Electromagnetics:

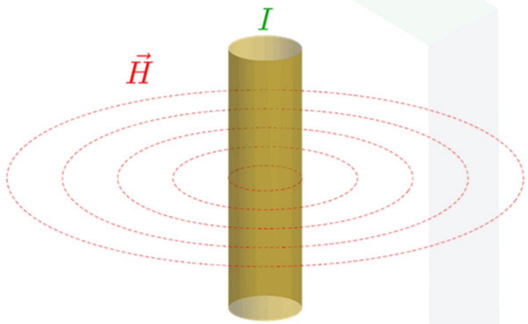
Electromagnetic Field Theory

Ampere's Circuit Law

Outline

- Ampere's Experiment
- Three Types of Current
- Ampere's Law in Integral Form
- Ampere's Law in Differential Form

Ampere's Experiment



Observations:

1. There exists a circulating magnetic field $\vec{H}$ around a conductor carrying a current I.
2. The current I can induce the magnetic field $\vec{H}$, or the magnetic field $\vec{H}$ can induce the current I.
3. The measured current I is in proportion to the circulation of $\vec{H}$.

$$I = \oint_L \vec{H} \bullet d\vec{\ell}$$

Three Types of Current

The total current I can be calculated by integrating the flux of the total electric current density $\vec{J}_\text{T}$ through a cross section S of the conductor.

$$I = \iint_S \vec{J}_\text{T} \bullet d\vec{s}$$

However, recall that there are three types of electric current.

$$\vec{J}_\text{T} = \underbrace{\vec{J}_\sigma + \vec{J}_\varepsilon}_{\vec{J}} + \vec{J}_D$$

$$\vec{J}_D = \frac{\partial \vec{D}}{\partial t} \equiv \text{displacement current}$$

$$\vec{J} \equiv \text{current due to free charges}$$

Putting all of this together gives

$$I = \iint_S \left[\vec{J} + \frac{\partial \vec{D}}{\partial t} \right] \bullet d\vec{s}$$

Ampere's Circuit Law in Integral Form

There are two ways of calculating the total current I in a conductor. Setting these equal gives

Ampere's Experiment → $\oint_L \vec{H} \bullet d\vec{\ell}$

Simple integration of current density → $\iint_S \left[\vec{J} + \frac{\partial \vec{D}}{\partial t} \right] \bullet d\vec{s}$

$$I = \oint_L \vec{H} \bullet d\vec{\ell} = \iint_S \left[\vec{J} + \frac{\partial \vec{D}}{\partial t} \right] \bullet d\vec{s}$$

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Apply Stoke's Theorem

Stoke's theorem allows a closed-contour line integral to be written as a surface integral.

$$\oint_L \vec{A} \bullet d\vec{\ell} = \iint_S \left(\nabla \times \vec{A} \right) \bullet d\vec{s}$$

Stoke's theorem

Applying this to Ampere's law in integral form gives us

$$I = \oint_L \vec{H} \bullet d\vec{\ell} = \iint_S \left[\vec{J} + \frac{\partial \vec{D}}{\partial t} \right] \bullet d\vec{s}$$

$$\downarrow$$

$$I = \iint_S \left(\nabla \times \vec{H} \right) \bullet d\vec{s} = \iint_S \left[\vec{J} + \frac{\partial \vec{D}}{\partial t} \right] \bullet d\vec{s}$$

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Ampere's Circuit Law in Differential Form

If the line L and surface S describe the same space, then the argument of both integrals must be equal. Setting these arguments equal gives Ampere's circuit law in differential form.

$$I = \iint_S \left(\nabla \times \vec{H}\right) \bullet d\vec{s} = \iint_S \left[\vec{J} + \frac{\partial \vec{D}}{\partial t}\right] \bullet d\vec{s}$$

$$\boxed{\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

Visualization of Ampere's Circuit Law in Differential Form

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$