Electromagnetics:
Electromagnetic Field Theory

Analysis & Design of Multi-Segment Transmission Lines

1

Lecture Outline

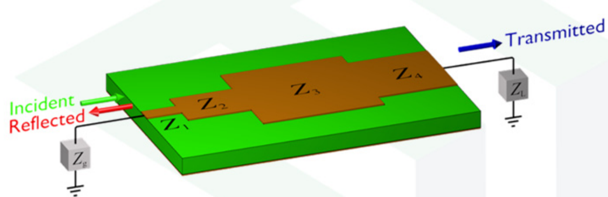
- Circuit/Wave Equivalence
- Analysis of Multi Segment Transmission Lines
- Digital Filter Analogy
- Example

2

Circuit Wave Equivalence

3

Circuit/Wave Equivalence (1 of 3)



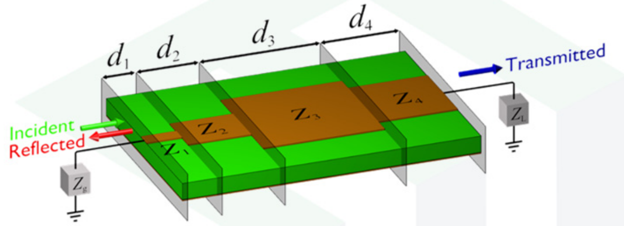
Here we have a “stepped-impedance” microwave circuit.

EMPossible

Slide 4

4

Circuit/Wave Equivalence (2 of 3)

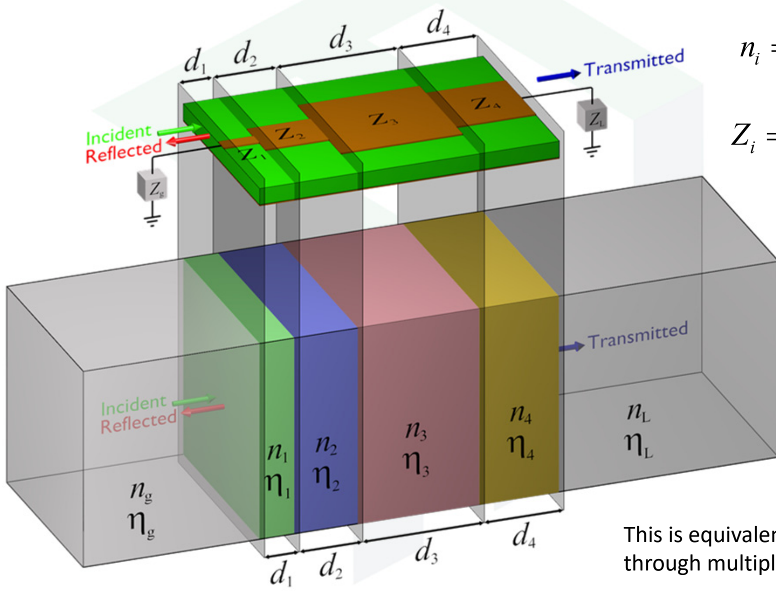


We view the circuit as a series of discrete segments that are uniform within the segment.

EMPossible Slide 5

5

Circuit/Wave Equivalence (3 of 3)



$$n_i = \sqrt{\mu_{r,i}\varepsilon_{r,i}}$$

$$Z_i = \eta_i = \sqrt{\frac{\mu_i}{\varepsilon_i}}$$

This is equivalent to waves propagating through multiple slabs of dielectric.

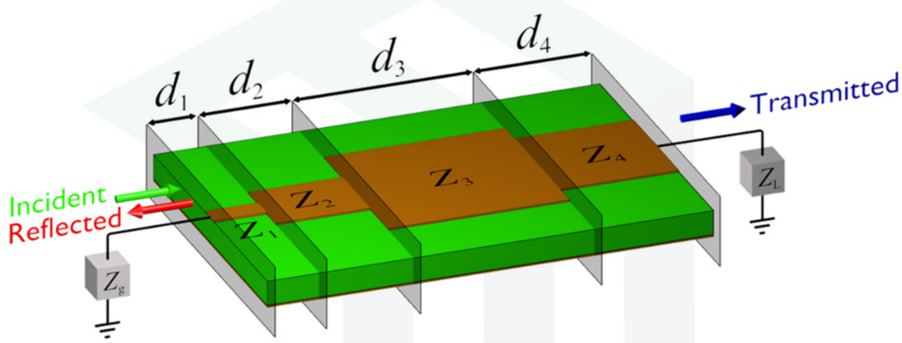
EMPossible Slide 6

6

Analysis of Multi-Segment Transmission Lines

7

Impedance Transformation Method (1 of 6)



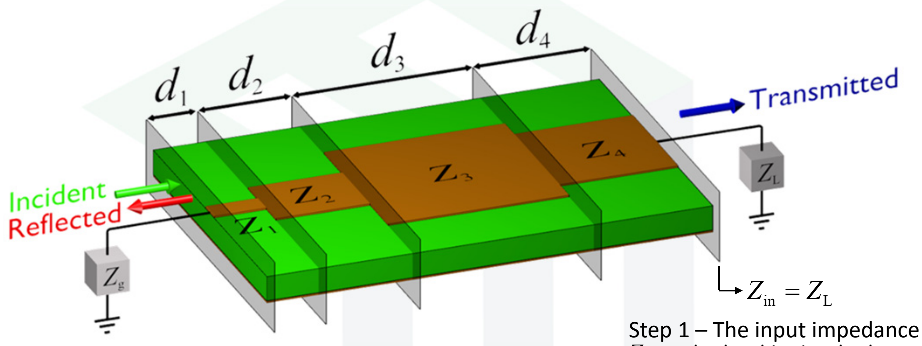
How do we calculate the reflection from this circuit?

EMPossible

Slide 8

8

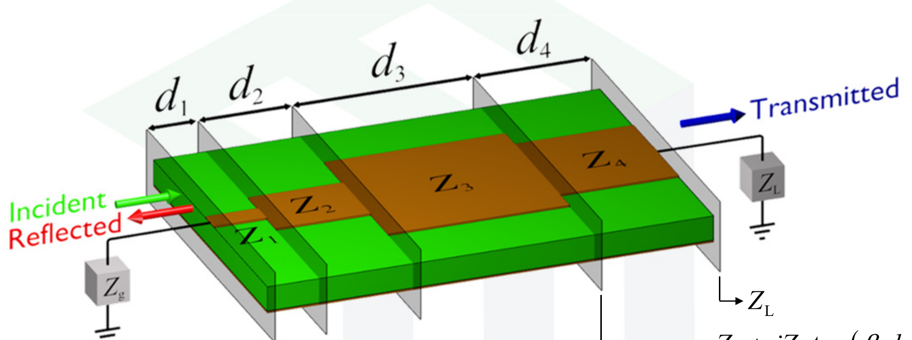
Impedance Transformation Method (2 of 6)



$$Z_{\text{in}} = Z_{\text{L}}$$

Step 1 – The input impedance Z_{in} at the load is simply the load impedance Z_{L}.

Impedance Transformation Method (3 of 6)

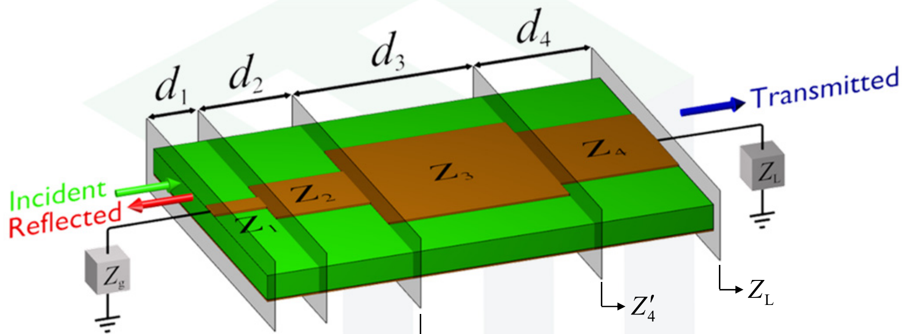


$$Z_{\text{L}}$$

$$Z_4' = Z_4 \frac{Z_{\text{L}} + jZ_4 \tan\left(\beta_4 d_4\right)}{Z_4 + jZ_{\text{L}} \tan\left(\beta_4 d_4\right)}$$

Step 2 – We use impedance transformation to calculate the impedance looking into the fourth segment.

Impedance Transformation Method (4 of 6)



$$Z_3' = Z_3 \frac{Z_4' + jZ_3 \tan(\beta_3 d_3)}{Z_3 + jZ_4' \tan(\beta_3 d_3)}$$

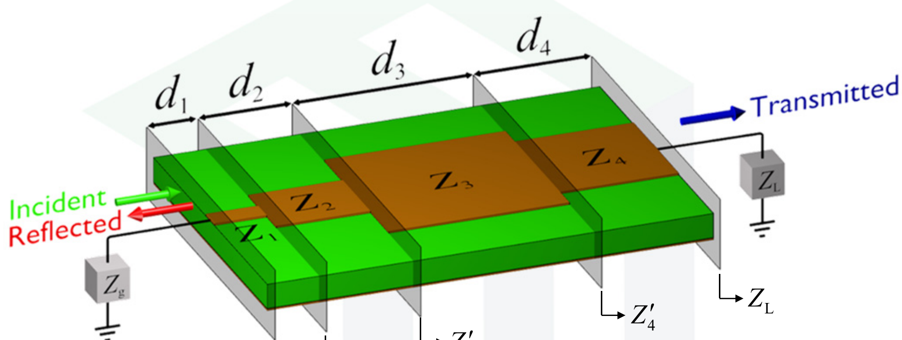
Step 3 – We use impedance transformation to calculate the impedance looking into the third segment.

EMPossible

Slide 11

11

Impedance Transformation Method (5 of 6)



$$Z_2' = Z_2 \frac{Z_3' + jZ_2 \tan(\beta_2 d_2)}{Z_2 + jZ_3' \tan(\beta_2 d_2)}$$

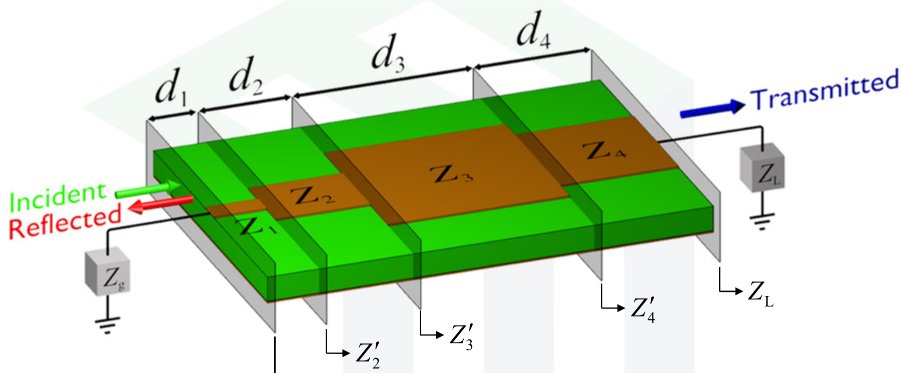
Step 4 – We use impedance transformation to calculate the impedance looking into the second segment.

EMPossible

Slide 12

12

Impedance Transformation Method (6 of 6)



$$Z_1' = Z_1 \frac{Z_2' + jZ_1 \tan(\beta_1 d_1)}{Z_1 + jZ_2' \tan(\beta_1 d_1)}$$

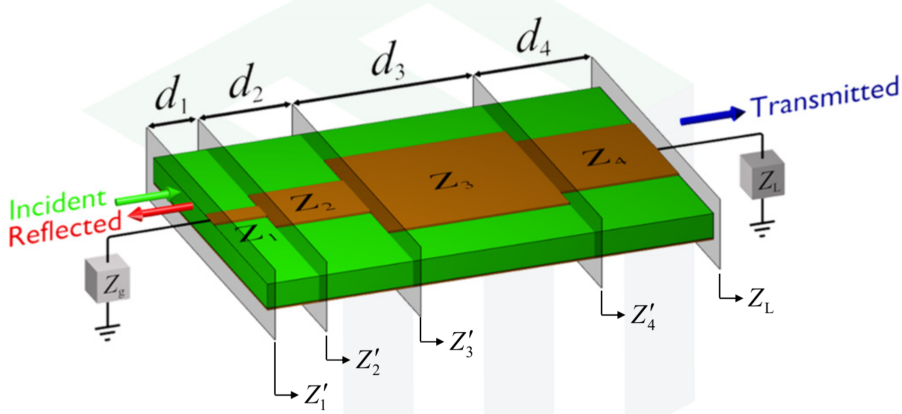
Step 5 – We use impedance transformation to calculate the impedance looking into the first segment.

EMPossible

Slide 13

13

Impedance Transformation Method (7 of 6)



Step 6 – We calculate the overall reflection as seen by the generator.

$$\Gamma = \frac{Z_1' - Z_g}{Z_1' + Z_g}$$

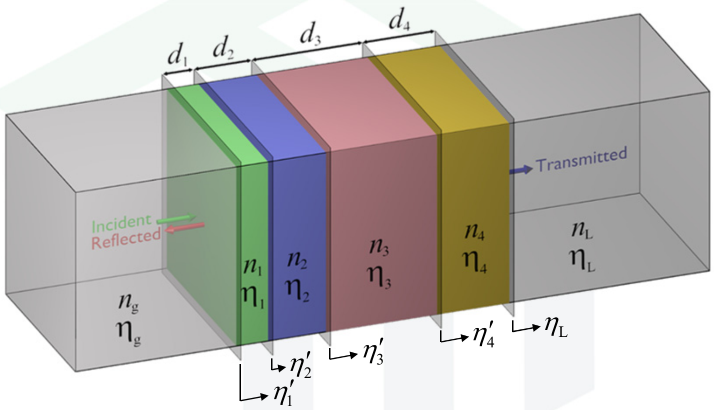
It is straightforward to extend this procedure to analyze circuits composed of any number of segments.

EMPossible

Slide 14

14

Impedance Transformation Method for Waves



Impedance Transformation for Waves

$$\eta'_i = \eta_i \frac{\eta'_{i+1} + j\eta_i \tan(\beta_i d_i)}{\eta_i + j\eta'_{i+1} \tan(\beta_i d_i)}$$

Overall Reflection

$$r = \frac{\eta'_1 - \eta_g}{\eta'_1 + \eta_g}$$

$$R = |r|^2$$

EMPossible Slide 15

15

Notes on the Impedance Transformation Method

- Very fast and simple to implement!
- Difficult to modify the method to calculate transmission when the materials have loss or gain.
- When gain and loss can be ignored, $R + T = 1$.
- Method cannot directly visualize the fields inside of the device.

EMPossible Slide 16

16

Digital Filter Analogy

Conditions for Analogy

The reflection coefficient Γ_i for a wave in medium i incident on medium $i+1$ is given by

$$\Gamma_i = \frac{Z_{i+1} - Z_i}{Z_{i+1} + Z_i} = \frac{\eta_{i+1} - \eta_i}{\eta_{i+1} + \eta_i}$$

Notice this is written for both transmission lines and dielectric slabs.

The overall reflection coefficient Γ_{in} from a series of M segments is very complicated. However, it can be greatly simplified given two conditions:

1. The electrical length of each segment is the same.

$$\psi \cong \beta_1 d_1 \cong \cdots \cong \beta_M d_M$$

2. The reflection coefficients Γ_i at the interfaces are small, allowing us to ignore waves reflected more than once.

$$\left|\Gamma_i\right| < \ \sim 0.1$$

Expression for Overall Reflection

Given the two conditions on the previous slide, the overall reflection coefficient Γ_{in} from M layers can be written as

$$\Gamma_{\text{in}} = \Gamma_0 + \Gamma_1 e^{-j2\psi} + \Gamma_2 e^{-j4\psi} + \cdots + \Gamma_M e^{-j2M\psi}$$

$$\Gamma_i = \frac{Z_{i+1} - Z_i}{Z_{i+1} + Z_i} = \frac{\eta_{i+1} - \eta_i}{\eta_{i+1} + \eta_i} \qquad \psi \cong \beta_1 d_1 \cong \cdots \cong \beta_M d_M$$

Digital FIR Filters

The general form of the transfer function for a finite impulse response (FIR) digital filter is

$$H(z) = \sum_{k=0}^{M} h(k) z^{-k} = h(0) + h(1) z^{-1} + h(2) z^{-2} + \cdots + h(M) z^{-M}$$

If we excite the digital filter with an impulse function at $t = 0$, the we observe $h(0)$ at time 0, $h(1)$ at time 1, $h(2)$ at time 2, and so on.

The frequency-domain response of a digital FIR filter is

$$H(\omega) = \sum_{k=0}^{M} h(k) e^{-j2k\psi} = h(0) + h(1) e^{-j2\psi} + h(2) e^{-j4\psi} + \cdots + h(M) e^{-j2M\psi}$$

The Analogy

If we compare the overall reflection coefficient Γ_{in} of our multi-segment/multi-layer device, we see that it has the same form as a digital filter.

$$\Gamma_{\text{in}} = \Gamma_0 + \Gamma_1 e^{-j2\psi} + \Gamma_2 e^{-j4\psi} + \cdots + \Gamma_M e^{-j2M\psi}$$
$$H = h(0) + h(1)e^{-j2\psi} + h(2)e^{-j4\psi} + \cdots + h(M)e^{-j2M\psi}$$

This means we can design multi-segment/multi-layer filters just like we design a digital filter.

Much is known about designing digital filters and all of this knowledge and all of the tools can be used to design filters for electromagnetic waves!

EMPossible

Slide 21

21

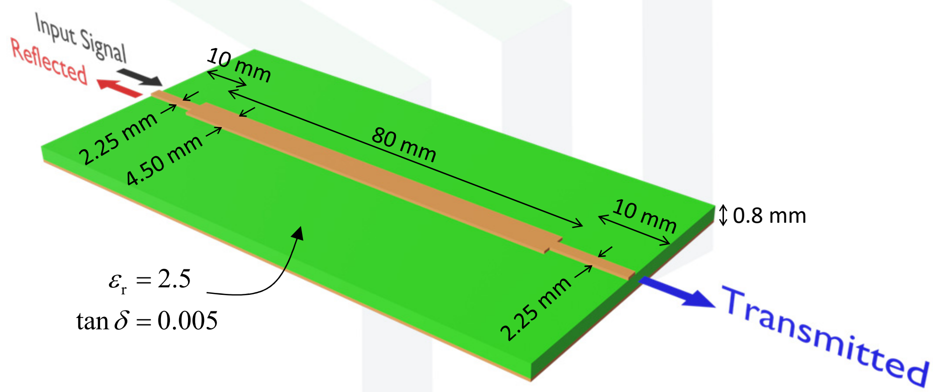
Example

Slide 22

22

Stepped-Impedance Microstrip

A stepped impedance microstrip circuit is designed into an FR4 printed circuit board as illustrated below. Simulate its reflectance and transmittance from 0 GHz up to 10 GHz.



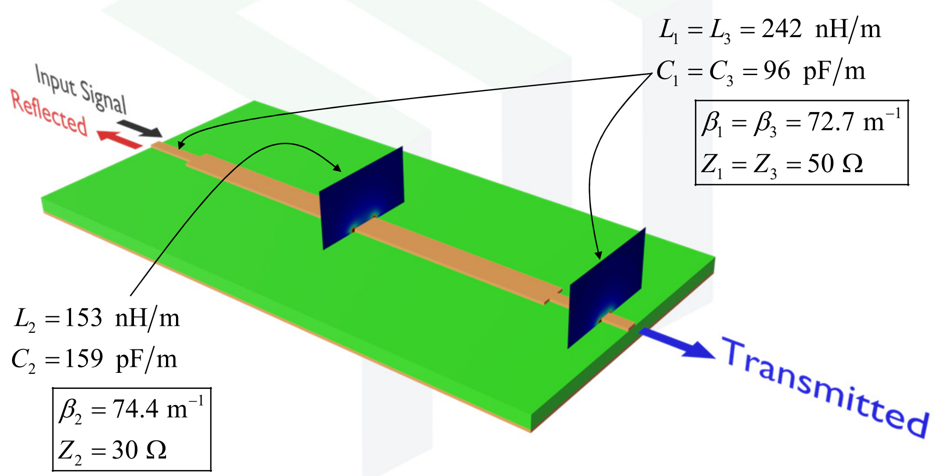
EMPossible

Slide 23

23

Step 1 – Analyze TL Regions

An electromagnetic simulation is performed to calculate the parameters characterizing the transmission lines in each segment of the filter.



β reported at 2.4 GHz

EMPossible

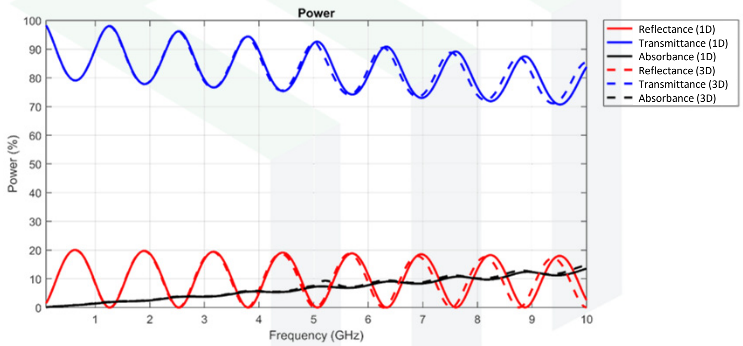
Slide 24

24

Step 2 – Simulate Multi-Segment Device

Simulation (1D): ~10 sec

Simulation (3D): ~4 min



Each frequency ω is an entirely different simulation.
β must be recalculated at each frequency ω.

$$\beta_i = \omega\sqrt{L_i C_i}$$

EMPossible