



Electromagnetics:
Electromagnetic Field Theory

Gauss' Law

Slide 1

Outline

- Fields and Charges
- Field Lines
- Two Ways to Calculate Total Charge Q
- Gauss' Law in Differential Form

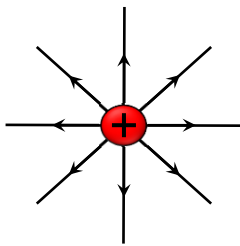
Slide 2

Fields & Charges (1 of 2)

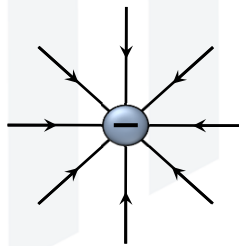
The electric field induced by charges is best described by the electric flux density \vec{D} because this quantity is most closely associated with charge.

$$\vec{D} = \frac{Q}{4\pi r^2} \hat{a}_r$$

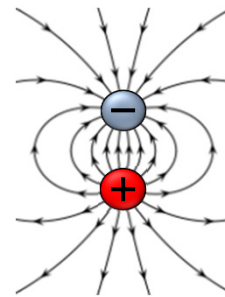
Electric field lines diverge from positive charges.



Electric field lines converge on negative charges.



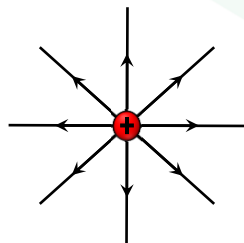
...or both!



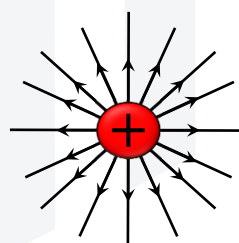
Fields & Charges (2 of 2)

The magnitude of the charge is usually conveyed by the density of the field lines.

Small charge

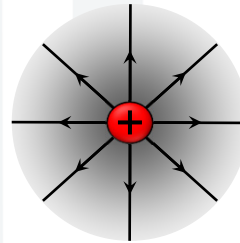
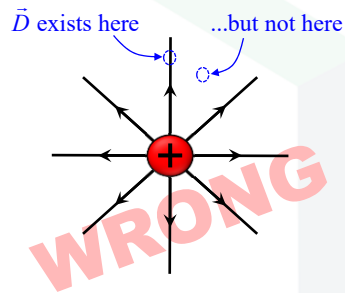


Large charge



A Note About Field Lines

It is in some ways misleading to draw electric field lines because it incorrectly implies that the field exists at some points but not others.

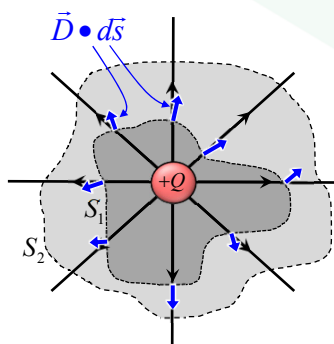


The electric field is a smooth and continuous phenomenon, but that is hard to convey in a diagram.

How to Calculate Total Charge Q

Method 1: Surface integral of electric flux $\vec{D} \cdot d\vec{s}$

Since the density of field lines convey the amount of charge, it makes sense that the total charge Q enclosed within a surface S can be calculated by integrating the flux of the field lines through the surface.



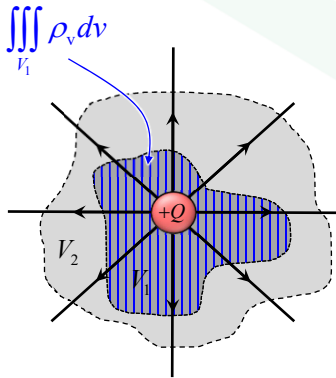
$$Q = \oiint_{S_1} \vec{D} \cdot d\vec{s} = \oiint_{S_2} \vec{D} \cdot d\vec{s}$$

The choice of surface S does not matter as long as the surface completely encloses the charge. Usually the shape of the surface is chosen as a constant-coordinate surface (i.e. boxes, cylinders or spheres) to make the math as simple as possible.

How to Calculate Total Charge Q

Method 2: Volume integral of electric charge density ρ_v

The total charge Q can be calculated by integrating the electric charge density ρ_v within a volume V that completely encompasses the charge.



$$Q = \iiint_{V_1} \rho_v dv = \iiint_{V_2} \rho_v dv$$

The choice of volume V is the same as the choice of the surface for Method 1. the shape of the volume does not matter as long it completely encompasses the charge. It is usually chosen in a manner to simplify the math.

Gauss' Law in Integral Form

Both methods calculate the same total charge so they can be set equal.



$$Q = \oiint_S \vec{D} \cdot d\vec{s} = \iiint_V \rho_v dv$$

Apply Divergence Theorem

The *divergence theorem* allows a closed-contour surface integral to be written as a volume integral.

$$\oiint_S \vec{A} \cdot d\vec{s} = \iiint_V (\nabla \cdot \vec{A}) dv \quad \text{Divergence theorem}$$

Applying this to Gauss' law gives

$$Q = \oiint_S \vec{D} \cdot d\vec{s} = \iiint_V \rho_v dv$$

↓

$$Q = \iiint_V (\nabla \cdot \vec{D}) dv = \iiint_V \rho_v dv$$

Gauss' Law in Differential Form

If the surface S and volume V describe the same space, the argument of both integrals must be equal. Setting these arguments equal gives Gauss' law in differential form.

$$Q = \iiint_V (\nabla \cdot \vec{D}) dv = \iiint_V \rho_v dv$$

$$\boxed{\nabla \cdot \vec{D} = \rho_v}$$