Electromagnetics:
Electromagnetic Field Theory

Faraday’s Law

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Outline

- Faraday’s Experiment
- Two Ways to Calculate Induced EMF
- Faraday’s Law in Integral Form
- Faraday’s Law in Differential Form

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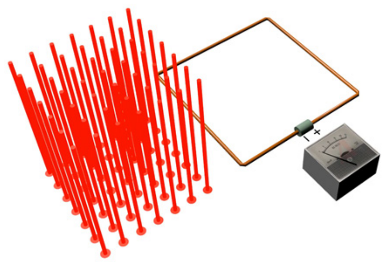
Faraday's Experiment



Observations:

1. The more (or stronger) the magnetic flux, the higher the voltage reading.

   $V_{\text{emf}} \propto \psi \qquad \psi = \iint_S \vec{B} \bullet d\vec{s}$

2. The more turns of the loop, the higher the voltage reading.

   $V_{\text{emf}} \propto N \qquad N \equiv \#\ \text{turns}$

3. The faster the time rate of change of the magnetic flux, the higher the voltage reading.

   $V_{\text{emf}} \propto \dfrac{d\psi}{dt}$

Calculate Induced Voltage (1 of 2)

Method 1: By experiment.

Faraday performed an experiment and determined that

$$V_{\text{emf}} = -N\frac{d\psi}{dt}$$

The total magnetic flux ψ accounting for the number of turns N is

$$N\psi = \iint_S \vec{B} \bullet d\vec{s}$$

$\psi \equiv$ magnetic flux
$N\psi \equiv$ magnetic flux linkage

Flux linkage is a property of a two-terminal device. It is not equivalent to flux. Flux can exist without the loop. Further, if the loops do not have the same orientation, they will not "link" to the magnetic field the same.

Flux and flux linkage are not the same thing, but often used synonymously because most devices are designed so that each loop links the same to the magnetic field and the math reduces to them being nearly equivalent.

Combing the above equations leads an expression for V_{emf} in terms of just the magnetic flux density $\vec{B}$.

$$V_{\text{emf}} = -N\frac{d\psi}{dt} = -\frac{d}{dt}(N\psi) = -\frac{\partial}{\partial t}\iint_S \vec{B} \bullet d\vec{s} = -\iint_S \left[\frac{\partial \vec{B}}{\partial t}\right] \bullet d\vec{s}$$

Calculate Induced Voltage (2 of 2)

Method 2: Use Kirchoff's voltage law

The voltage across the terminals of the resistor can be calculated using Kirchoff's voltage law. For electromagnetics, Kirchoff's voltage law becomes a line integral. Assuming the resistor is very small compared to the loop, we get

$$V_{\text{emf}} = \oint_L \vec{E} \bullet d\vec{\ell} \qquad V = E \cdot \ell$$

Faraday's Law in Integral Form

Both methods calculate the same voltage so they can be set equal.

Method 2 → $\oint_L \vec{E} \bullet d\vec{\ell}$ Method 1 → $-\iint_S \left[\frac{\partial \vec{B}}{\partial t}\right] \bullet d\vec{s}$

$$\boxed{V_{\text{emf}} = \oint_L \vec{E} \bullet d\vec{\ell} = -\iint_S \left[\frac{\partial \vec{B}}{\partial t}\right] \bullet d\vec{s}}$$

Apply Stoke's Theorem

Stoke's theorem allows us to write a closed-contour line integral as a surface integral.

$$\oint_L \vec{A} \bullet d\vec{\ell} = \iint_S \left(\nabla \times \vec{A}\right) \bullet d\vec{s}$$

Applying this to Faraday's law in integral form gives us

$$V_{\text{emf}} = \oint_L \vec{E} \bullet d\vec{\ell} = -\iint_S \left[\frac{\partial \vec{B}}{\partial t}\right] \bullet d\vec{s}$$

$$\downarrow$$

$$V_{\text{emf}} = \iint_S \left(\nabla \times \vec{E}\right) \bullet d\vec{s} = -\iint_S \left[\frac{\partial \vec{B}}{\partial t}\right] \bullet d\vec{s}$$

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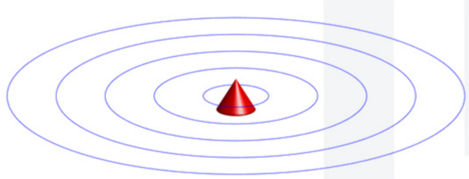
Faraday's Law in Differential Form

If the line L and surface S describe the same space, the argument of both integrals must be equal. Setting these arguments equal gives Faraday's law in differential form.

$$V_{\text{emf}} = \iint_S \left(\nabla \times \vec{E}\right) \bullet d\vec{s} = \iint_S \left[-\frac{\partial \vec{B}}{\partial t}\right] \bullet d\vec{s}$$



$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

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