



Electrostatics:

Resistor Examples

EE3321

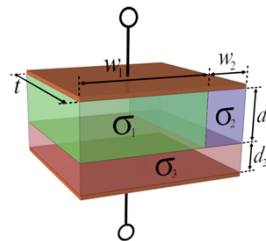
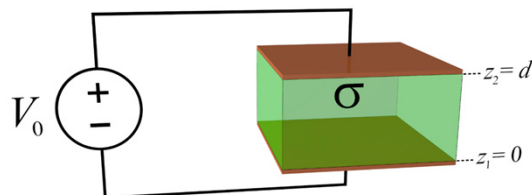
Electromagnetic Field Theory



Outline



- Parallel plate resistor
- Inhomogeneous resistor



Example #1: Parallel Plate Resistor



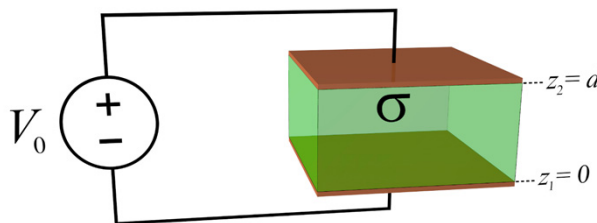
Example #1



Step 1 – Choose coordinate system.

Cartesian

Step 2 – Assume V_0



Example #1

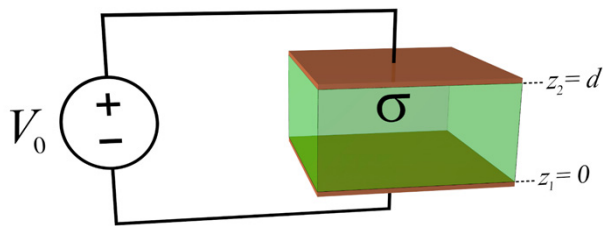


Step 3 – Solve Laplace's equation

$$\nabla^2 V = 0$$

$$\frac{d^2 V}{dz^2} = 0 \quad V(0) = 0 \text{ and } V(d) = V_0$$

$$V(z) = \frac{V_0 z}{d} \quad 0 \leq z \leq d$$



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Example #1

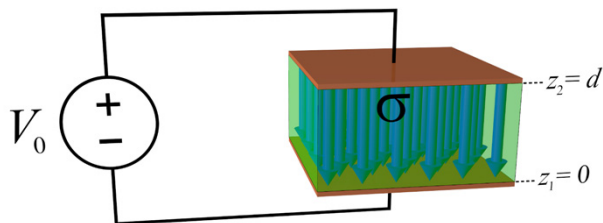


Step 4 – Calculate \vec{E}

$$\vec{E} = -\nabla V = -\frac{dV}{dz} \hat{a}_z$$

$$\vec{E} = -\frac{d}{dz} \left(\frac{V_0 z}{d} \right) \hat{a}_z \quad 0 \leq z \leq d$$

$$\vec{E} = -\frac{V_0}{d} \hat{a}_z \quad 0 \leq z \leq d$$



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Example #1



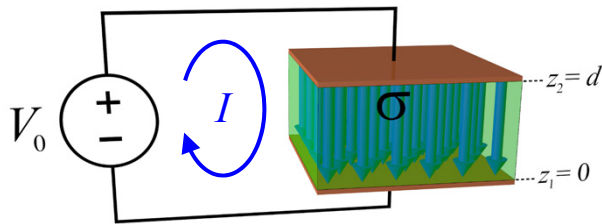
Step 5 – Calculate Current I

$$I = \iint_S \sigma \vec{E} \cdot d\vec{s} = \iint_S \sigma \left(-\frac{V_0}{d} \hat{a}_z \right) \cdot d\vec{s}$$

$$I = -\sigma \frac{V_0}{d} \underbrace{\iint_S \hat{a}_z \cdot d\vec{s}}_S = -\frac{\sigma V_0}{d} S$$

The sign can be ignored since the direction of the current is known.

$$I = \frac{\sigma V_0}{d} S$$



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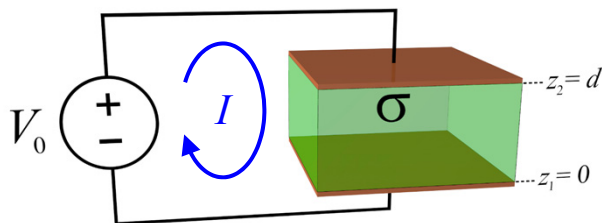
Example #1



Step 6 – Calculate R using $R = V_0/I$.

$$R = \frac{V_0}{I} = \frac{V_0}{\frac{\sigma V_0}{d} S} = \frac{d}{\sigma S}$$

$$R = \frac{d}{\sigma S}$$



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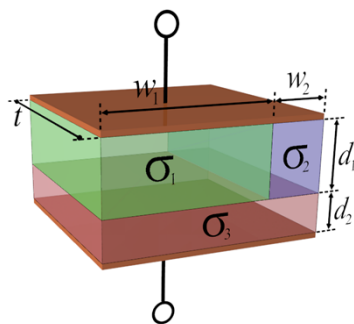
Example #2: Inhomogeneous Resistor



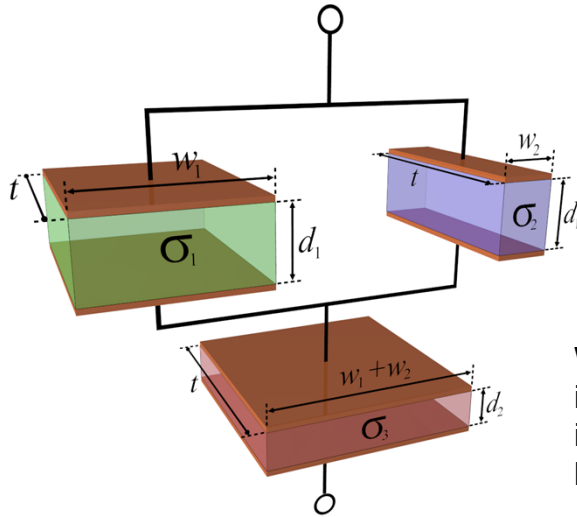
Example #2



Suppose we have an inhomogeneous resistor.



Example #2

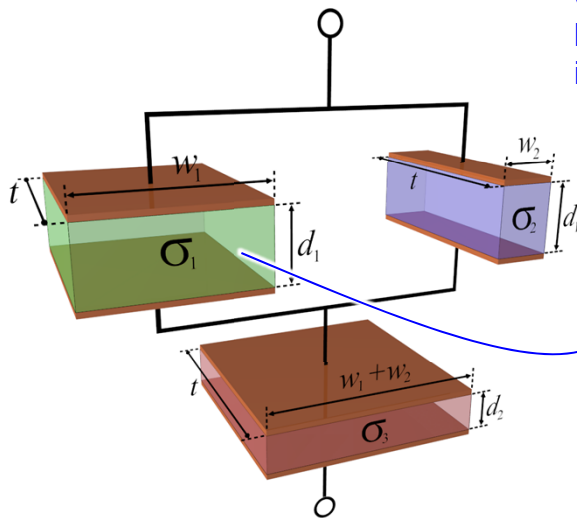


We split the inhomogeneous resistor into a combination of homogeneous resistors.

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Example #2



Calculate each homogeneous resistor independently.

$$R_1 = \frac{d_1}{\sigma_1 S_1} = \frac{d_1}{\sigma_1 t w_1}$$

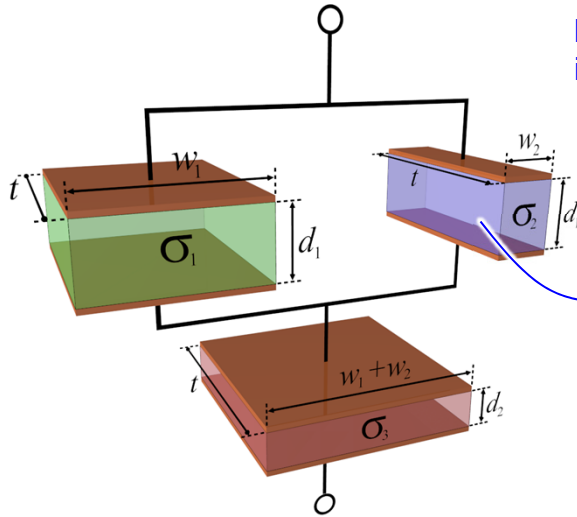
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Example #2



Calculate each homogeneous resistor independently.



$$R_1 = \frac{d_1}{\sigma_1 S_1} = \frac{d_1}{\sigma_1 t w_1}$$

$$R_2 = \frac{d_1}{\sigma_2 S_2} = \frac{d_1}{\sigma_2 t w_2}$$

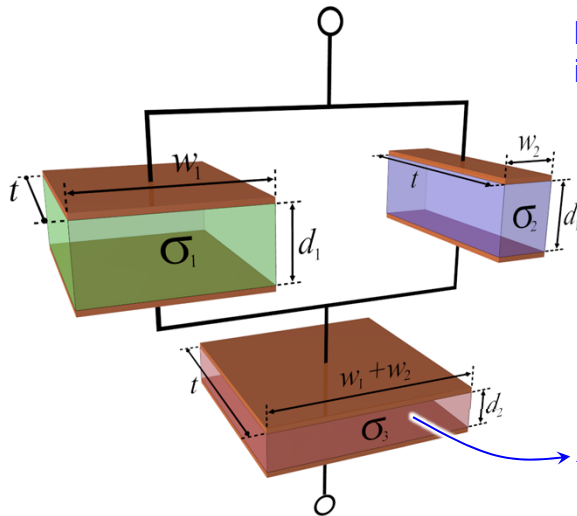
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Example #2



Calculate each homogeneous resistor independently.



$$R_1 = \frac{d_1}{\sigma_1 S_1} = \frac{d_1}{\sigma_1 t w_1}$$

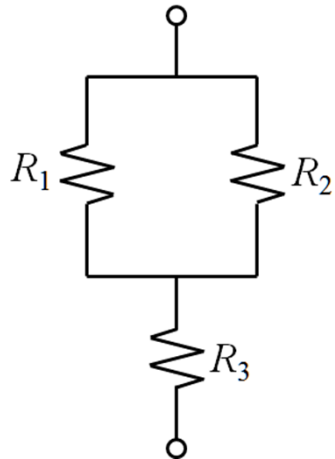
$$R_2 = \frac{d_1}{\sigma_2 S_2} = \frac{d_1}{\sigma_2 t w_2}$$

$$R_3 = \frac{d_2}{\sigma_3 S_3} = \frac{d_2}{\sigma_3 t (w_1 + w_2)}$$

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Example #2



We view our resistor as a series/parallel combination of resistors.

The equivalent resistance is

$$R_{\text{eq}} = R_1 \parallel R_2 + R_3$$

$$= \left(\frac{d_1}{\sigma_1 t w_1} \parallel \frac{d_1}{\sigma_2 t w_2} \right) + \frac{d_2}{\sigma_3 t (w_1 + w_2)}$$

$$R_{\text{eq}} = \frac{1}{t} \left[\frac{d_1 \sigma_1 \sigma_2 w_1 w_2}{\sigma_1 w_1 + \sigma_2 w_2} + \frac{d_2}{\sigma_3 (w_1 + w_2)} \right]$$