Advanced Computation:
Computational Electromagnetics

Method of Moments for Thin Wire Antennas

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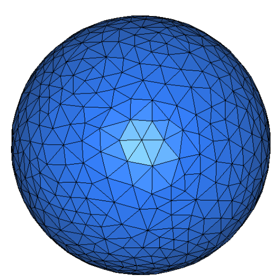
Outline

- Introduction
- Pocklington's and Hallen's Integral Equations
- Method of Moments Solution to Pocklington's Equation
- Thin Wire Excitations
- Impedance Loading



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Introduction

The Method of Moments

$$\mathbf{Lf} = \mathbf{g}$$

$$\mathbf{f} = \sum_n a_n \mathbf{v}_n \qquad \sum_n a_n \mathbf{L}\mathbf{v}_n = \mathbf{g}$$

$$\sum_n a_n \langle \mathbf{v}_m, \mathbf{L}\mathbf{v}_n \rangle = \langle \mathbf{v}_m, \mathbf{g} \rangle$$

$$\begin{bmatrix} \langle \mathbf{v}_1, \mathbf{L}\mathbf{v}_1 \rangle & \langle \mathbf{v}_1, \mathbf{L}\mathbf{v}_2 \rangle & \\ \langle \mathbf{v}_2, \mathbf{L}\mathbf{v}_1 \rangle & \langle \mathbf{v}_2, \mathbf{L}\mathbf{v}_2 \rangle & \\ & & \ddots \end{bmatrix} \begin{bmatrix} a_1 \\ a_1 \\ \vdots \\ a_N \end{bmatrix} = \begin{bmatrix} \langle \mathbf{v}_1, \mathbf{Lg} \rangle \\ \langle \mathbf{v}_2, \mathbf{Lg} \rangle \\ \vdots \\ \langle \mathbf{v}_N, \mathbf{Lg} \rangle \end{bmatrix}$$

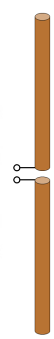
Galerkin Method

- Converts a linear equation to a matrix equation

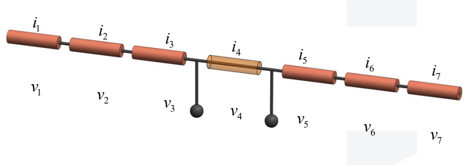
Integral Equation

- Usually uses PEC approximation
- Usually based on current

$$E_z^{\text{inc}} = \frac{j}{\omega\varepsilon} \int_{-L/2}^{L/2} I_z(z') \left(k^2 + \frac{\partial^2}{\partial z^2} \right) \frac{e^{-jkr}}{4\pi r} dz'$$



The Method of Moments



$$\begin{bmatrix} z_{11} & z_{12} & z_{13} & z_{14} & z_{15} & z_{16} & z_{17} \\ z_{21} & z_{22} & z_{23} & z_{24} & z_{25} & z_{26} & z_{27} \\ z_{31} & z_{32} & z_{33} & z_{34} & z_{35} & z_{36} & z_{37} \\ z_{41} & z_{42} & z_{43} & z_{44} & z_{45} & z_{46} & z_{47} \\ z_{51} & z_{52} & z_{53} & z_{54} & z_{55} & z_{56} & z_{57} \\ z_{61} & z_{62} & z_{63} & z_{64} & z_{65} & z_{66} & z_{67} \\ z_{71} & z_{72} & z_{73} & z_{74} & z_{75} & z_{76} & z_{77} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{bmatrix}$$

Pocklington's and Hallen's Integral Equations

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Maxwell's Equations

Maxwell's equations in the frequency-domain can be written as

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$

$$\nabla \times \vec{H} = \vec{J} + j\omega\varepsilon\vec{E}$$

$$\nabla \bullet \left(\varepsilon\vec{E}\right) = 0$$

$$\nabla \bullet \left(\mu\vec{H}\right) = 0$$

- Frequency-domain
- Differential form
- Constitutive relations have been substituted

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Definition of Magnetic Vector Potential

Since $\nabla \bullet \left(\mu \vec{H}\right) = 0$, the term $\mu \vec{H}$ is solenoidal. This means it only forms loops so it can be written in terms of the curl of some other vector function $\vec{A}$.

$$\mu \vec{H} = \nabla \times \vec{A}$$

We call that other vector function $\vec{A}$ the magnetic vector potential because it shares many attributes of the electric scalar potential.

The magnetic vector potential is not a physical quantity, but is useful in simplifying the mathematics of some electromagnetic analyses.

Definition of Electric Scalar Potential

We can substitute the magnetic vector potential into $\nabla \times \vec{E} = -j\omega\mu\vec{H}$ to arrive at

$$\nabla \times \vec{E} = -j\omega\mu\left(\frac{\nabla \times \vec{A}}{\mu}\right)$$

$$\nabla \times \vec{E} + j\omega\nabla \times \vec{A} = 0$$

$$\nabla \times \left(\vec{E} + j\omega\vec{A}\right) = 0$$

The term inside the parentheses has zero curl. This means it is "conservative" and behaves like a static electric field. We define the electric scalar potential Φ from this quantity.

$$\vec{E} + j\omega\vec{A} = -\nabla\Phi$$

Lorentz Gauge Condition

We now have more variables than we have degrees of freedom, so we need to "fix the gauge." We do this be relating some of the variables.

We have yet to specify anything about the divergence of the magnetic vector potential. For convenience and to fix the gauge, we let

$$\nabla \bullet \vec{A} = -j\omega\mu\varepsilon\Phi$$

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Wave Equation in Terms of Potentials (1 of 2)

First, we write $\nabla \times \vec{H} = \vec{J} + j\omega\varepsilon\vec{E}$ in terms of the magnetic vector potential.

$$\nabla \times \left(\frac{\nabla \times \vec{A}}{\mu} \right) = \vec{J} + j\omega\varepsilon\vec{E}$$

To simplify this further, we must <u>assume our antenna is embedded in a homogeneous medium</u>. Now our equation reduces to

$$\nabla\left(\nabla \bullet \vec{A}\right) - \nabla^2\vec{A} = \mu\vec{J} + j\omega\mu\varepsilon\vec{E}$$

We now substitute the electric scalar potential into this equation.

$$\nabla^2\vec{A} + \omega^2\mu\varepsilon\vec{A} - \nabla\left(j\omega\mu\varepsilon\Phi + \nabla \bullet \vec{A}\right) = -\mu\vec{J}$$

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Wave Equation in Terms of Potentials (2 of 2)

Next, we eliminate the electric scalar potential using the Lorentz gauge condition.

$$\nabla^2 \vec{A} + \omega^2 \mu\varepsilon \vec{A} = -\mu \vec{J}$$

We now have a vector wave equation that relates the magnetic vector potential and current. This is ideal for antenna analysis!

Recognizing that $\beta^2 = \omega^2 \mu\varepsilon \vec{A}$, the vector wave equation can also be written as

$$\nabla^2 \vec{A} + \beta^2 \vec{A} = -\mu \vec{J}$$

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Z-Axis Thin Wire

The vector wave equation can be written in matrix form as

$$\nabla^2 \vec{A} + \omega^2 \mu\varepsilon \vec{A} = -\mu \vec{J} \quad \rightarrow \quad \nabla^2 \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} + \beta^2 \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = -\mu \begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix}$$

For thin wire structures, we assume the currents are restricted to the direction of the wire. For a z-axis oriented thin wire, the wave equation reduces to

$$\nabla^2 \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} + \beta^2 \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = -\mu \begin{bmatrix} 0 \\ 0 \\ J_z \end{bmatrix}$$

Therefore, $A_x = A_y = 0$

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Revised Equations

Given the z-axis thin wire approximation, the Lorentz gauge condition reduces to

$$\nabla \bullet \vec{A} = -j\omega\mu\varepsilon\Phi \quad \rightarrow \quad \frac{\partial A_z}{\partial z} = -j\omega\mu\varepsilon\Phi$$

Our definition of electric scalar potential reduces to

$$\vec{E} + j\omega\vec{A} = -\nabla\Phi \quad \rightarrow \quad E_z + j\omega A_z = -\frac{\partial \Phi}{\partial z}$$

Combining the above equations, we arrive at

$$E_z = \frac{1}{j\omega\mu\varepsilon}\left(\frac{\partial^2 A_z}{\partial z^2} + \omega^2 \mu\varepsilon A_z\right)$$

This is not our differential equation to solve. This is how we will calculate the electric field from the magnetic vector potential.

What is a Green's Function

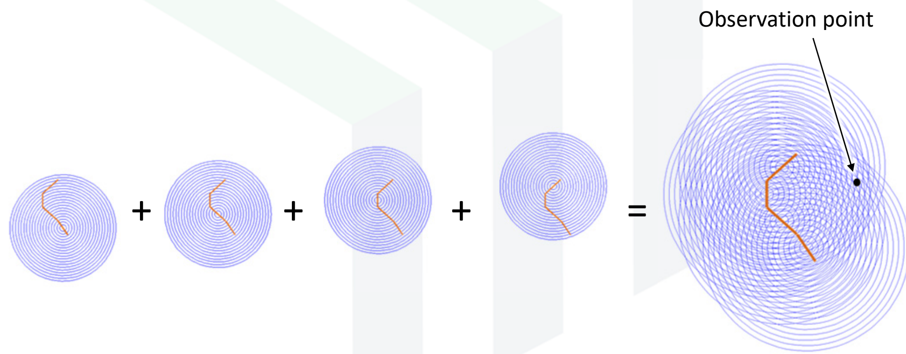
Suppose a device can be decomposed into many identical small elements.

If the response of one of these elements can be obtained, then the overall solution is the superposition of the response of all of the tiny elements comprising the device.

The response of one of these tiny elements is called the *Green's function*. The overall solution is obtained by integrating the Green's function over the domain of the device.

Illustration of a Green's Function

We can visualize integrating a Green's function this way…



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Green's Function for Small Current Element (1 of 2)

Our wave equation for the z-axis thin wire is

$$\nabla^2 A_z + \beta^2 A_z = -\mu J_z$$

Away from the wire, $J_z = 0$ and the differential equation reduces to

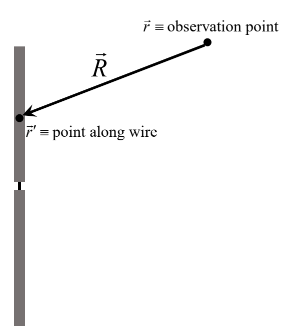
$$\nabla^2 \psi + \beta^2 \psi = 0$$

This has a solution of

$$\vec{G}\left(\vec{r},\vec{r}'\right) = \frac{e^{-j\beta R}}{4\pi R} \qquad R = \left|\vec{r} - \vec{r}'\right|$$

$\vec{r} \equiv$ observation point

$\vec{r}' \equiv$ location of point source of current



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Green's Function for Small Current Element (2 of 2)

The total magnetic vector potential is obtained by integrating the Green's function everywhere. The kernel of this integration will be zero except where there is current so the integral only has to be performed over the volume of space where there is current.

$$A_z = \iiint_v \mu J_z G dv$$

For z-axis thin wires, this becomes

$$A_z(\rho,\phi,z) = \mu \int_{-L/2}^{L/2} \int_0^{2\pi} \frac{I_z(z')}{2\pi} \frac{e^{-jkR}}{4\pi R} d\phi' dz'$$

We have assumed the current is uniform in the cross section of the wire.

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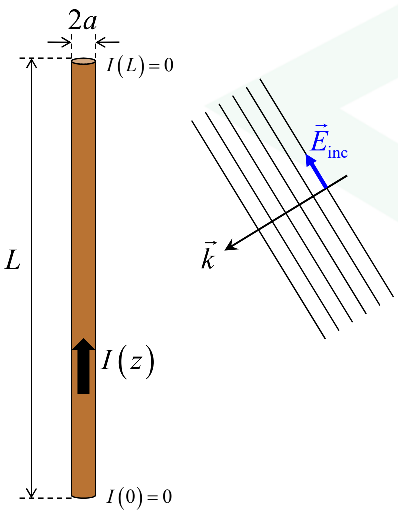
Thin Wire Approximation



Assume the wire is very thin relative to its length.

$$a \ll L$$

The incident wave excites a current on the thin wire.

$$\vec{J}(\vec{r}) = \frac{I_z(z)}{2\pi a}\hat{z}$$

We assume there is no dependence on the wire azimuthal angle ϕ.

We assume that the current goes to zero at the extreme ends of the wire.

$$I_z(0) = I_z(L) = 0$$

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Magnetic Vector Potential (1 of 3)

The magnetic vector potential on the surface of the wire due to the current in the wire is written in terms of a surface integral.

$$A_z(\rho,\phi,z) = \mu \int_{-L/2}^{L/2} \int_0^{2\pi} \frac{I_z(z')}{2\pi} \frac{e^{-jkR}}{4\pi R} d\phi' dz'$$

Where R is the distance from the point in the integral to the observation point.

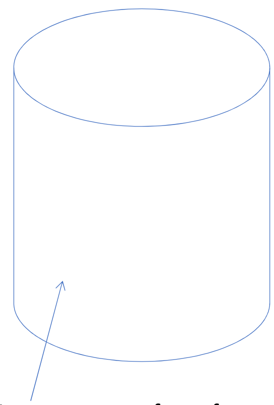
$$R = \sqrt{(z-z')^2 + (\rho-\rho')^2}$$

Since the magnetic vector potential is written on the surface of the wire, $\rho'=a$.

$$(\rho-\rho')^2 = \rho^2 + a^2 - 2\rho a \cos(\phi'-\phi)$$

Due to the cylindrical symmetry, we can replace $\phi'-\phi$ with just ϕ' without loss of generality.

$$(\rho-\rho')^2 = \rho^2 + r^2 - 2\rho r \cos\phi'$$



We integrate on surface of cylinder

Magnetic Vector Potential (2 of 3)

The magnetic vector potential can now be written as

$$A_z(\rho,\phi,z) = \mu \int_{-L/2}^{L/2} \frac{I_z(z')}{2\pi} \int_0^{2\pi} \frac{e^{-jkR}}{4\pi R} d\phi' dz'$$

$$R = \sqrt{(z-z')^2 + \rho^2 + a^2 - 2\rho a \cos\phi'}$$

If a is very small,

$$R \cong \sqrt{(z-z')^2 + \rho^2}$$

Then there is no ϕ dependence and the magnetic vector potential equation reduces to

$$A_z(\rho,z) = \mu \int_{-L/2}^{L/2} I_z(z') \frac{e^{-jkR}}{4\pi R} dz'$$

← "Thin wire approximation with reduced kernel"

The surface integral has been reduced to a line integral.

Magnetic Vector Potential (3 of 3)

For the line integral, we assume the testing points are located on the z-axis. When this is the case, $\rho=a$. and the magnetic vector potential can be written as only a function of z.

$$A_z\left(z\right)=\mu\int_{-L/2}^{L/2} I_z\left(z'\right)\frac{e^{-jkR}}{4\pi R}dz'$$

$$R \cong \sqrt{\left(z-z'\right)^2+a^2}$$

Incident and Radiated Field

The radiated field on the surface of the wire is obtained from the magnetic vector potential on the surface of the wire.

$$E_z^{\text{rad}}=-j\omega A_z-\frac{j}{\omega\mu\varepsilon}\frac{\partial^2}{\partial z^2}A_z=-\frac{j}{\omega\mu\varepsilon}\left[k^2+\frac{\partial^2}{\partial z^2}\right]A_z$$

$$k^2=k_0^2\mu_r\varepsilon_r$$

Boundary conditions require that the field on the surface of the wire is zero. The field on the surface is the sum of the incident and radiated fields.

$$E_z^{\text{total}}=E_z^{\text{rad}}+E_z^{\text{inc}}=0$$

Therefore, we can write

PEC Approximation

$$E_z^{\text{inc}}=-E_z^{\text{rad}}=\frac{j}{\omega\mu\varepsilon}\left[k^2+\frac{\partial^2}{\partial z^2}\right]A_z$$

Hallen's Integral Equation

Recall the following equations:

$$A_z(z) = \mu \int_{-L/2}^{L/2} I_z(z') \frac{e^{-jkR}}{4\pi R} dz'$$

$$E_z^{\text{inc}} = \frac{j}{\omega\mu\varepsilon}\left[k^2 + \frac{\partial^2}{\partial z^2}\right] A_z$$

Substituting the first equation into the second leads to Hallen's integral equation.

$$E_z^{\text{inc}}(z) = \frac{j}{\omega\varepsilon}\left[k^2 + \frac{\partial^2}{\partial z^2}\right] \int_{-L/2}^{L/2} I_z(z') \frac{e^{-jkR}}{4\pi R} dz'$$

$$R \cong \sqrt{(z-z')^2 + a^2}$$

Pocklington's Integral Equation

Starting with Hallen's integral equation

$$E_z^{\text{inc}}(z) = \frac{j}{\omega\varepsilon}\left[k^2 + \frac{\partial^2}{\partial z^2}\right] \int_{-L/2}^{L/2} I_z(z') \frac{e^{-jkR}}{4\pi R} dz'$$

We move the differential operator under the integral sign.

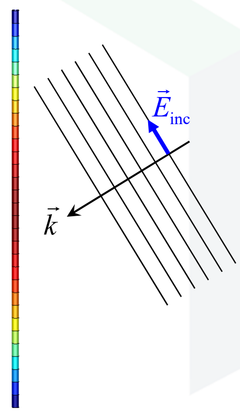
$$E_z^{\text{inc}}(z) = \frac{j}{\omega\varepsilon} \int_{-L/2}^{L/2} I_z(z') \left[k^2 + \frac{\partial^2}{\partial z^2}\right] \frac{e^{-jkR}}{4\pi R} dz'$$

While Pocklington's equation is the most famous and easier to solve, it is not as well behaved as Hallen's equation resulting in slower convergence and poorer accuracy.

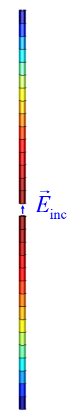
Visualizing Pocklington's Integral Equation

$$E_z^{\text{inc}}(z) = \frac{j}{\omega\varepsilon}\int_{-L/2}^{L/2} I_z(z')\left[k^2+\frac{\partial^2}{\partial z^2}\right]\frac{e^{-jkr}}{4\pi r}dz'$$

We formulated Pocklington's integral equation with being a receiving antenna in mind.



We will implement the equation as a transmitting antenna.



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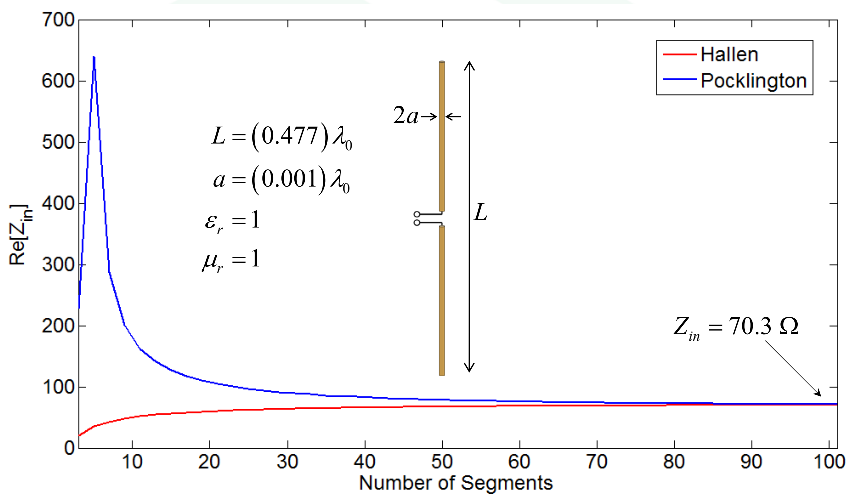
Convergence Comparison



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Method of Moments Solution to Pocklington's Equation

Setup for the Galerkin Method

We start with Pocklington's integral equation in the following form.

$$\int_{-L/2}^{L/2} I_z(z')\left[k^2+\frac{\partial^2}{\partial z^2}\right]\frac{e^{-jkR}}{4\pi R}dz'=-j\omega\varepsilon E_z^{\text{inc}}(z)$$

This has the form of $\mathrm{L}\{f\}g$ where

$$\mathrm{L}\{f\}=\frac{1}{-j\omega\varepsilon}\int_{-L/2}^{L/2} I_z(z')\left[k^2+\frac{\partial^2}{\partial z^2}\right]\frac{e^{-jkR}}{4\pi R}dz'$$

$$f=I(z)$$

$$g=E_z^{\text{inc}}(z)$$

Recall the Galerkin Method

$$L\{f\} = g$$

Step 1 – Expand unknown into set of basis functions

$$L\left\{\sum_n a_n v_n\right\} = g$$

Step 2 – Test both sides against basis functions.

$$\left\langle v_m, L\left\{\sum_n a_n v_n\right\}\right\rangle = \left\langle v_m, g\right\rangle$$

Step 3 – Construct matrix equation

$$\sum_n a_n \left\langle v_m, L\{v_n\}\right\rangle = \left\langle v_m, g\right\rangle$$

$$\begin{bmatrix} \langle v_1, L\{v_1\}\rangle & \langle v_1, L\{v_2\}\rangle & & \\ \langle v_2, L\{v_1\}\rangle & \langle v_2, L\{v_2\}\rangle & & \\ & & \ddots & \\ & & & \langle v_M, L\{v_N\}\rangle \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_M \end{bmatrix} = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_M \end{bmatrix}$$

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Galerkin Method – Step 1

Pocklington's integral equation…

$$\int_{-L/2}^{L/2} I_z(z')\left[k^2 + \frac{\partial^2}{\partial z^2}\right]\frac{e^{-jkR}}{4\pi R}dz' = -j\omega\varepsilon E_z^{\text{inc}}(z)$$

We expand the current function into a set of basis functions.

$$I_z(z) = \sum_n a_n \cdot v_n(z)$$

Substituting this into Pocklington's equation yields

$$\int_{-L/2}^{L/2}\left[\sum_n a_n \cdot v_n(z')\right]\left[k^2 + \frac{\partial^2}{\partial z^2}\right]\frac{e^{-jkR}}{4\pi R}dz' = -j\omega\varepsilon E_z^{\text{inc}}(z)$$

$$\sum_n a_n \int_{v_n} v_n(z')\left[k^2 + \frac{\partial^2}{\partial z^2}\right]\frac{e^{-jkR}}{4\pi R}dz' = -j\omega\varepsilon E_z^{\text{inc}}(z)$$

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Galerkin Method – Step 2

We test both sides of the equation with the basis functions using the inner product.

$$\sum_n a_n \int_{v_n} v_n(z') \left[k^2 + \frac{\partial^2}{\partial z^2} \right] \frac{e^{-jkR}}{4\pi R} dz' = -j\omega\varepsilon E_z^{\text{inc}}(z)$$

$$\left\langle v_m(z), \sum_n a_n \int_{v_n} v_n(z') \left[k^2 + \frac{\partial^2}{\partial z^2} \right] \frac{e^{-jkR}}{4\pi R} dz' \right\rangle = \left\langle v_m(z), -j\omega\varepsilon E_z^{\text{inc}}(z) \right\rangle$$

$$\sum_n a_n \left\langle v_m(z), \int_{v_n} v_n(z') \left[k^2 + \frac{\partial^2}{\partial z^2} \right] \frac{e^{-jkR}}{4\pi R} dz' \right\rangle = -j\omega\varepsilon \left\langle v_m(z), E_z^{\text{inc}}(z) \right\rangle$$

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Galerkin Method – Step 3

Next, we construct a matrix equation from the inner products.

$$\sum_n a_n \left\langle v_m(z), \int_{v_n} v_n(z') \left[k^2 + \frac{\partial^2}{\partial z^2} \right] \frac{e^{-jkR}}{4\pi R} dz' \right\rangle = -j\omega\varepsilon \left\langle v_m(z), E_z^{\text{inc}}(z) \right\rangle$$

$$\rightarrow \quad [z_{mn}][a_n] = [g_m]$$

$$z_{mn} = \int_{v_m} v_m(z) \int_{v_n} v_n(z') \left[k^2 + \frac{\partial^2}{\partial z^2} \right] \frac{e^{-jkR}}{4\pi R} dz' dz$$

$$g_m = -j\omega\varepsilon \int_{v_m} v_m(z) E_z^{\text{inc}}(z) dz$$

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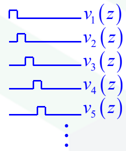
Pulse Basis Functions (1 of 3)

We let our basis functions be pulse functions defined only on the segments.

$$v_m(z) = \begin{cases} 0 & z \text{ is outside } m^{\text{th}} \text{ segment} \\ 1 & z \text{ is inside } m^{\text{th}} \text{ segment} \end{cases}$$



This is called point-matching.

Using these basis functions, we have

$$z_{mn} = \int_{v_m} v_m(z) \int_{v_n} v_n(z') \left[k^2 + \frac{\partial^2}{\partial z^2} \right] \frac{e^{-jkR}}{4\pi R} dz' dz \qquad R = \sqrt{(z_m - z')^2 + a^2}$$

$$= k^2 \int_{z_m - \frac{\Delta z}{2}}^{z_m + \frac{\Delta z}{2}} \frac{e^{-jkR}}{4\pi R} dz' + \left[(z_m - z') \frac{1 + jkR}{R^3} e^{-jkr} \right]_{z' = z_m - \frac{\Delta z}{2}}^{z' = z_m + \frac{\Delta z}{2}}$$

$$g_m = -j\omega\varepsilon \int_{v_m} v_m(z) E_z^{\text{inc}}(z) dz$$

$$= -j\omega\varepsilon E_z^{\text{inc}}(z_m)$$

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Pulse Basis Functions (2 of 3)

When calculating the impedance elements, we must evaluate the following integral as part of those calculations.

$$\int_{z_n - \frac{\Delta z}{2}}^{z_n + \frac{\Delta z}{2}} \frac{e^{-jkR}}{4\pi R} dz' \qquad R = \sqrt{(z_m - z')^2 + a^2}$$

When $m = n$, we can use a small argument approximation.

$$\int_{z_m - \frac{\Delta z}{2}}^{z_m + \frac{\Delta z}{2}} \frac{e^{-jkR}}{4\pi R} dz' \approx \int_{-\Delta z/2}^{\Delta z/2} \frac{1 - jkR}{4\pi R} dz' = \frac{1}{4\pi} \ln \left[\frac{\sqrt{1 + (2a/\Delta z)^2} + 1}{\sqrt{1 + (2a/\Delta z)^2} - 1} \right] - \frac{jk\Delta z}{4\pi}$$

Otherwise, we must numerically evaluate the integral.

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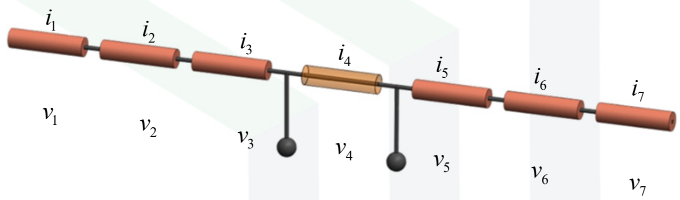
Pulse Basis Functions (3 of 3)

We can now interpret [a] as a column vector containing the currents in each segment of the antenna.



$$[a] = [i]$$

$$[z_{mn}][a_n] = [g_m]$$

Transformation to True Impedance Matrix

The matrix equation is

$$[z_{mn}][a_n] = [g_m]$$

The a_n coefficients are the currents in each segment. The g_m coefficients are scaled electric fields. Based on this, it is more intuitive to write the matrix equation as

$$[z_{mn}][i_n] = \left[-j\omega\varepsilon E_z^{\text{inc}}(z_m)\right]$$

We would like the units on the right-hand side to be voltage so that the $[Z]$ matrix is true impedance. Voltage is related to the electric field through

$$E_z^{\text{inc}}(z_m) = \frac{V_m}{\Delta z}$$

The final matrix equation in terms of element voltage and current is

$$\frac{j\Delta z}{\omega\varepsilon}[z_{mn}][i_n] = [V_m] \qquad \underbrace{\frac{j\Delta z\eta}{k}[z_{mn}]}_{\text{True } \mathbf{Z}}[i_n] = [V_m]$$

Singularity in Pocklington's Equation

Recall the equation to compute the impedance elements.

$$z_{mn}=\frac{1}{4\pi}\ln\left(\frac{\sqrt{1+\left(2a/\Delta z\right)^2}+1}{\sqrt{1+\left(2a/\Delta z\right)^2}-1}\right)-\frac{jk\Delta z}{4\pi}+\left[\left(z_m-z'\right)\frac{1+jkR}{R^3}e^{-jkr}\right]_{z'=z_n-\frac{\Delta z}{2}}^{z'=z_n+\frac{\Delta z}{2}}$$

Strong singularity!!

- Slow convergence
- Poor accuracy

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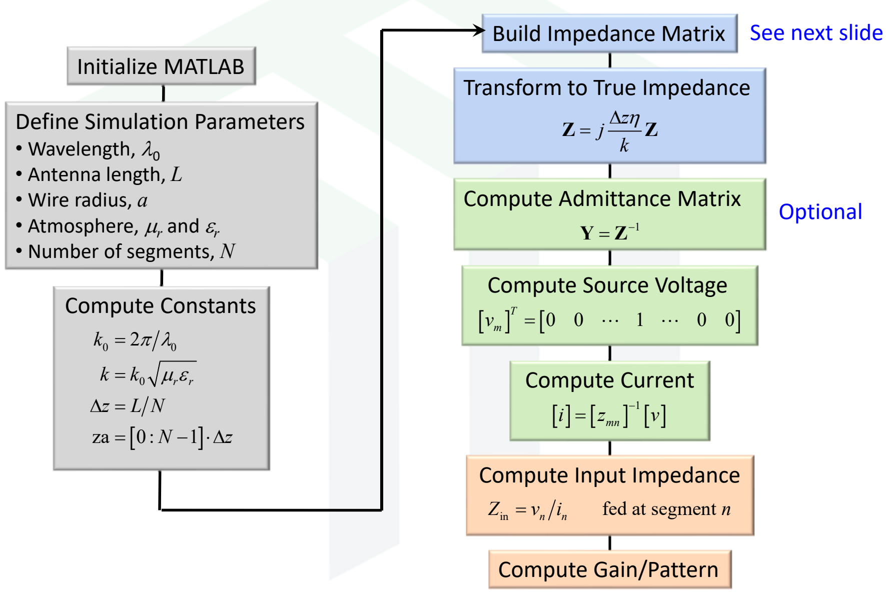
Implementation



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Building the Impedance Matrix

Compute Diagonal Term

$$z'_{mm} = \frac{1}{4\pi}\ln\left(\frac{\sqrt{1+\left(2a/\Delta z\right)^2}+1}{\sqrt{1+\left(2a/\Delta z\right)^2}-1}\right) - \frac{jk\Delta z}{4\pi}$$

Loop Over all m and n

Calculation Step #1

$$z'_{mn} = \begin{cases} z'_{mm} & \text{for } m = n \\ \displaystyle\int_{z_n-\frac{\Delta z}{2}}^{z_n+\frac{\Delta z}{2}} \frac{e^{-jkR}}{4\pi R}dz' & \text{for } m \neq n \end{cases}$$

Calculation Step #2

$$r_1 = \sqrt{\left(z_m - z_n + \Delta z/2\right)^2 + a^2}$$

$$t_1 = \left(z_m - z_n + \Delta z/2\right)\frac{1+jkr_1}{r_1^3}e^{-jkr_1}$$

$$r_2 = \sqrt{\left(z_m - z_n - \Delta z/2\right)^2 + a^2}$$

$$t_1 = \left(z_m - z_n - \Delta z/2\right)\frac{1+jkr_2}{r_2^3}e^{-jkr_2}$$

$$z_{mn} = k^2 z'_{mn} + t_2 - t_1$$

Thin Wire Excitations

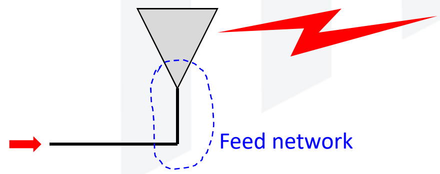
What is an Excitation?

Many antenna parameters are most easily calculated when the antenna is treated as a transmitting device.

The excitation of the antenna is the manner in which energy is “fed” into the antenna from an external source so that it can be radiated.

The properties of an antenna depend very much on how and where energy is applied to the structure.

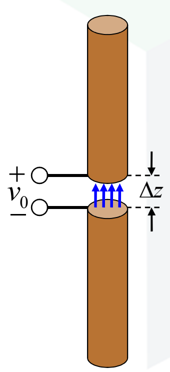
The feed system of an antenna is a hugely complex subject so our approach will be to model the feed method and not the feed itself.



The Delta-Gap Source

The detla-gap source models the feed as if the incident field exists only in the small gap at the antenna terminals.

This is the simplest source to implement. It performs well for computing radiation patterns, but is usually less accurate for impedance calculations.

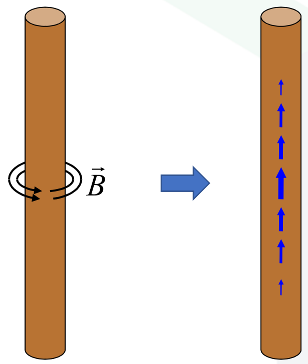


$$\vec{E}_z^{\text{inc}}(z) = \begin{cases} \dfrac{v_0}{\Delta z}\hat{z} & \text{at the gap} \\ 0 & \text{elsewhere} \end{cases}$$

The Magnetic Frill Source

The magnetic frill source models the feed magnetic field circulating around the thin wire at the feed.

This source is slightly more difficult to implement and involves more computations for the source, but is more accurate.



$$\vec{E}_z^{\text{inc}}(z) = \frac{1}{2\ln(b/a)}\left(\frac{e^{-jkr_a}}{r_a} - \frac{e^{-jkr_b}}{r_b}\right)$$

$$r_a = \sqrt{z^2 + a^2}$$

$$r_b = \sqrt{z^2 + b^2}$$

$$b \approx 3a$$

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Impedance Loading

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Incorporating Impedance Loading

Pocklington's Integral Equation

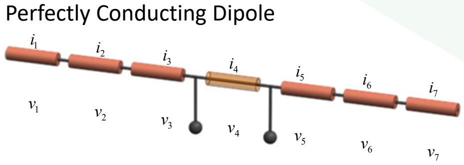
$$\frac{j}{\omega\varepsilon\Delta z}\int_L I(z')\left(\frac{\partial}{\partial z}+k^2\right)\frac{e^{-jkr}}{4\pi r}dz' = V(z)$$

Method of Moments ⇒

Matrix Equation

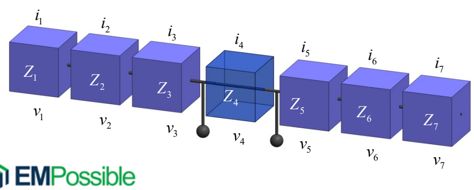
$$[Z][I]=[V]$$

Perfectly Conducting Dipole



$$\begin{bmatrix} z_{11} & z_{12} & z_{13} & z_{14} & z_{15} & z_{16} & z_{17} \\ z_{21} & z_{22} & z_{23} & z_{24} & z_{25} & z_{26} & z_{27} \\ z_{31} & z_{32} & z_{33} & z_{34} & z_{35} & z_{36} & z_{37} \\ z_{41} & z_{42} & z_{43} & z_{44} & z_{45} & z_{46} & z_{47} \\ z_{51} & z_{52} & z_{53} & z_{54} & z_{55} & z_{56} & z_{57} \\ z_{61} & z_{62} & z_{63} & z_{64} & z_{65} & z_{66} & z_{67} \\ z_{71} & z_{72} & z_{73} & z_{74} & z_{75} & z_{76} & z_{77} \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \end{bmatrix}=\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{bmatrix}$$

Impedance Loaded Dipole



$$\begin{bmatrix} z_{11}+Z_1 & z_{12} & z_{13} & z_{14} & z_{15} & z_{16} & z_{17} \\ z_{21} & z_{22}+Z_2 & z_{23} & z_{24} & z_{25} & z_{26} & z_{27} \\ z_{31} & z_{32} & z_{33}+Z_3 & z_{34} & z_{35} & z_{36} & z_{37} \\ z_{41} & z_{42} & z_{43} & z_{44}+Z_4 & z_{45} & z_{46} & z_{47} \\ z_{51} & z_{52} & z_{53} & z_{54} & z_{55}+Z_5 & z_{56} & z_{57} \\ z_{61} & z_{62} & z_{63} & z_{64} & z_{65} & z_{66}+Z_6 & z_{67} \\ z_{71} & z_{72} & z_{73} & z_{74} & z_{75} & z_{76} & z_{77}+Z_7 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \end{bmatrix}=\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{bmatrix}$$

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