Electromagnetics:
Electromagnetic Field Theory

Charge Distributions

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Outline

- Charge Distributions
- Fields Around Charge Distributions
- Recipe to Calculate Field Around Charge Distributions

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Charge Distributions

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Point Charge

Size: infinitely small

Total Charge: Q

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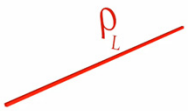
Line Charge



Size: Length L

Charge Density: Coulombs/Meter (C/m)

Total Charge:

$$Q = \begin{cases} \int_L \rho_\ell \, d\ell & \text{general case} \\ \rho_\ell L & \text{uniform } \rho_\ell \end{cases}$$

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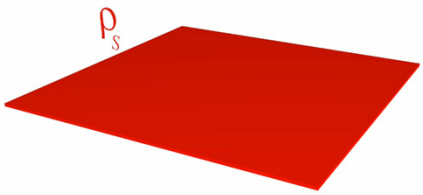
Surface Charge



Size: Length L
Width W

Charge Density: Coulombs/Meter^2 (C/m^2)

Total Charge:

$$Q = \begin{cases} \iint_S \rho_s \, ds & \text{general case} \\ \rho_s S & \text{uniform } \rho_s \end{cases}$$

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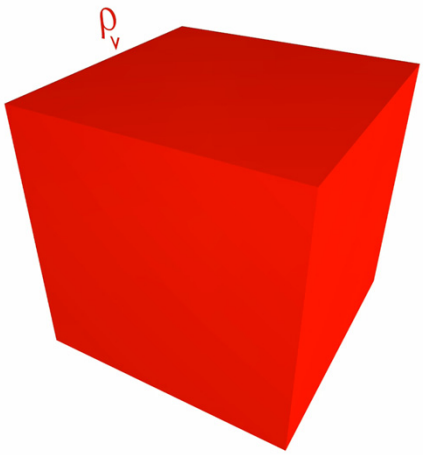
Volume Charge



Size: Length L
Width W
Height H

Charge Density: Coulombs/Meter^3 (C/m^3)

Total Charge:

$$Q = \begin{cases} \iiint_v \rho_v dv & \text{general case} \\ \rho_v v & \text{uniform } \rho_v \end{cases}$$

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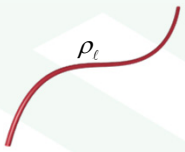
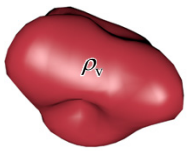
Charge Distribution Summary

Point Charge	Line Charge	Sheet Charge	Volume Charge
			
Charge $Q\ (\mathrm{C})$	Line Charge Density $\rho_\ell\ (\mathrm{C/m})$	Surface Charge Density $\rho_s\ (\mathrm{C/m^2})$	Volume Charge Density $\rho_v\ (\mathrm{C/m^3})$
Total Charge $Q_{\text{Total}} = \sum_{i=1}^{N} Q_i$	Total Charge $Q_{\text{Total}} = \int_\ell \rho_\ell d\ell \cong \rho_\ell L$	Total Charge $Q_{\text{Total}} = \iint_S \rho_s ds \cong \rho_s S$	Total Charge $Q_{\text{Total}} = \iiint_v \rho_v dv = \rho_v V$
Total Field $\vec{D}_{\text{Total}} = \sum_{i=1}^{N} \frac{Q_i}{4\pi R_i^2} \hat{a}_{R_i}$	Total Field $\vec{D}_{\text{Total}} = \int_\ell \frac{\rho_\ell d\ell}{4\pi R^2} \hat{a}_R$	Total Field $\vec{D}_{\text{Total}} = \iint_s \frac{\rho_s ds}{4\pi R^2} \hat{a}_R$	Total Field $\vec{D}_{\text{Total}} = \iiint_v \frac{\rho_v dv}{4\pi R^2} \hat{a}_R$

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Fields Around Charge Distributions

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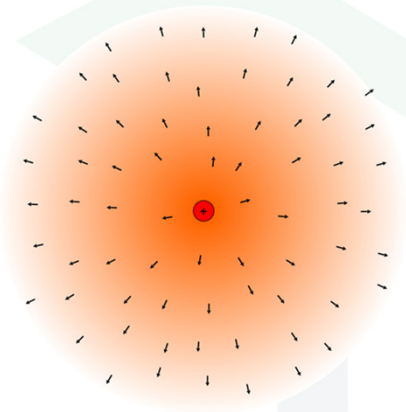
Point Charge



$$\vec{D} = \frac{Q}{4\pi r^2}\hat{a}_r$$

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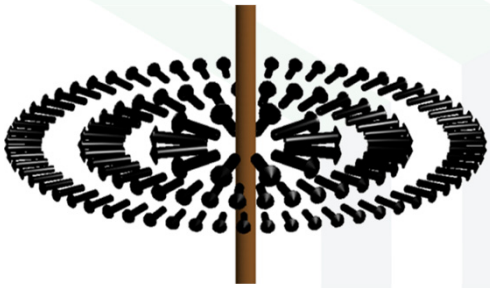
Infinite Line Charge



$$\vec{D} = \frac{\rho_L}{2\pi\rho}\hat{a}_\rho$$

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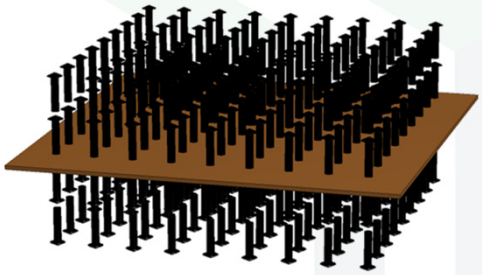
Infinite Surface Charge



$$\vec{D}_{\text{total}} = \frac{\rho_S}{2}\hat{a}_z$$

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Recipe to Calculate Field Around Charge Distributions

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Recipe for Solving Problems

1. Draw the problem and label with all dimensions and parameters.
2. Choose a coordinate system to make math easier.
3. Write general equation.

	Point	Line	Surface	Volume
Q_{Total}	$\sum_{i=1}^{N} Q_i$	$\int_{\ell} \rho_\ell d\ell$	$\iint_{S} \rho_s ds$	$\iiint_{V} \rho_v dv$
D_{Total}	$\sum_{i=1}^{N} \frac{Q_i}{4\pi R_i^2} \hat{a}_{R_i}$	$\int_{\ell} \frac{\rho_\ell d\ell}{4\pi R^2} \hat{a}_R$	$\iint_{S} \frac{\rho_s ds}{4\pi R^2} \hat{a}_R$	$\iiint_{V} \frac{\rho_v dv}{4\pi R^2} \hat{a}_R$

4. Write expressions for each term in the integral.
5. Choose limits of integration.
6. Solve the integral.
7. Interpret the result.

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