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Electromagnetics:
Electromagnetic Field Theory

Coordinate Systems

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Outline

- Coordinate Systems & Unit Vectors
- Conversion Between Different Coordinate Systems

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Coordinate Systems & Unit Vectors

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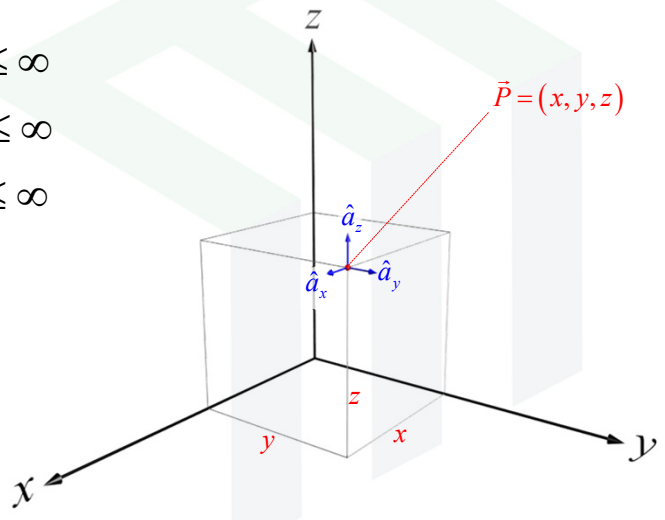
Cartesian Coordinates (x, y, z)

$$-\infty \le x \le \infty$$
$$-\infty \le y \le \infty$$
$$-\infty \le z \le \infty$$



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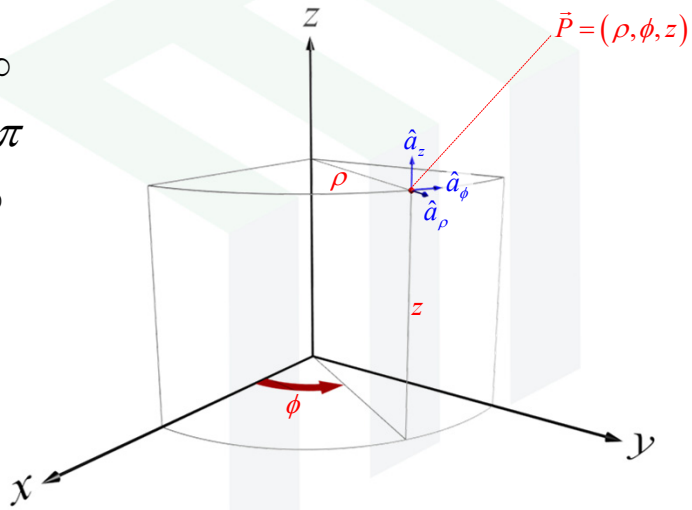
Cylindrical Coordinates (ρ, ϕ, z)

$$0 \le \rho \le \infty$$
$$0 \le \phi < 2\pi$$
$$-\infty \le z \le \infty$$



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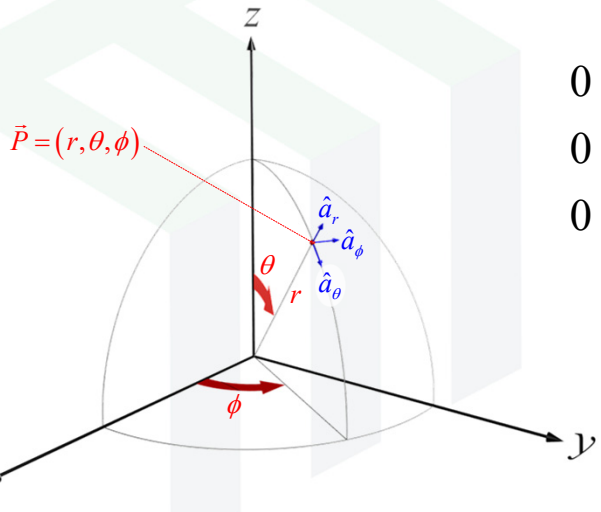
Spherical Coordinates (r, θ, ϕ)



$$0 \le r \le \infty$$
$$0 \le \theta \le \pi$$
$$0 \le \phi < 2\pi$$

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Conversion Between Coordinate Systems

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Cartesian ↔ Cylindrical

Coordinate Conversion

$$\rho = \sqrt{x^2 + y^2}$$

$$\phi = \text{atan}\,2\left(\frac{y}{x}\right)$$

$$z = z$$

$$x = \rho\cos\phi$$

$$y = \rho\sin\phi$$

$$z = z$$

Vector Conversion

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

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Cartesian ↔ Spherical

Coordinate Conversion

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{x^2 + y^2}}{z}\right)$$

$$\phi = \text{atan}\,2\left(y/x\right)$$

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

Vector Conversion

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

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