Electromagnetics:
Electromagnetic Field Theory

Examples & Applications

Lecture Outline

- Scattering at an Interface
- Sunrises & Sunsets
- Rainbows
- Polarized Sunglasses

Scattering at an Interface

Numerical Example (1 of 11)

A left-hand circularly polarized (LCP) wave is incident from air ($n_{\text{air}} = 1.0$) onto water ($n_{\text{H2O}} = 1.327$) at 53° off the normal. Determine the following:

1. The critical angle θ_{c} for both TE and TM polarizations
2. The Brewster's angle θ_{B} for both TE and TM polarizations
3. Angle of transmission θ_{t} of both TE and TM polarizations
4. Impedance of both media η_1 and η_2
5. Reflection coefficient r for both TE and TM polarizations.
6. Transmission coefficient t for both TE and TM polarizations.
7. Overall reflectance R of the wave
8. Overall transmittance T of the wave
9. Does $R + T$ = 100%? If not, why not?
10. Polarization of the reflected wave.

Numerical Example (2 of 11)

A left-hand circularly polarized (LCP) wave is incident from air ($n_{\text{air}} = 1.0$) onto water ($n_{\text{H2O}} = 1.327$) at 53° off the normal.

1 – The critical angle θ_c for both TE and TM polarizations

The critical angle θ_c is the same for both polarizations.

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) \qquad n_1 > n_2$$

There is no critical angle because this example has $n_1 < n_2$.

Aside: there is a critical angle for waves propagating from water to air.

$$\theta_c = \sin^{-1}\left(\frac{1.0}{1.327}\right) = 48.9°$$

Numerical Example (3 of 11)

A left-hand circularly polarized (LCP) wave is incident from air ($n_{\text{air}} = 1.0$) onto water ($n_{\text{H2O}} = 1.327$) at 53° off the normal.

2 – The Brewster's angle θ_B for both TE and TM polarizations

Assuming there is no magnetic response, we only have a Brewster's angle for the TM polarization component of the wave.

$$\tan\theta_{\text{B,TM}} = \frac{n_2}{n_1}$$

$$\theta_{\text{B,TM}} = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

$$= \tan^{-1}\left(\frac{1.327}{1.0}\right) \quad \rightarrow \quad \boxed{\theta_{\text{B,TM}} = 53°}$$

Numerical Example (4 of 11)

A left-hand circularly polarized (LCP) wave is incident from air ($n_{\text{air}} = 1.0$) onto water ($n_{\text{H2O}} = 1.327$) at 53° off the normal.

3 – Angle of transmission θ_t of both TE and TM polarizations

Both polarizations will have the same angle of transmission. It is calculated using Snell's law.

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

$$\theta_t = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_i\right)$$

$$= \sin^{-1}\left(\frac{1.0}{1.327}\sin 53^\circ\right) \quad \rightarrow \quad \boxed{\theta_t = 37^\circ}$$

Numerical Example (5 of 11)

A left-hand circularly polarized (LCP) wave is incident from air ($n_{\text{air}} = 1.0$) onto water ($n_{\text{H2O}} = 1.327$) at 53° off the normal.

4 – Impedance of both mediums η_1 and η_2

Assuming no magnetic response, $\mu_{r,1} = \mu_{r,2} = 1$

Therefore, the impedances are

$$\eta_1 = \frac{\eta_0}{n_1} = \frac{376.73\ \Omega}{1.0} \quad \rightarrow \quad \boxed{\eta_1 = 376.73\ \Omega}$$

$$\eta_2 = \frac{\eta_0}{n_2} = \frac{376.73\ \Omega}{1.327} \quad \rightarrow \quad \boxed{\eta_2 = 283.90\ \Omega}$$

Numerical Example (6 of 11)

A left-hand circularly polarized (LCP) wave is incident from air ($n_{\text{air}} = 1.0$) onto water ($n_{\text{H2O}} = 1.327$) at 53° off the normal.

5 – Reflection coefficient r for both TE and TM polarizations.

$$r_{\text{TE}} = \frac{\eta_2 \cos\theta_{\text{i}} - \eta_1 \cos\theta_{\text{t}}}{\eta_2 \cos\theta_{\text{i}} + \eta_1 \cos\theta_{\text{t}}} = \frac{(283.90\ \Omega)\cos 53° - (376.73\ \Omega)\cos 37°}{(283.90\ \Omega)\cos 53° + (376.73\ \Omega)\cos 37°}$$

$$\boxed{r_{\text{TE}} = -0.2756}$$

$$r_{\text{TM}} = \frac{\eta_2 \cos\theta_{\text{t}} - \eta_1 \cos\theta_{\text{i}}}{\eta_2 \cos\theta_{\text{t}} + \eta_1 \cos\theta_{\text{i}}} = \frac{(283.90\ \Omega)\cos 37° - (376.73\ \Omega)\cos 53°}{(283.90\ \Omega)\cos 37° + (376.73\ \Omega)\cos 53°}$$

$$\boxed{r_{\text{TM}} = 7.3\times10^{-6} \approx 0}$$

Numerical Example (7 of 11)

A left-hand circularly polarized (LCP) wave is incident from air ($n_{\text{air}} = 1.0$) onto water ($n_{\text{H2O}} = 1.327$) at 53° off the normal.

6 – Transmission coefficient t for both TE and TM polarizations.

$$t_{\text{TE}} = 1 + r_{\text{TE}} = 1 + (-0.2756) \quad \rightarrow \quad \boxed{t_{\text{TE}} = 0.7244}$$

$$1 + r_{\text{TM}} = \frac{\cos\theta_{\text{t}}}{\cos\theta_{\text{i}}} t_{\text{TM}}$$

$$t_{\text{TM}} = (1 + r_{\text{TM}}) \frac{\cos\theta_{\text{i}}}{\cos\theta_{\text{t}}}$$

$$= (1+0)\frac{\cos 53°}{\cos 37°} \quad \rightarrow \quad \boxed{t_{\text{TM}} = 0.7536}$$

Numerical Example (8 of 11)

A left-hand circularly polarized (LCP) wave is incident from air ($n_{\text{air}} = 1.0$) onto water ($n_{\text{H2O}} = 1.327$) at 53° off the normal.

7 – Overall reflectance R of the wave

The reflectance for both polarizations separately are

$$R_{\text{TE}} = \left|r_{\text{TE}}\right|^2 = \left|-0.2756\right|^2 \quad \rightarrow \quad \boxed{R_{\text{TE}} = 0.076}$$

$$R_{\text{TM}} = \left|r_{\text{TM}}\right|^2 = \left|0\right|^2 \quad \rightarrow \quad \boxed{R_{\text{TM}} = 0}$$

The applied wave is circularly polarized so both TE and TM have equal power in them. Therefore, the overall reflectance is

$$R = (50\%)R_{\text{TE}} + (50\%)R_{\text{TM}} = (50\%)(0.076) + (50\%)(0) \quad \rightarrow \quad \boxed{R = 0.038}$$

Numerical Example (9 of 11)

A left-hand circularly polarized (LCP) wave is incident from air ($n_{\text{air}} = 1.0$) onto water ($n_{\text{H2O}} = 1.327$) at 53° off the normal.

8 – Overall transmittance T of the wave

The transmittance for both polarizations separately are

$$T_{\text{TE}} = \left|t_{\text{TE}}\right|^2 \frac{\eta_1}{\eta_2}\frac{\cos\theta_{\text{t}}}{\cos\theta_{\text{i}}} = \left|0.7244\right|^2 \frac{376.73\ \Omega}{283.90\ \Omega}\frac{\cos 37^\circ}{\cos 53^\circ} \quad \rightarrow \quad \boxed{T_{\text{TE}} = 0.9240}$$

$$T_{\text{TM}} = \left|t_{\text{TM}}\right|^2 \frac{\eta_1}{\eta_2}\frac{\cos\theta_{\text{t}}}{\cos\theta_{\text{i}}} = \left|0.7536\right|^2 \frac{376.73\ \Omega}{283.90\ \Omega}\frac{\cos 37^\circ}{\cos 53^\circ} \quad \rightarrow \quad \boxed{T_{\text{TM}} = 1.00}$$

The applied wave is circularly polarized so both TE and TM have equal power in them. Therefore, the overall transmittance is

$$T = (50\%)T_{\text{TE}} + (50\%)T_{\text{TM}} = (50\%)(0.9240) + (50\%)(1.00) \quad \rightarrow \quad \boxed{T = 0.9620}$$

Numerical Example (10 of 11)

A left-hand circularly polarized (LCP) wave is incident from air ($n_{\text{air}} = 1.0$) onto water ($n_{\text{H2O}} = 1.327$) at 53° off the normal.

9 – Does $R + T$ = 100%? If not, why not?

$$R + T = 0.038 + 0.9620 = \underline{1.0}$$

Yes! This power is conserved.

Numerical Example (11 of 11)

A left-hand circularly polarized (LCP) wave is incident from air ($n_{\text{air}} = 1.0$) onto water ($n_{\text{H2O}} = 1.327$) at 53° off the normal.

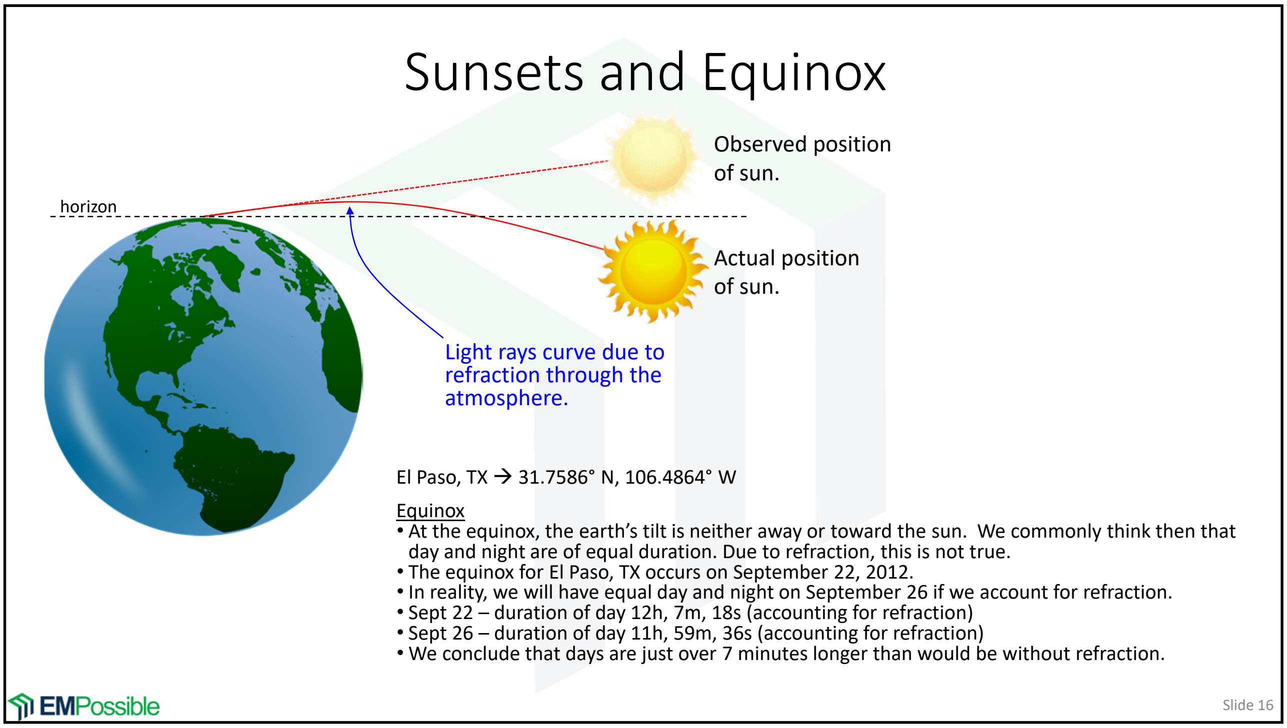
10 – Polarization of the reflected wave.

The wave is incident at the Brewster's angle where the TM polarization is completely transmitted.

This means it is only the TE wave that gets partially reflected.

The reflected wave can only be <u>TE polarized</u>.

Sunrises & Sunsets



Sunsets and Equinox

horizon

Observed position of sun.

Actual position of sun.

Light rays curve due to refraction through the atmosphere.

El Paso, TX → 31.7586° N, 106.4864° W

Equinox

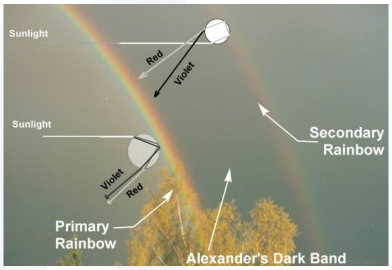
- At the equinox, the earth’s tilt is neither away or toward the sun. We commonly think then that day and night are of equal duration. Due to refraction, this is not true.
- The equinox for El Paso, TX occurs on September 22, 2012.
- In reality, we will have equal day and night on September 26 if we account for refraction.
- Sept 22 – duration of day 12h, 7m, 18s (accounting for refraction)
- Sept 26 – duration of day 11h, 59m, 36s (accounting for refraction)
- We conclude that days are just over 7 minutes longer than would be without refraction.

EMPossible

Rainbows

Rainbows (1 of 2)

There are actually a lot of physics involved with rainbows.

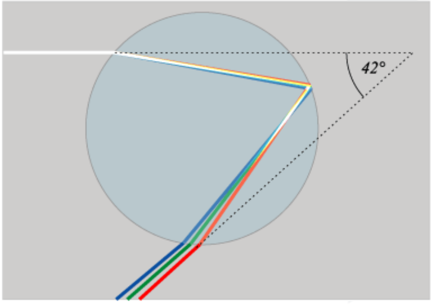


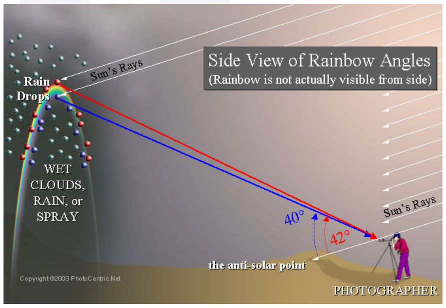
There are always multiple rainbows. Very often they are just too dim to see.

Rainbows (2 of 2)

Rainbows form due to:

1. **Total Internal Reflection** – Light reflects twice inside of a raindrop and exits at around 41° away from the incident light.
2. **Dispersion** – The refractive index of water is slightly different for each color of light, so the angle of light leaving the raindrop is different for different colors. Thus, the colors spread apart as the sun light propagates away from the raindrop.





Polarized Sunglasses

Polarized Sunglasses (1 of 2)

Polarized sunglasses reduce glare (i.e. reflections from surfaces)

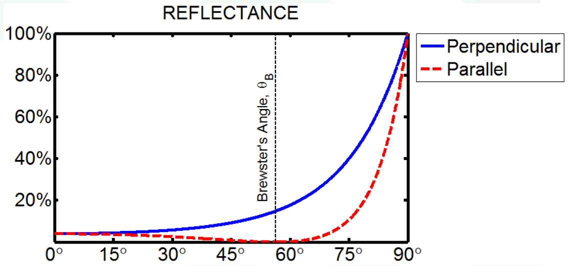
Without polarized sunglasses

With polarized sunglasses

Light tends to become partially TE polarized upon reflection from water, glass, and most man-made objects. Polarized sunglasses block this polarization allowing you to see the surface and what is behind it instead of the reflected light. Some glare remains because the reflected light is only partially polarized.

Polarized Sunglasses (2 of 2)

Consider Fresnel reflection from air to water.



The TE polarization is more strongly reflected, thus polarized sunglasses block this polarization.