Electromagnetics:
Electromagnetic Field Theory

Gauss' Law for Magnetic Fields

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Outline

- No Magnetic Charge
- Gauss' Law for Magnetic Fields in Integral Form
- Gauss' Law for Magnetic Fields in Differential Form

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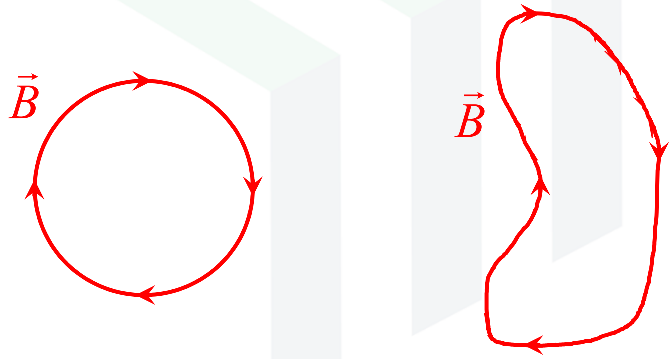
No Magnetic Charge

Since there exists no isolated magnetic charge, the magnetic field cannot have a beginning or an end. The magnetic field can only form loops.



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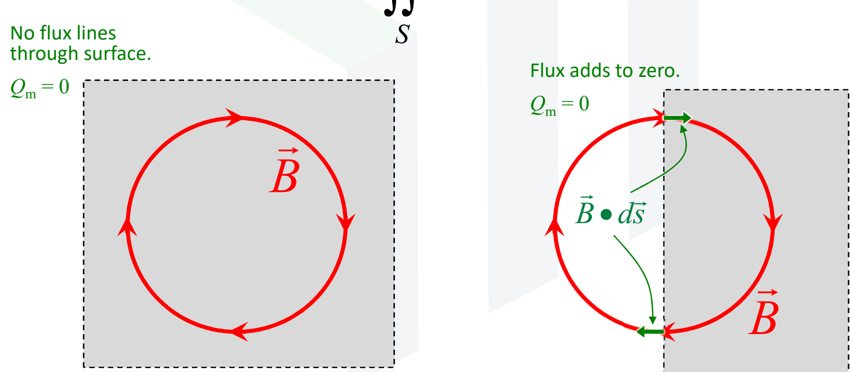
Prove Zero Magnetic Charge

Surface integral of magnetic flux

Following what we did for electric fields, we calculate total magnetic charge enclosed within a surface S by integrating the flux of the magnetic field lines at the surface.

$$0 = \oiint_S \vec{B} \bullet d\vec{s}$$



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Gauss' Law for Magnetic Fields in Integral Form

The result from last slide is Gauss' law for magnetic fields.

$$\oiint_S \vec{B} \bullet d\vec{s} = 0$$

Apply Divergence Theorem

The *divergence theorem* allows us to write a closed-contour surface integral as a volume integral.

$$\oiint_S \vec{A} \bullet d\vec{s} = \iiint_V \left(\nabla \bullet \vec{A} \right) dv$$

Applying this to Gauss' law for magnetic fields gives us

$$Q_{\text{m}} = \oiint_S \vec{B} \bullet d\vec{s} = \iiint_V \rho_{\text{m}} dv$$

$$\downarrow$$

$$Q_{\text{m}} = \iiint_V \left(\nabla \bullet \vec{B} \right) dv = \iiint_V \rho_{\text{m}} dv$$

Gauss' Law for Magnetic Fields in Differential Form

If the surface S and volume V describe the same space, the argument of both integrals must be equal. Setting these arguments equal gives Gauss' law for magnetic fields in differential form.

$$Q_{\mathrm{m}} = \iiint_V \left(\nabla \bullet \vec{B} \right) dv = \iiint_V \rho_{\mathrm{m}} dv$$

$$\nabla \bullet \vec{B} = \rho_{\mathrm{m}}$$

However, there is no magnetic charge so $\rho_{\mathrm{m}} = 0$.

$$\boxed{\nabla \bullet \vec{B} = 0}$$

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