Electromagnetics:
Electromagnetic Field Theory

Important Concepts

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Outline

- Right-Handed Coordinate Systems
- Flux
- Stoke's Theorem
- Divergence Theorem
- Curl of Gradient is Zero
- Product Rule for Divergence

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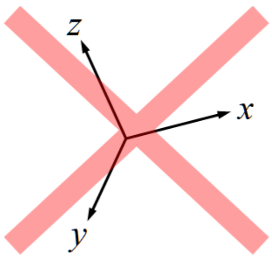
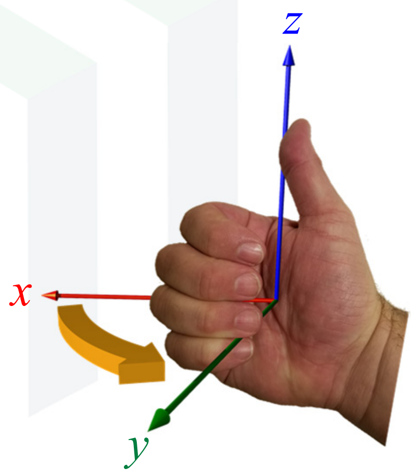
Right-Handed Coordinate Systems

Always construct your coordinate system to be right-handed.

$$\hat{a}_x \times \hat{a}_y = \hat{a}_z$$





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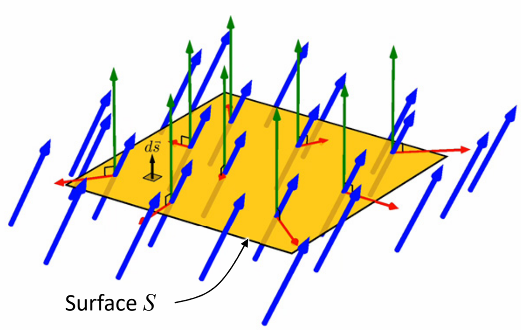
Flux ψ

Flux is the total amount of a vector field that passes *straight through* a surface.

- Vector Field $\vec{A}$
- Normal component $\rightarrow$ counted as flux
- Tangential component $\rightarrow$ ignored

$$\psi = \iint_S \vec{A} \bullet d\vec{s}$$



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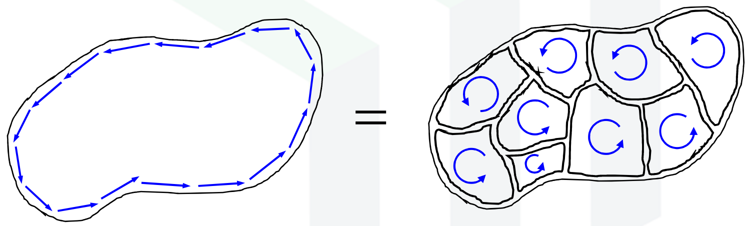
Stoke's Theorem

$$\oint_L \vec{F} \bullet d\vec{\ell} = \iint_S \left(\nabla \times \vec{F} \right) \bullet d\vec{s}$$



Stoke's theorem allows us to write a closed-contour line integral as a surface integral.

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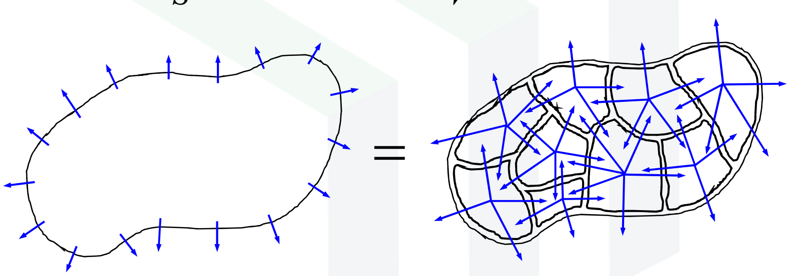
Divergence Theorem

$$\oiint_S \vec{F} \bullet d\vec{s} = \iiint_V \left(\nabla \bullet \vec{F} \right) dv$$



The divergence theorem allows us to write a closed-contour surface integral as a volume integral.

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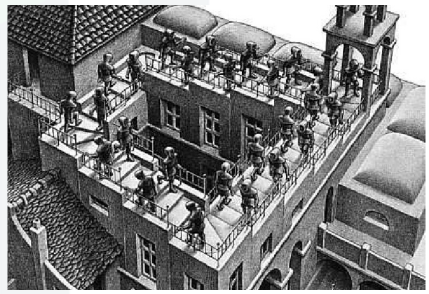
Curl of Gradient is Zero

$$\nabla \times \left(\nabla f \right) = 0$$

Why?

1. The gradient ∇f always points in the direction that f increases.
2. If a vector field as curl, then that vector field forms closed loops.
3. How can a function always be increasing around a closed loop?



M. C. Escher (1898 – 1972)

The curl of the gradient of any scalar function is always zero.

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Product Rule for Divergence

$$\nabla \bullet \left(f\vec{A} \right) = f\left(\nabla \bullet \vec{A} \right) + \vec{A} \bullet \nabla f$$

$$\begin{aligned}
\nabla \bullet \left(f\vec{A} \right) &= \nabla \bullet \left(fA_x\hat{a}_x + fA_y\hat{a}_y + fA_z\hat{a}_z \right) \\
&= \frac{\partial}{\partial x}\left(fA_x \right) + \frac{\partial}{\partial y}\left(fA_y \right) + \frac{\partial}{\partial z}\left(fA_z \right) \\
&= \left(f\frac{\partial A_x}{\partial x} + A_x\frac{\partial f}{\partial x} \right) + \left(f\frac{\partial A_y}{\partial y} + A_y\frac{\partial f}{\partial y} \right) + \left(f\frac{\partial A_z}{\partial z} + A_z\frac{\partial f}{\partial z} \right) \\
&= \underbrace{f\left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right)}_{f\left(\nabla \bullet \vec{A} \right)} + \underbrace{\left(A_x\frac{\partial f}{\partial x} + A_y\frac{\partial f}{\partial y} + A_z\frac{\partial f}{\partial z} \right)}_{\vec{A} \bullet \nabla f}
\end{aligned}$$

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