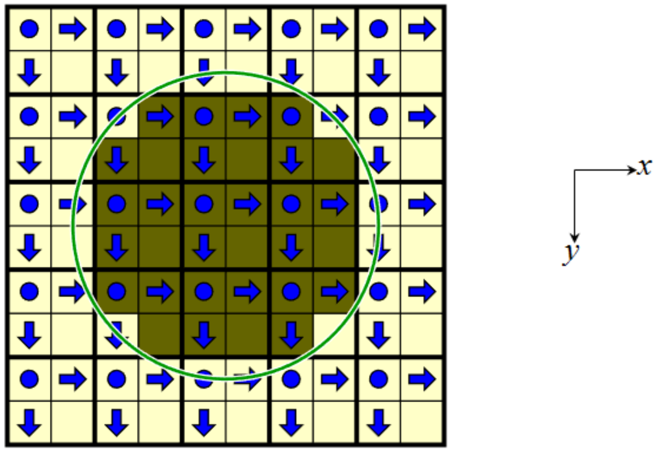
2×→1× (3 of 4): *Recall Field Staggering*

Recall the relation between the 2× and 1× grids as well as the location of the field components.



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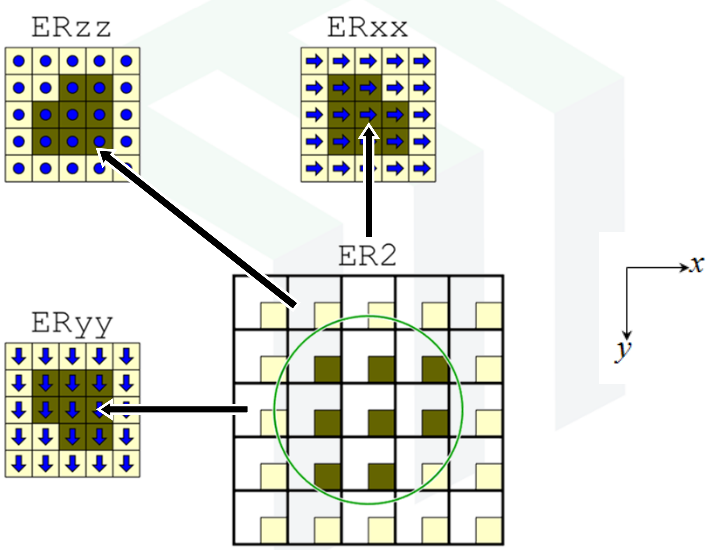
2×→1× (4 of 4): *Parse to 1× Grid*



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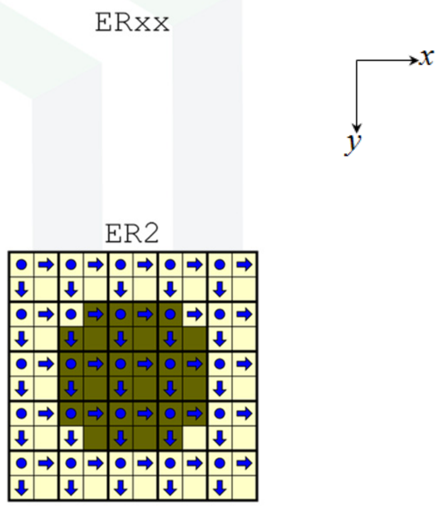
Extract ERxx from ER2

```
ERxx = ER2(2:2:Nx2,1:2:Ny2)
```



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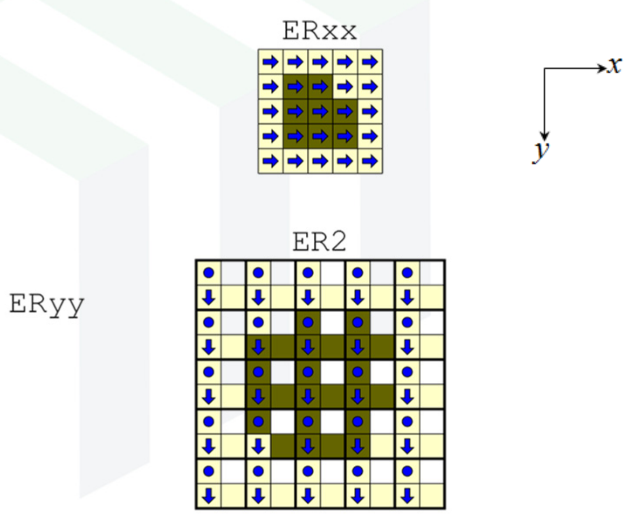
Extract ERyy from ER2

```
ERyy = ER2(1:2:Nx2,2:2:Ny2)
```



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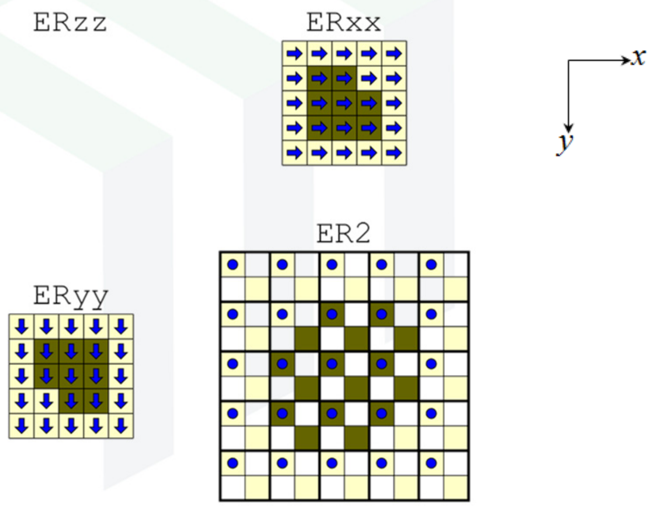
Extract `ERzz` from `ER2`

```
ERzz = ER2(1:2:Nx2,1:2:Ny2)
```



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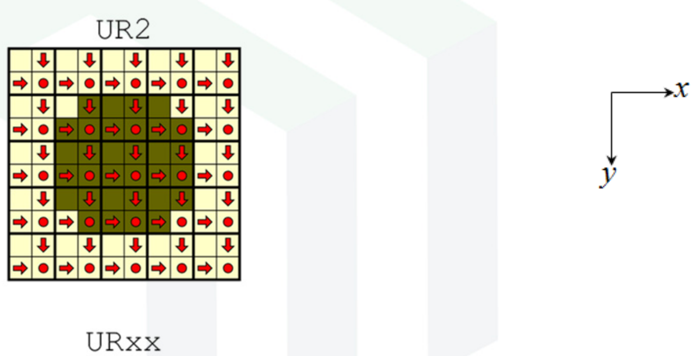
Extract `URxx` from `UR2`

```
URxx = UR2(1:2:Nx2,2:2:Ny2)
```



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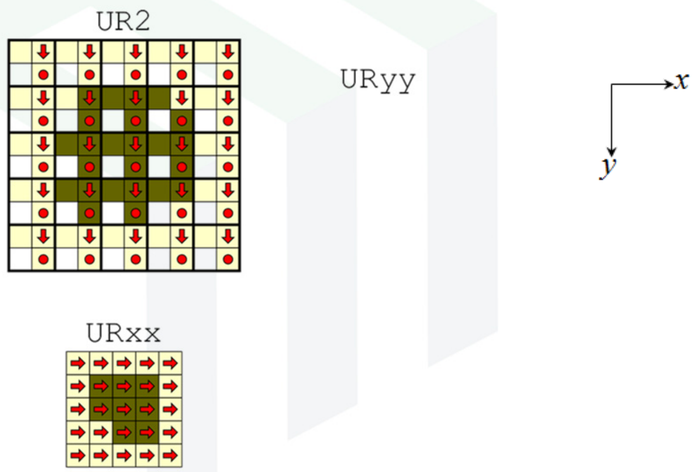
Extract URyy from UR2

```
URyy = UR2(2:2:Nx2,1:2:Ny2)
```



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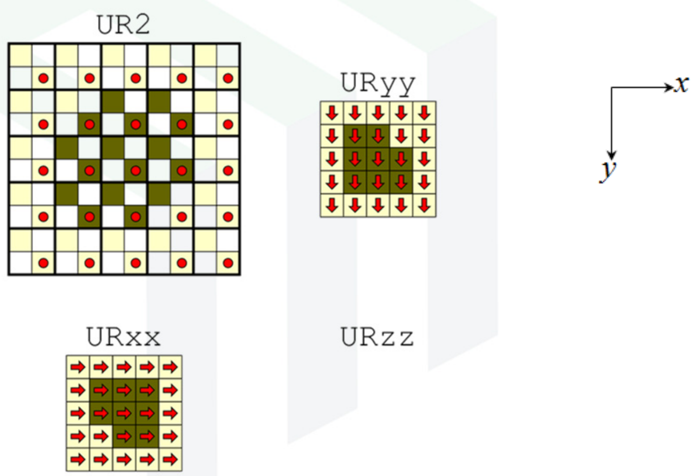
Extract URzz from UR2

```
URzz = UR2(2:2:Nx2,2:2:Ny2)
```



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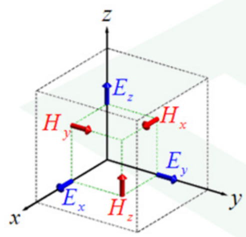
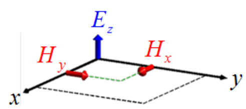
MATLAB Code for Parsing Onto 1× Grid

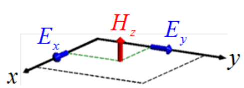



```
ERxx = ER2(2:2:Nx2,1:2:Ny2,1:2:Nz2);
ERyy = ER2(1:2:Nx2,2:2:Ny2,1:2:Nz2);
ERzz = ER2(1:2:Nx2,1:2:Ny2,2:2:Nz2);

URxx = UR2(1:2:Nx2,2:2:Ny2,2:2:Nz2);
URyy = UR2(2:2:Nx2,1:2:Ny2,2:2:Nz2);
URzz = UR2(2:2:Nx2,2:2:Ny2,1:2:Nz2);
```



E_z Mode

```
ERzz = ER2(1:2:Nx2,1:2:Ny2);

URxx = UR2(1:2:Nx2,2:2:Ny2);
URyy = UR2(2:2:Nx2,1:2:Ny2);
```



H_z Mode

```
ERxx = ER2(2:2:Nx2,1:2:Ny2);
ERyy = ER2(1:2:Nx2,2:2:Ny2);

URzz = UR2(2:2:Nx2,2:2:Ny2);
```

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Calculating the Grid Parameters

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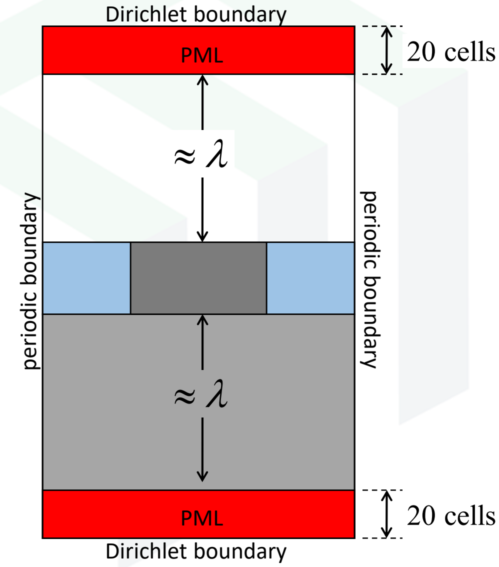
Grid Strategy



Consideration #1: Wavelength λ_0

The grid resolution must be sufficient to resolve the shortest wavelength.

First, determine the smallest wavelength:

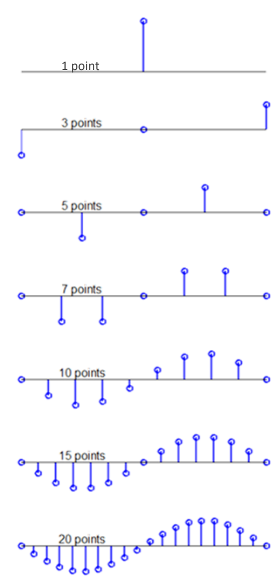
$$\lambda_{\min} = \frac{\min\left[\lambda_0\right]}{\max\left[n(x,y)\right]}$$

$\min[\lambda_0]$ is the shortest wavelength of interest in the simulation. $\max[n(x,y)]$ is the largest refractive index found anywhere in the grid.

Second, resolve the wave with at least N_λ cells.

$$\Delta_\lambda \approx \frac{\lambda_{\min}}{N_\lambda} \qquad N_\lambda \geq 10$$

N_λ	Comments
10 to 20	Low contrast dielectrics
20 to 30	High contrast dielectrics
40 to 60	Most metallic structures
100 to 200	Plasmonic devices



Consideration #2: Mechanical Features

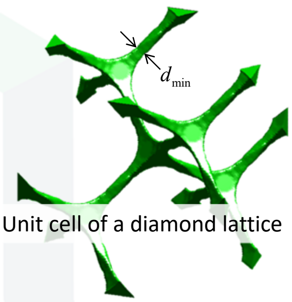
The grid resolution must be sufficient to resolve the smallest mechanical features of the device.

First, determine the smallest feature:

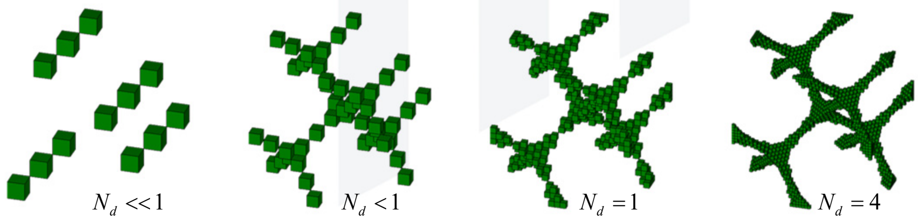
$$d_{\min}$$

Second, resolve $d_{\min}$ by at least N_d cells (usually 1 to 4)

$$\Delta_d \approx \frac{d_{\min}}{N_d} \qquad N_d \geq 1$$



Unit cell of a diamond lattice



Calculating the Initial Grid Resolution

1. Must resolve the minimum wavelength.

$$\Delta_\lambda \leq \frac{\min(\lambda_0) \div \max\left[n(x,y)\right]}{N_\lambda} \qquad N_\lambda \geq 10$$

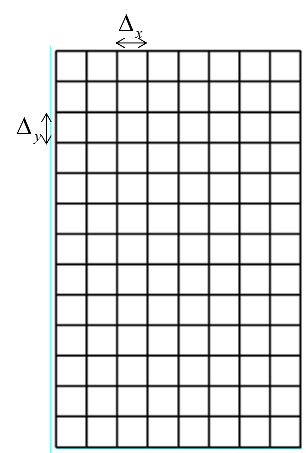
Note: If you are performing a parameter sweep over frequency or wavelength, $\min(\lambda_0)$ is the shortest wavelength in the sweep.

2. Must resolve the minimum structural dimension.

$$\Delta_d \leq \frac{\ell_{\min}}{N_d} \qquad N_d \geq 1$$

Proceed with the smallest of the above quantities to be our initial grid resolution

$$\Delta_x = \Delta_y = \min\left[\Delta_\lambda, \Delta_d\right]$$

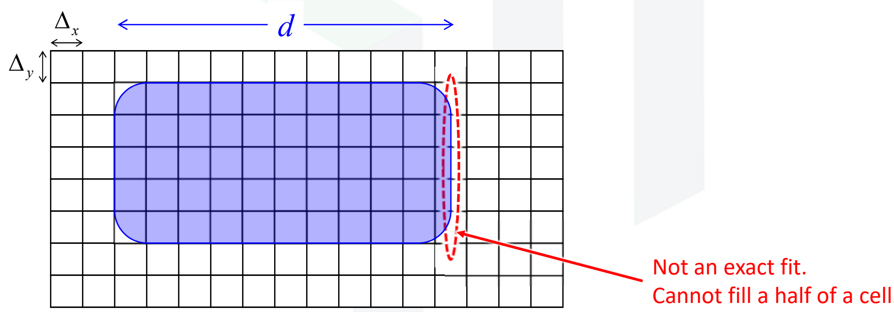


Resolving Critical Dimensions (1 of 3)

The actual dimensions of the device we wish to simulate have not actually yet been considered.

This means we likely cannot resolve the exact dimensions of a device.

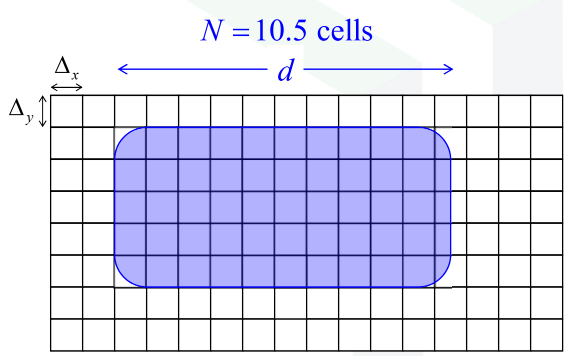
Suppose we wish to place a device of length d onto a grid.



Resolving Critical Dimensions (2 of 3)

To fix this, we first calculate how many cells N are needed to resolve the most important dimension. In this case, let this be d.

$$N = \frac{d}{\Delta_x}$$



Second, we determine how many cells we wish to exactly resolve d. We do this by rounding N up to the nearest integer.

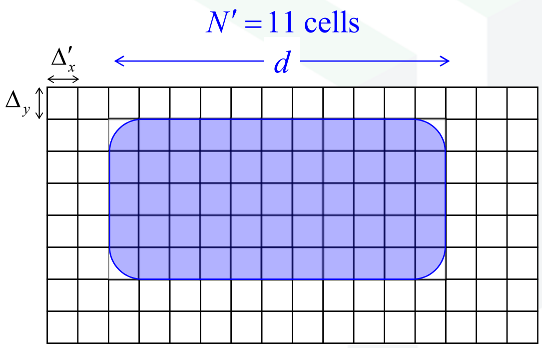
$$N' = \text{ceil}\left[\frac{d}{\Delta_x}\right]$$

$N' = 11$ cells

Resolving Critical Dimensions (3 of 3)

Third, we adjust the value of Δ_x to represent the dimension d exactly.

$$\Delta'_x = \frac{d}{N'}$$



This step can be called "snapping" the grid to a critical dimension.

Unfortunately, using a uniform grid, it is only possible to do this for one dimension per axis.

"Snap" Grid to Critical Dimensions

Decide what dimensions along each axis are critical.

$$d_x \quad \text{and} \quad d_y$$

→ Typically this is a lattice constant or grating period along x
→ Typically this is a film thickness or grating depth along y

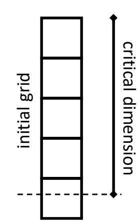
Compute how many grid cells comprise d_x and d_y and round UP.

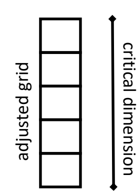
$$M_x = \text{ceil}\left(d_x / \Delta_x\right)$$
$$M_y = \text{ceil}\left(d_y / \Delta_y\right)$$

Adjust grid resolution to fit these critical dimensions in grid EXACTLY.

$$\Delta'_x = d_x / M_x$$
$$\Delta'_y = d_y / M_y$$

For simplicity, drop the prime symbol ' from Δ_x and Δ_y.





Compute Total Grid Size

- Don't forget to add cells for PML!
- Must often add "space" between PML and device.

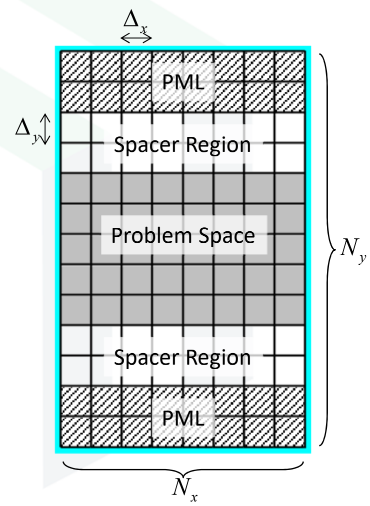
Note: This is particularly important when modeling devices with large evanescent fields.

$$N_x = \frac{\Lambda_x}{\Delta_x}$$

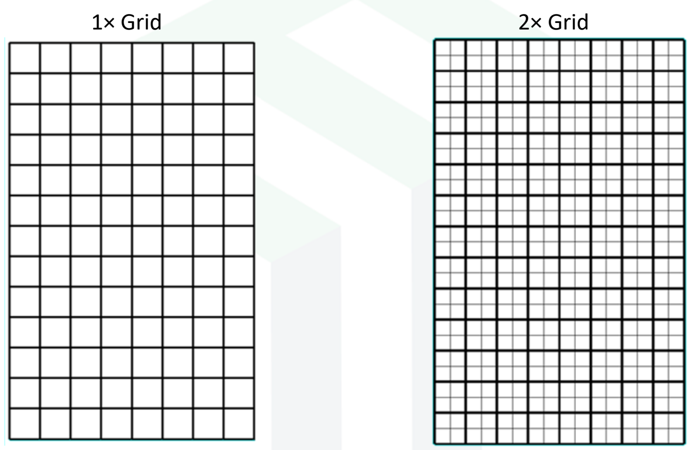
Easiest to make N_x odd. Reason discussed later.

$$N_y = \frac{\Lambda_y}{\Delta_y} + 2N_{\text{PML}} + 2N_{\text{SPACE}}$$

$$N_{\text{SPACE}} \cong \text{ceil}\left(\frac{\lambda_{\max}}{n_{\text{buff}}\Delta_y}\right)$$



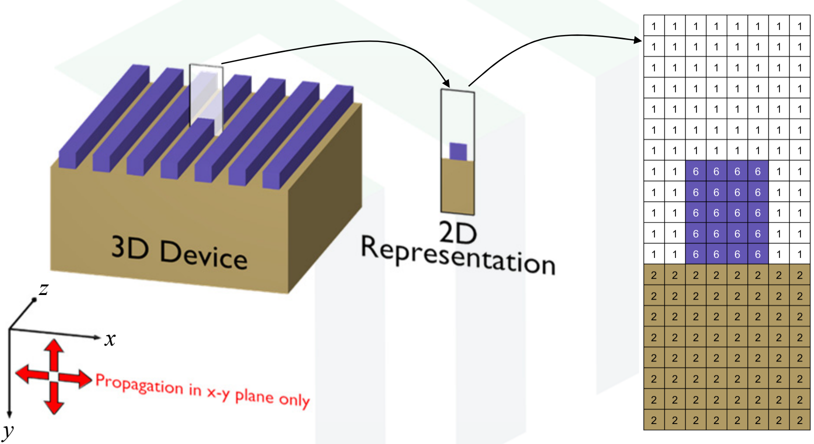
Compute 2× Grid Parameters




```
Nx2 = 2*Nx;   dx2 = dx/2;
Ny2 = 2*Ny;   dy2 = dy/2;
```

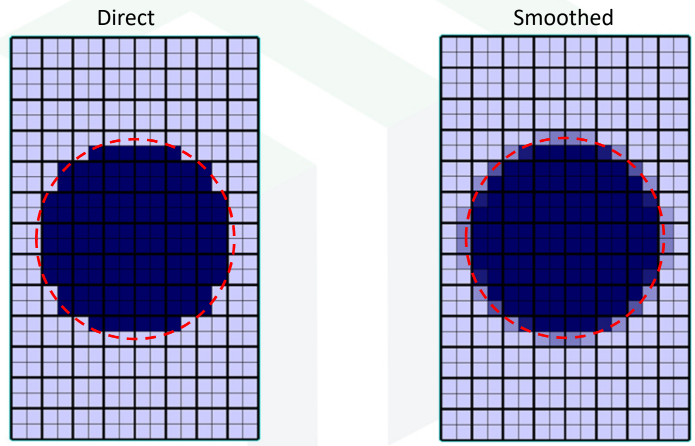
Constructing a Device on the Grid

Reducing 3D Problems to 2D



Representation on a Cartesian grid

Averaging At the Edges



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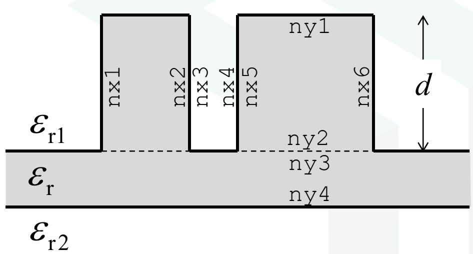
Building Rectangular Structures

For rectangular structures, considering calculating start and stop indices.



Be **very** careful with how many points on the grid you fill in!

```
ny2 = ny1 + round(d/dy2) - 1;
```

Without subtracting 1 here, filling in `ny1` to `ny2` would include an extra cell. This can introduce error into your results.

```
% BUILD DEVICE
ER2 = er1*ones(Nx2,Ny2);          %fill everywhere with er1
ER2(nx1:nx2,ny1:ny2) = er;        %add tooth 1
ER2(nx5:nx6,ny1:ny2) = er;        %add tooth 2
ER2(:,ny3:ny4)       = er;        %add substrate
ER2(:,ny4+1:Ny2)     = er2;       %fill transmission region
```

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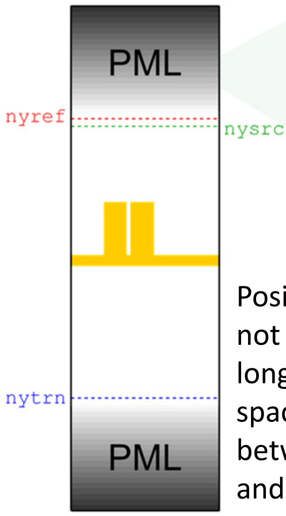
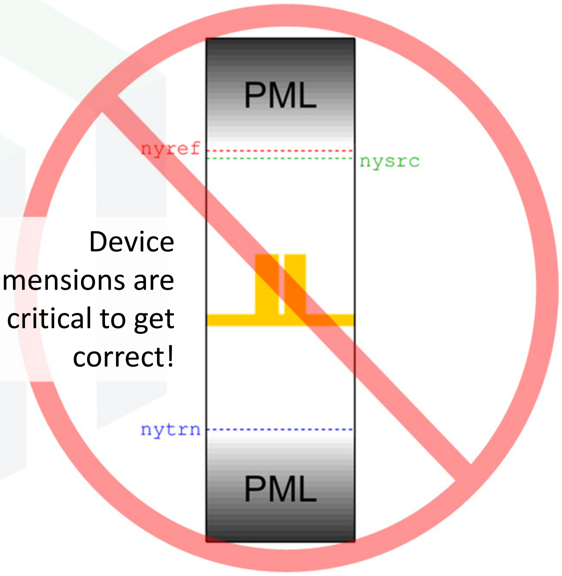
Critical and Non-Critical Parameters



Position offset is not critical as long as sufficient space remains between device and PMLs.

Device dimensions are critical to get correct!



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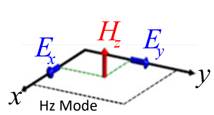
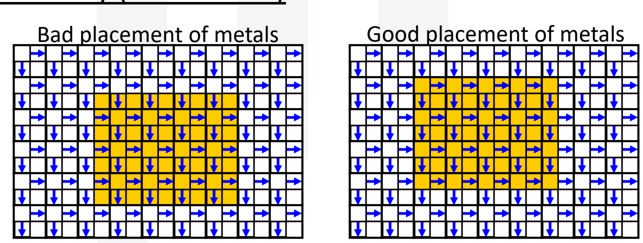
Oh Yeah, Metals!

Perfect Electric Conductors

$$\varepsilon_{\mathrm{r}} = -10000 \qquad \text{or} \qquad \begin{bmatrix} \ddots & & & & \\ 0 & 0 & 1 & 0 & 0 \\ & & & \ddots & \end{bmatrix} \begin{bmatrix} E_1 \\ \vdots \\ E_m \\ \vdots \\ E_M \end{bmatrix} = \begin{bmatrix} f_1 \\ \\ 0 \\ \\ f_M \end{bmatrix} \longrightarrow E_m = 0$$

Include Tangential Fields at Boundary (TM modes!)





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Walkthrough of the FDFD Algorithm

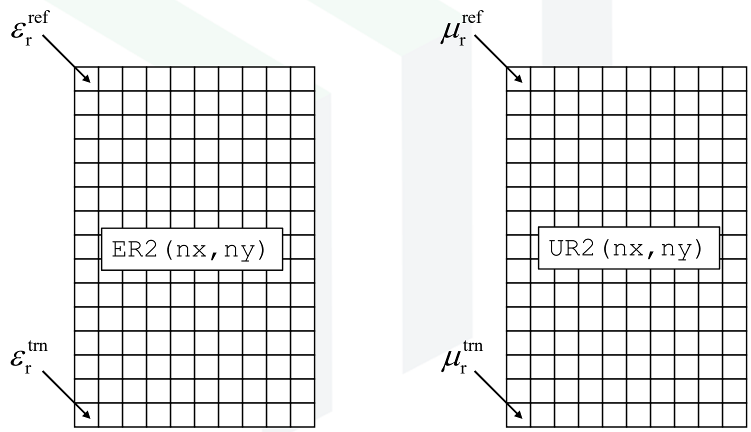
Input to the FDFD Algorithm

The FDFD algorithm requires the following information:

- The materials on the 2× grid: `ER2(nx,ny)` and `UR2(nx,ny)`
- The grid resolution: `dx` and `dy`
- The size of the PML on the 1× grid: `NYLO` and `NYHI`
- The source wavelength, λ_0
- Angle of incidence, θ
- Mode/polarization: 'E' or 'H'

(1) Determine the Material Properties in the Reflected and Transmitted Regions

If these parameters are not provided in the dashboard, or a dashboard does not exist, they can be pulled directly off of the grid.



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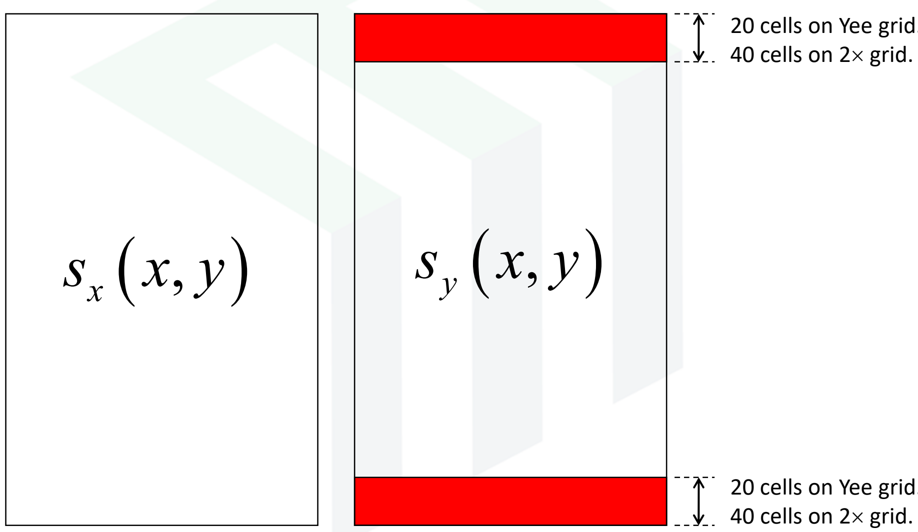
(2) Compute the PML Parameters on 2× Grid



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(3) Incorporate the PML

We can incorporate the PML parameters into [μ] and [ε] as follows

$$\nabla \times \vec{E} = k_0 [\mu'_r] \vec{\tilde{H}}$$
$$\nabla \times \vec{\tilde{H}} = k_0 [\varepsilon'_r] \vec{E}$$

$$[\varepsilon'_r] = \begin{bmatrix} \frac{s_y s_z}{s_x}\varepsilon_{xx} & 0 & 0 \\ 0 & \frac{s_x s_z}{s_y}\varepsilon_{yy} & 0 \\ 0 & 0 & \frac{s_x s_y}{s_z}\varepsilon_{zz} \end{bmatrix} \qquad [\mu'_r] = \begin{bmatrix} \frac{s_y s_z}{s_x}\mu_{xx} & 0 & 0 \\ 0 & \frac{s_x s_z}{s_y}\mu_{yy} & 0 \\ 0 & 0 & \frac{s_x s_y}{s_z}\mu_{zz} \end{bmatrix}$$

For 2D simulations, $s_z = 1$ and we have

$$\varepsilon'_{xx} = \frac{s_y}{s_x}\varepsilon_r \qquad \mu'_{xx} = \frac{s_y}{s_x}\mu_r$$
$$\varepsilon'_{yy} = \frac{s_x}{s_y}\varepsilon_r \qquad \mu'_{yy} = \frac{s_x}{s_y}\mu_r$$
$$\varepsilon'_{zz} = s_x s_y \varepsilon_r \qquad \mu'_{zz} = s_x s_y \mu_r$$

Note: the PML is incorporated into the 2× grid.

```
% INCORPORATE PML
URxx = UR2./sx.*sy;
URyy = UR2.*sx./sy;
URzz = UR2.*sx.*sy;
ERxx = ER2./sx.*sy;
ERyy = ER2.*sx./sy;
ERzz = ER2.*sx.*sy;
```

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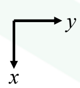
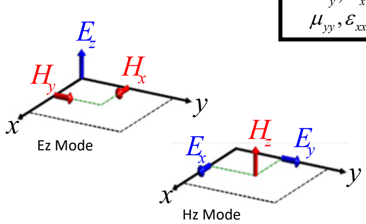
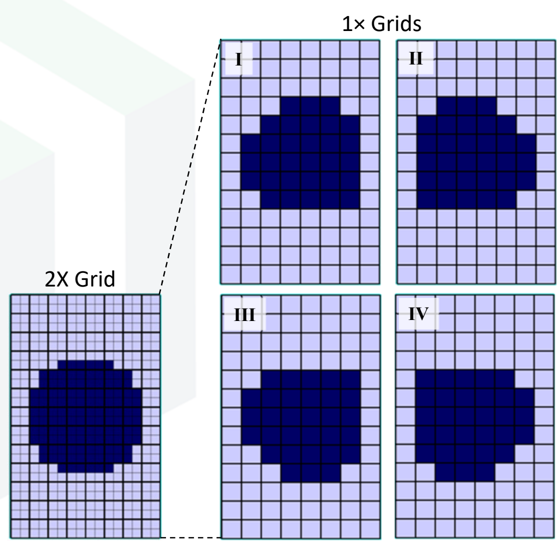
(4) Overlay Materials Onto 1× Grids

Field and materials assignments

I	II
E_z, ε_{zz}	H_x, E_y, $\mu_{xx}, \varepsilon_{yy}$
III	**IV**
H_y, E_x, $\mu_{yy}, \varepsilon_{xx}$	H_z, μ_{zz}








```
% OVERLAY MATERIALS ONTO 1X GRID
URxx = URxx(1:2:Nx2,2:2:Ny2);
URyy = URyy(2:2:Nx2,1:2:Ny2);
URzz = URzz(2:2:Nx2,2:2:Ny2);
ERxx = ERxx(2:2:Nx2,1:2:Ny2);
ERyy = ERyy(1:2:Nx2,2:2:Ny2);
ERzz = ERzz(1:2:Nx2,1:2:Ny2);
```

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(5) Compute the Wave Vector Terms

$$k_0 = \frac{2\pi}{\lambda_0}$$

$$\vec{k}_{\text{inc}} = k_0 n_{\text{ref}} \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix}$$

This is a vector quantity

$$k_x(m) = k_{x,\text{inc}} - m\frac{2\pi}{\Lambda_x} \qquad m = \cdots, -2, -1, 0, 1, 2, \cdots$$

N_x points total.
For proper symmetry, N_x should be odd.

```
m = [-floor(Nx/2):floor(Nx/2)]';
```

$$k_{y,\text{ref}}(m) = \sqrt{(k_0 n_{\text{ref}})^2 - k_x^2(m)}$$

$$k_{y,\text{trn}}(m) = \sqrt{(k_0 n_{\text{trn}})^2 - k_x^2(m)}$$

These equations come from the dispersion equation for the reflected and transmitted regions.

Recall that there used to be a negative sign here. We are able to drop it as long as we also drop the negative sign when calculating diffraction efficiency.

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(6) Construct Diagonal Materials Matrices

$$\boldsymbol{\varepsilon}_{xx} = \begin{bmatrix} \varepsilon_{xx}(1) & & & 0 \\ & \varepsilon_{xx}(2) & & \\ & & \ddots & \\ 0 & & & \varepsilon_{xx}(N) \end{bmatrix} \qquad \boldsymbol{\mu}_{xx} = \begin{bmatrix} \mu_{xx}(1) & & & 0 \\ & \mu_{xx}(2) & & \\ & & \ddots & \\ 0 & & & \mu_{xx}(N) \end{bmatrix}$$

$$\boldsymbol{\varepsilon}_{yy} = \begin{bmatrix} \varepsilon_{yy}(1) & & & 0 \\ & \varepsilon_{yy}(2) & & \\ & & \ddots & \\ 0 & & & \varepsilon_{yy}(N) \end{bmatrix} \qquad \boldsymbol{\mu}_{yy} = \begin{bmatrix} \mu_{yy}(1) & & & 0 \\ & \mu_{yy}(2) & & \\ & & \ddots & \\ 0 & & & \mu_{yy}(N) \end{bmatrix}$$

$$\boldsymbol{\varepsilon}_{zz} = \begin{bmatrix} \varepsilon_{zz}(1) & & & 0 \\ & \varepsilon_{zz}(2) & & \\ & & \ddots & \\ 0 & & & \varepsilon_{zz}(N) \end{bmatrix} \qquad \boldsymbol{\mu}_{zz} = \begin{bmatrix} \mu_{zz}(1) & & & 0 \\ & \mu_{zz}(2) & & \\ & & \ddots & \\ 0 & & & \mu_{zz}(N) \end{bmatrix}$$

```
ERxx = diag(sparse(ERxx(:)));
```

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(7) Construct the Derivative Matrices

$$\mathbf{D}_{x'}^{\mathrm{e}},\ \mathbf{D}_{y'}^{\mathrm{e}},\ \mathbf{D}_{x'}^{\mathrm{h}},\ \text{and}\ \mathbf{D}_{y'}^{\mathrm{h}}$$

```
[DEX,DEY,DHX,DHY] = yeeder(NGRID,k0*RES,BC,kinc/k0);
```

Frequency (or wavelength) information is incorporated into FDFD here

Be sure this function uses sparse matrices from the very beginning.

`A = sparse(M,M)` – creates a sparse M×M matrix of zeros.
`A = spdiags(b,d,A)` – Adds array `b` to diagonal `d` in matrix `A`.

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(8) Compute the Wave Matrix A

E Mode

$$\mathbf{A}_{\mathrm{E}} = \mathbf{D}_{x'}^{\mathrm{h}}\boldsymbol{\mu}_{yy}^{-1}\mathbf{D}_{x'}^{\mathrm{e}} + \mathbf{D}_{y'}^{\mathrm{h}}\boldsymbol{\mu}_{xx}^{-1}\mathbf{D}_{y'}^{\mathrm{e}} + \boldsymbol{\varepsilon}_{zz}$$

H Mode

$$\mathbf{A}_{\mathrm{H}} = \mathbf{D}_{x'}^{\mathrm{e}}\boldsymbol{\varepsilon}_{yy}^{-1}\mathbf{D}_{x'}^{\mathrm{h}} + \mathbf{D}_{y'}^{\mathrm{e}}\boldsymbol{\varepsilon}_{xx}^{-1}\mathbf{D}_{y'}^{\mathrm{h}} + \boldsymbol{\mu}_{zz}$$

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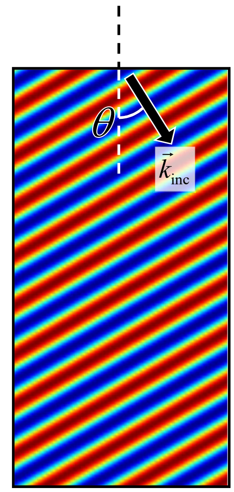
(9) Compute the Source Field

The source has an amplitude of 1.0

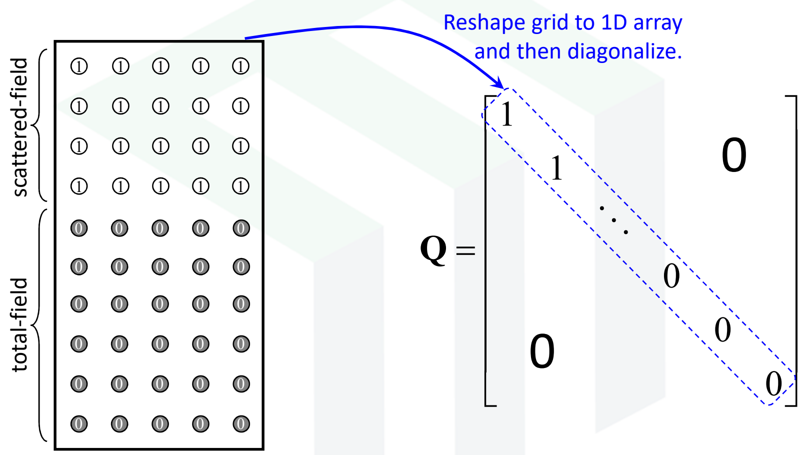
$$f_{\text{src}}(x,y) = \exp\left(j\vec{k}_{\text{inc}} \bullet \vec{r}\right)$$
$$= \exp\left[j\left(k_{x,\text{inc}}x + k_{y,\text{inc}}y\right)\right]$$

Don't forget to make $\mathbf{f}_{\text{src}}$ a column vector.



1× grid

(10) Compute the Scattered-Field Masking Matrix, Q




```
Q = diag(sparse(Q(:)));
```

It is good practice to make the scattered-field region at least one cell larger than the y-low PML.

(11) Compute the source vector, b

$$\mathbf{Af} = \mathbf{b} \qquad \mathbf{b} = (\mathbf{QA} - \mathbf{AQ})\mathbf{f}_{\text{src}}$$

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(12) Compute the Field f

$$\mathbf{f} = \mathbf{A}^{-1}\mathbf{b}$$

Aside

In MATLAB, `f = A\b` employs a direct LU decomposition to calculate `f`. This is very stable and robust, but a half-full matrix is created so memory can explode for large problems.

Iterative solutions can be faster and require much less memory, but they are less stable and may never converge to a solution. Correcting these problems requires significant modification to the FDFD algorithm taught here.

Don't forget to `reshape()` **f** from a column vector to a 2D grid after the calculation.

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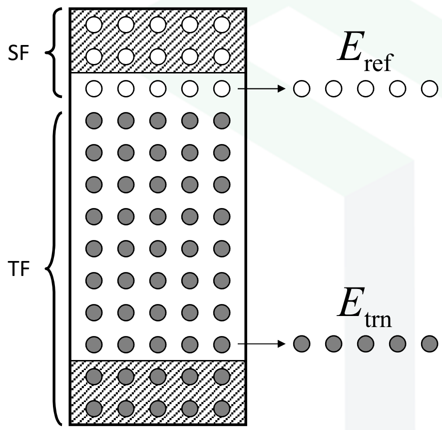
(13) Extract Transmitted and Reflected Fields



The reflected field is extracted from inside the scattered-field, but outside the PML.

The transmitted field is extracted from the grid after the device, but outside the PML.

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(14) Remove the Phase Tilt

Recall Bloch's theorem,

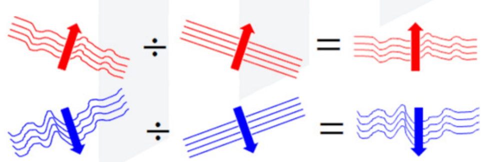
$$\vec{E}(\vec{r}) = \vec{A}_{\vec{k}_{\text{inc}}}(\vec{r}) \cdot e^{j\vec{k}_{\text{inc}} \bullet \vec{r}}$$

This implies the transmitted and reflected fields have the following form

$$E_{\text{ref}}(x) = A_{\text{ref}}(x) e^{jk_{x,\text{inc}}x}$$
$$E_{\text{trn}}(x) = A_{\text{trn}}(x) e^{jk_{x,\text{inc}}x}$$

We remove the phase tilt to calculate the amplitude terms.

$$A_{\text{ref}}(x) = E_{\text{ref}}(x) e^{-jk_{x,\text{inc}}x}$$
$$A_{\text{trn}}(x) = E_{\text{trn}}(x) e^{-jk_{x,\text{inc}}x}$$



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(15) Calculate the Complex Amplitudes of the Spatial Harmonics

Recall that the plane wave spectrum is the Fourier transform of the field.

We calculate the FFT of the field amplitude arrays.

$$S_{\text{ref}}(m) = \text{FFT}\left\{A_{\text{ref}}(x)\right\}$$

$$S_{\text{trn}}(m) = \text{FFT}\left\{A_{\text{trn}}(x)\right\}$$

Some FFT algorithms (like MATLAB) require that you divide by the number of points and shift after calculation.

```
Sref = flipud(fftshift(fft(Aref)))/Nx;
Strn = flipud(fftshift(fft(Atrn)))/Nx;
```

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(16) Calculate Diffraction Efficiencies

The source wave was given unit amplitude so

$$\left|S_{\text{inc}}\right|^2 = 1$$

The diffraction efficiencies of the reflected modes are then

$$R(m) = \left|S_{\text{ref}}(m)\right|^2 \frac{\text{Re}\left[k_y^{\text{ref}}(m)/\mu_{\text{r,inc}}\right]}{\text{Re}\left[k_y^{\text{inc}}/\mu_{\text{r,inc}}\right]} \quad \text{E Mode}$$

$$R(m) = \left|\vec{U}_{\text{ref}}(m)\right|^2 \frac{\text{Re}\left[k_y^{\text{ref}}(m)/\varepsilon_{\text{r,inc}}\right]}{\text{Re}\left[k_y^{\text{inc}}/\varepsilon_{\text{r,inc}}\right]} \quad \text{H Mode}$$

Recall that there used to be a negative sign here. We dropped it because we also dropped the sign when calculating $k_{y,\text{ref}}(m)$.

The diffraction efficiencies of the transmitted modes are then

$$T(m) = \left|S_{\text{trn}}(m)\right|^2 \frac{\text{Re}\left[k_y^{\text{trn}}/\mu_{\text{r,trn}}\right]}{\text{Re}\left[k_y^{\text{inc}}/\mu_{\text{r,inc}}\right]}$$

(E Mode)

$$T(m) = \left|U_{\text{trn}}(m)\right|^2 \frac{\text{Re}\left[k_y^{\text{trn}}/\varepsilon_{\text{r,trn}}\right]}{\text{Re}\left[k_y^{\text{inc}}/\varepsilon_{\text{r,inc}}\right]}$$

(H Mode)

$S(m) \equiv$ amplitudes of E mode spatial harmonics

$U(m) \equiv$ amplitudes of H mode spatial harmonics

Note: these equations assume that $\left|\vec{S}_{\text{inc}}\right| = 1$.

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(17) Calculate Reflectance, Transmittance, and Conservation of Power

The overall reflectance is

$$\text{REF} = \sum_m R(m)$$

The overall transmittance is

$$\text{TRN} = \sum_m T(m)$$

Conservation of power is computed as

$$\text{REF} + \text{TRN} + \text{ABS} = 100\%$$

If no loss or gain is incorporated, then ABS = 0 and we will have

$$\text{REF} + \text{TRN} = 100\%$$

$$\begin{aligned} &\text{REF} + \text{TRN} < 100\% && \text{loss} \\ &\text{REF} + \text{TRN} = 100\% && \text{no loss or gain} \\ &\text{REF} + \text{TRN} > 100\% && \text{gain} \end{aligned}$$

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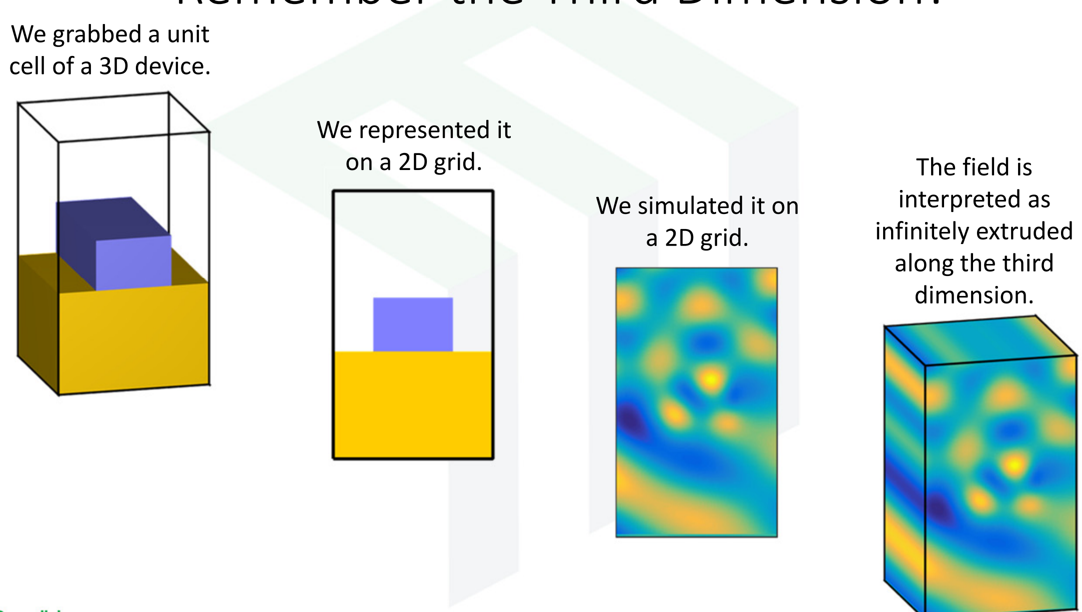
Remember the Third Dimension!



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Examples for Benchmarking

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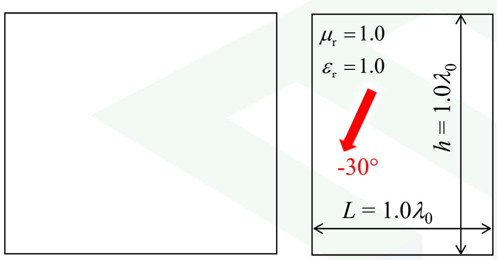
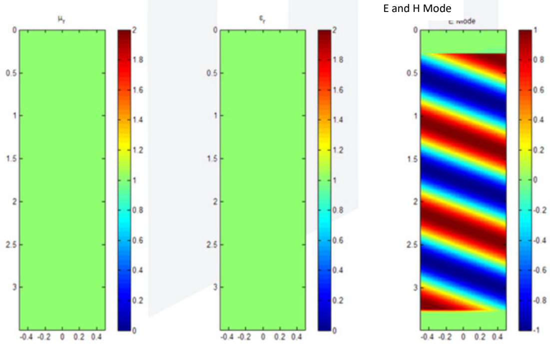
Air Simulation



	E Mode		*H* Mode	
m	*R*	*T*	*R*	*T*
-4	0%	0%	0%	0%
-3	0%	0%	0%	0%
-2	0%	0%	0%	0%
-1	0%	0%	0%	0%
0	0%	100%	0%	100%
1	0%	0%	0%	0%
2	0%	0%	0%	0%
3	0%	0%	0%	0%
4	0%	0%	0%	0%

E and H Mode



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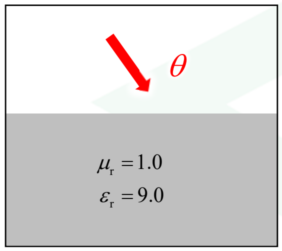
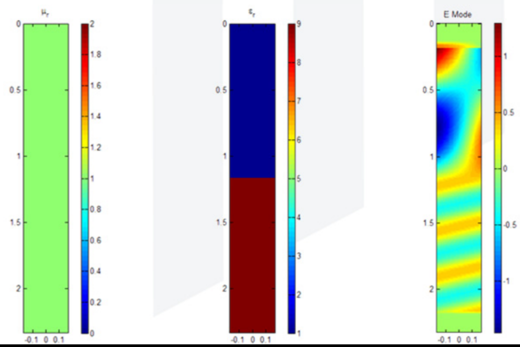
Dielectric Slab Grating



	E Mode		H Mode	
θ	R	T	R	T
-60°	49.5%	50.6%	5.0%	95.4%
-25°	28.5%	71.7%	21.8%	78.4%
0°	25.1%	75.0%	25.1%	75.1%
15°	26.3%	73.8%	24.0%	76.3%
45°	37.2%	62.9%	13.9%	86.4%
80°	77.7%	21.8%	8.3%	91.9%

Note: you can come up with your own benchmarking examples using the transfer matrix method!



$\theta = 45°$

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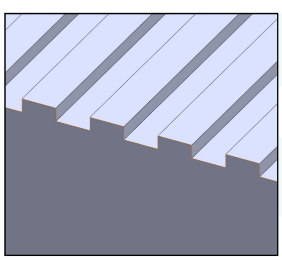
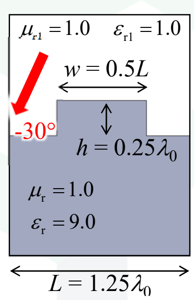
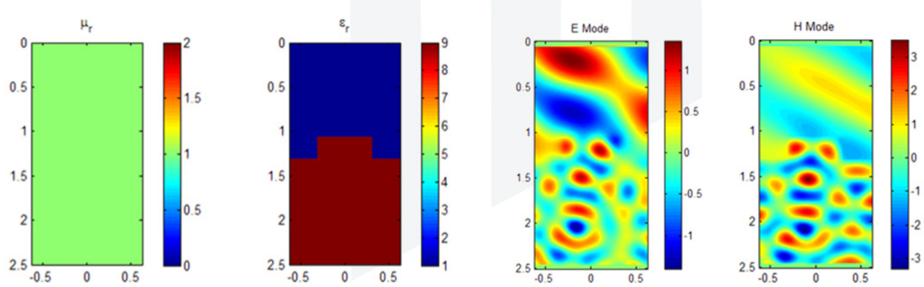
Binary Diffraction Grating





	E Mode		H Mode	
m	R	T	R	T
-4	0%	4.6%	0%	2.4%
-3	0%	5.3%	0%	7.5%
-2	0%	2.6%	0%	3.1%
-1	6.9%	22.1%	14.1%	38.8%
0	12.7%	7.8%	0.9%	5.4%
1	0%	17.7%	0%	17.7%
2	0%	16.4%	0%	7.6%
3	0%	3.9%	0%	2.4%
4	0%	0%	0%	0%



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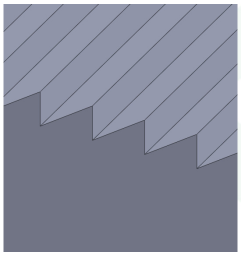
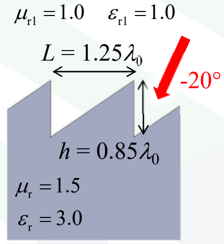
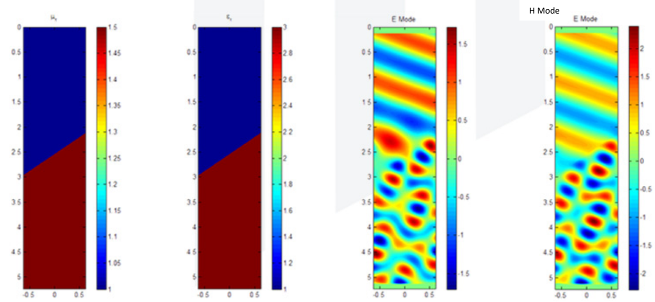
Sawtooth Diffraction Grating





	E Mode		*H* Mode	
<u>*m*</u>	<u>*R*</u>	<u>*T*</u>	<u>*R*</u>	<u>*T*</u>
-4	0%	0%	0%	0%
-3	0%	0%	0%	0%
-2	0%	5.6%	0%	0.4%
-1	0%	39.2%	0%	53.1%
0	0%	0.8%	0%	3.4%
1	0%	44.2%	0%	29.5%
2	0%	10.1%	0%	13.6%
3	0%	0%	0%	0%
4	0%	0%	0%	0%



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