



Electromagnetics:
Electromagnetic Field Theory

Ampere's Circuit Law

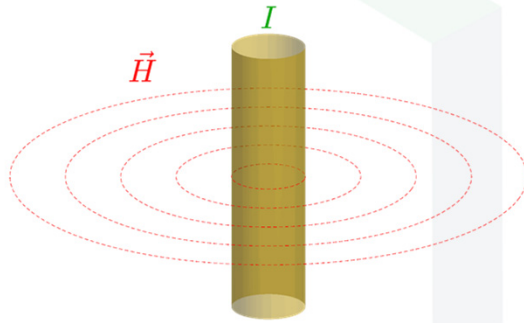
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Outline

- Ampere's Experiment
- Three Types of Current
- Ampere's Law in Integral Form
- Ampere's Law in Differential Form

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Ampere's Experiment



Observations:

1. There exists a circulating magnetic field \vec{H} around a conductor carrying a current I .
2. The current I can induce the magnetic field \vec{H} , or the magnetic field \vec{H} can induce the current I .
3. The measured current I is in proportion to the circulation of \vec{H} .

$$I = \oint_L \vec{H} \cdot d\vec{\ell}$$

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Three Types of Current

The total current I can be calculated by integrating the flux of the electric current density \vec{J} through a cross section of the conductor.

$$I = \iint_S \vec{J}_{\text{total}} \cdot d\vec{s}$$

However, recall that there are three types of electric current.

$$\vec{J}_{\text{total}} = \underbrace{\vec{J}_{\sigma} + \vec{J}_{\epsilon}}_{\vec{J}} + \vec{J}_D$$

$$\vec{J}_D = \frac{\partial \vec{D}}{\partial t} \equiv \text{displacement current}$$

$$\vec{J} \equiv \text{current due to free charges}$$

Putting this together gives

$$I = \iint_S \left[\vec{J} + \frac{\partial \vec{D}}{\partial t} \right] \cdot d\vec{s}$$

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Ampere's Circuit Law in Integral Form

We have two ways of calculating the total current I in a conductor. Setting these equal gives...

Ampere's
Experiment

Simple integration
of current density

$$I = \oint_L \vec{H} \cdot d\vec{\ell} = \iint_S \left[\vec{J} + \frac{\partial \vec{D}}{\partial t} \right] \cdot d\vec{s}$$

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Apply Stoke's Theorem

Stoke's theorem allows us to write a closed-contour line integral as a surface integral.

$$\oint_L \vec{A} \cdot d\vec{\ell} = \iint_S (\nabla \times \vec{A}) \cdot d\vec{s}$$

Applying this to Ampere's law in integral form gives us

$$I = \oint_L \vec{H} \cdot d\vec{\ell} = \iint_S \left[\vec{J} + \frac{\partial \vec{D}}{\partial t} \right] \cdot d\vec{s}$$

↓

$$I = \iint_S (\nabla \times \vec{H}) \cdot d\vec{s} = \iint_S \left[\vec{J} + \frac{\partial \vec{D}}{\partial t} \right] \cdot d\vec{s}$$

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Ampere's Circuit Law in Differential Form

If the line L and surface S describe the same space, the argument of both integrals must be equal. Setting these arguments equal gives Ampere's circuit law in differential form.

$$I = \iint_S (\nabla \times \vec{H}) \cdot d\vec{s} = \iint_S \left[\vec{J} + \frac{\partial \vec{D}}{\partial t} \right] \cdot d\vec{s}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Visualization of Ampere's Circuit Law in Differential Form

