Electromagnetics:
Electromagnetic Field Theory

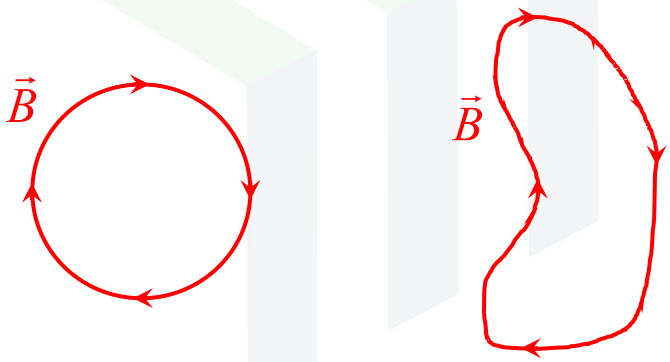
Gauss' Law for Magnetic Fields

Outline

- No Magnetic Charge
- Gauss' Law for Magnetic Fields in Integral Form
- Gauss' Law for Magnetic Fields in Differential Form

No Magnetic Charge

Since there exists no isolated magnetic charge, the magnetic flux cannot diverge from a point or converge to a point. This means that magnetic flux cannot have a beginning or an end. Therefore, magnetic flux can only form loops.

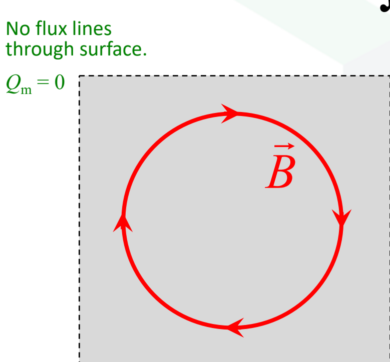


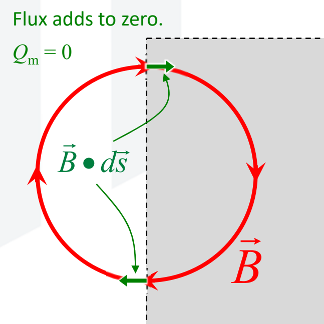
Prove Zero Magnetic Charge

Surface integral of magnetic flux $\vec{B} \cdot d\vec{s}$

Following what was done for electric flux, the total magnetic charge enclosed by a surface S if calculated by integrating the magnetic flux through the surface. Total magnetic charge must be zero.

$$0 = \oint\!\!\!\oint_S \vec{B} \bullet d\vec{s}$$





Gauss' Law for Magnetic Fields in Integral Form

The result from last slide is Gauss' law for magnetic fields.

$$\oint\!\!\!\oint_S \vec{B} \bullet d\vec{s} = 0$$

Apply Divergence Theorem

The *divergence theorem* a closed-contour surface integral to be written as a volume integral.

$$\oint\!\!\!\oint_S \vec{A} \bullet d\vec{s} = \iiint_V \left(\nabla \bullet \vec{A}\right) dv$$

Divergence theorem

Applying this to Gauss' law for magnetic fields gives

$$Q_{\mathrm{m}} = \oint\!\!\!\oint_S \vec{B} \bullet d\vec{s} = \iiint_V \rho_{\mathrm{m}} dv$$

$$\downarrow$$

$$Q_{\mathrm{m}} = \iiint_V \left(\nabla \bullet \vec{B}\right) dv = \iiint_V \rho_{\mathrm{m}} dv$$

Gauss' Law for Magnetic Fields in Differential Form

If the surface S and volume V describe the same space, then the argument of both integrals must be equal. Setting these arguments equal gives Gauss' law for magnetic fields in differential form.

$$Q_{\mathrm{m}} = \iiint_V \left(\nabla \bullet \vec{B}\right) dv = \iiint_V \rho_{\mathrm{m}} dv$$

$$\nabla \bullet \vec{B} = \rho_{\mathrm{m}}$$

However, there is no magnetic charge so $\rho_{\mathrm{m}} = 0$.

$$\boxed{\nabla \bullet \vec{B} = 0}$$