Electromagnetics:

Electromagnetic Field Theory

Waves in Lossy Dielectrics

1

Lecture Outline

- Complex Wave Parameters
- Visualization of EM Waves
- Complex Wave Parameters for Special Cases
  - Lossy dielectrics (general case)
  - Good dielectrics
  - Good conductors

Slide 2

2

Complex Wave Parameters

The Complex Permittivity $\tilde{\varepsilon}$

There are two ways to specify the electrical properties of a material:

Complex Permittivity: $\tilde{\varepsilon} = \varepsilon' - j\varepsilon''$

Real Permittivity & Conductivity: ε and σ

The two systems above can be related using Maxwell's equations.

Complex Permittivity: $\nabla \times \vec{H} = j\omega\tilde{\varepsilon}\vec{E}$

Real Permittivity & Conductivity:

$$\begin{aligned}\nabla \times \vec{H} &= \vec{J} + j\omega\varepsilon\vec{E} \\ &= \sigma\vec{E} + j\omega\varepsilon\vec{E} \\ &= (\sigma + j\omega\varepsilon)\vec{E} \\ &= j\omega\left(\frac{\sigma}{j\omega} + \varepsilon\right)\vec{E}\end{aligned}$$

The relation is:

$$\tilde{\varepsilon} = \varepsilon' - j\varepsilon'' = \varepsilon + \frac{\sigma}{j\omega}$$

EMPossible

Parameter Values for Various Materials

Complex Permittivity

Material under test	Complex permittivity	
	ε′	ε″
Air in an empty container	0.937 ~ 1.010	−0.020 ~ +0.007
Water collected from a laboratory tap	10.030 ~ 11.949	16.783 ~ 14.759
Water, 25 ˚C [8]	10.032 ~ 7.674	17.671 ~ 12.461

Conductivity

Material	Conductivity σ
Copper	5.96×10^7 S/m
Gold	4.10×10^7 S/m
Nickel	1.43×10^7 S/m
Iron	1.00×10^7 S/m
Drinking Water	5×10^{-3} S/m
Air	~ 10^{-10} S/m
Teflon	~ 10^{-24} S/m

EMPossible

Slide 5

5

The Complex Permeability $\tilde{\mu}$

Similarly, the permeability $\tilde{\mu}$ can also be a complex number.

$$\tilde{\mu} = \mu' - j\mu''$$

It is unusual to see complex permeability $\tilde{\mu}$ used in practice.

EMPossible

Slide 6

6

The Complex Wave Number $\tilde{k}$

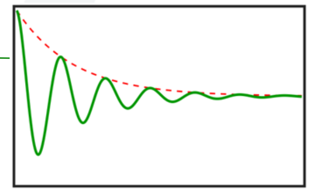
A wave travelling in the $+z$ direction can be written in terms of the complex wave number $\tilde{k}$ as

$$\vec{E}(z) = \vec{P}e^{-j\tilde{k}z} \qquad \tilde{k} = k' - jk''$$

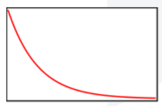
Substituting $\tilde{k} = k' - jk''$ into the wave solution gives

$$\vec{E}(z) = \vec{P}e^{-j(k'-jk'')z} = \vec{P}\underbrace{e^{-k''z}}_{\text{attenuation}}\underbrace{e^{-jk'z}}_{\text{oscillation}}$$

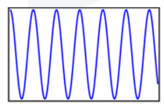
attenuation & oscillation



attenuation



oscillation



The Complex Propagation Constant, $\tilde{\gamma}$

A wave travelling the $+z$ direction can be written in terms of the complex propagation constant $\tilde{\gamma}$ as

$$\vec{E}(z) = \vec{P}e^{-\tilde{\gamma}z} \qquad \tilde{\gamma} = \gamma' + j\gamma''$$

Substituting this into the wave solution yields

$$\vec{E}(z) = \vec{E}_0 e^{-(\gamma'+j\gamma'')z} = \vec{E}_0 \underbrace{e^{-\gamma'z}}_{\text{attenuation}}\underbrace{e^{-j\gamma''z}}_{\text{oscillation}}$$

Attenuation Coefficient α and Phase Constant β

A wave travelling the $+z$ direction can also be written in terms of an attenuation coefficient α and a phase constant β and as

$$\vec{E}(z) = \vec{E}_0\, e^{-k''z} e^{-jk'z}$$

$$\vec{E}(z) = \vec{E}_0\, \underbrace{e^{-\alpha z}}_{\text{attenuation}}\, \underbrace{e^{-j\beta z}}_{\text{oscillation}}$$

$$\vec{E}(z) = \vec{E}_0\, e^{-\gamma' z} e^{-j\gamma'' z}$$

$$\vec{E}(z) = \vec{E}_0\, \underbrace{e^{-\alpha z}}_{\text{attenuation}}\, \underbrace{e^{-j\beta z}}_{\text{oscillation}}$$

This provides the physical meaning of the real and imaginary parts of the complex wave number $\tilde{k}$ and propagation constant $\tilde{\gamma}$.

$$\tilde{k} = \beta - j\alpha \qquad \alpha = -\text{Im}\left[\tilde{k}\right] \qquad \beta = \text{Re}\left[\tilde{k}\right]$$

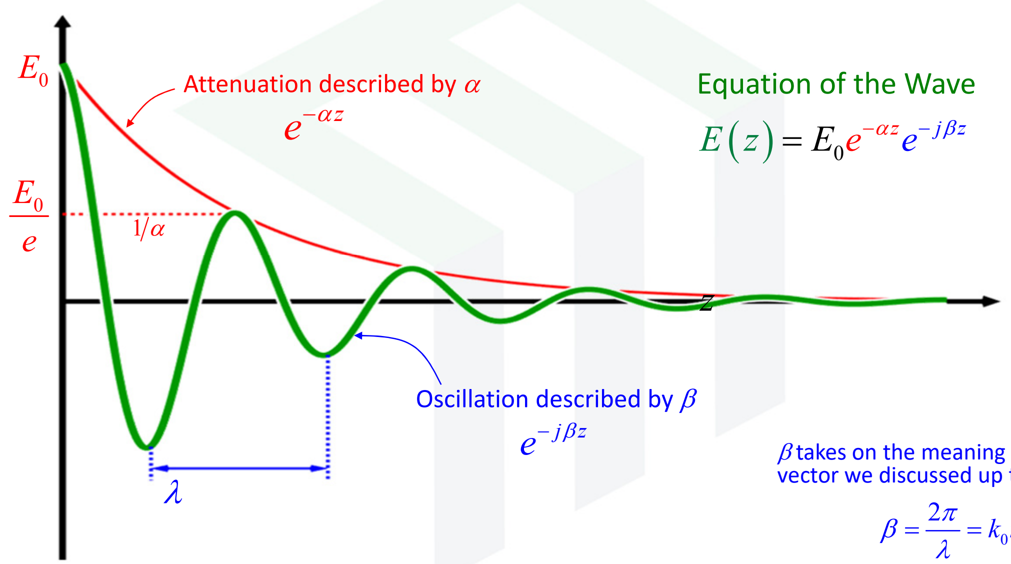
$$\tilde{\gamma} = \alpha + j\beta \qquad \alpha = \text{Re}\left[\tilde{\gamma}\right] \qquad \beta = \text{Im}\left[\tilde{\gamma}\right]$$

EMPossible

Slide 9

9

Physical Meaning of α and β



β takes on the meaning of the wave vector we discussed up to this point.

$$\beta = \frac{2\pi}{\lambda} = k_0 n$$

EMPossible

Slide 10

10

Calculating α and β from $\tilde{\mu}$, $\tilde{\varepsilon}$ and σ

Given complex permeability $\tilde{\mu}$ and complex permittivity $\tilde{\varepsilon}$,

$$k = \beta - j\alpha = \omega\sqrt{\tilde{\mu}\tilde{\varepsilon}} \quad \rightarrow \quad \boxed{\begin{aligned} \alpha &= -\mathrm{Im}\left[\omega\sqrt{\tilde{\mu}\tilde{\varepsilon}}\right] \\ \beta &= \mathrm{Re}\left[\omega\sqrt{\tilde{\mu}\tilde{\varepsilon}}\right] \end{aligned}}$$

Given real permeability μ, real permittivity ε and conductivity σ,

$$-\gamma^2 = \omega^2\tilde{\mu}\tilde{\varepsilon} = \omega^2\mu(\varepsilon + \sigma/j\omega)$$

$$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$$

$$(\alpha + j\beta)^2 = j\omega\mu(\sigma + j\omega\varepsilon)$$

$$(\alpha^2 - \beta^2) + j(2\alpha\beta) = (-\omega^2\mu\varepsilon) + j(\omega\mu\sigma)$$

$$\downarrow$$

$$\left.\begin{aligned} \alpha^2 - \beta^2 &= -\omega^2\mu\varepsilon \\ 2\alpha\beta &= \omega\mu\sigma \end{aligned}\right\}$$

$$\boxed{\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}}$$

$$\boxed{\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}}$$

α collects all loss information into a single parameter.

β collects all phase information into a single parameter.

Both α and β are a crazy mix of the fundamental parameters.

EMPossible

Slide 11

11

Absorption Coefficient, α_P

The absorption coefficient α_P describes how power decays as a function of position.

$$P(z) = P_0 e^{-\alpha_P z}$$

The attenuation coefficient α was previously defined as how the field amplitude decays as a function of position.

$$E(z) = E_0 e^{-\alpha z} e^{-j\beta z}$$

Given that $P \propto E^2$, the attenuation coefficient α and absorption coefficient α_P are related through

$$P(z) = \frac{|E(z)|^2}{\eta} = \frac{E_0^2}{\eta} e^{-2\alpha z}$$

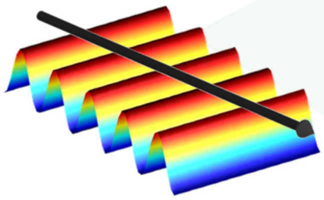
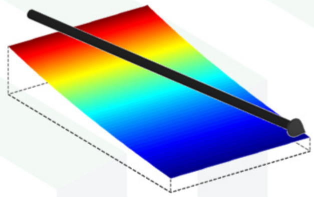
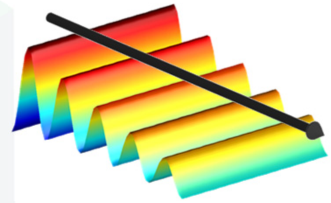
$$\alpha_P = 2\alpha$$

EMPossible

Slide 12

12

1D Waves with Complex Wave Number $\tilde{k}$

Purely Real $\tilde{k}$	Purely Imaginary $\tilde{k}$	Complex $\tilde{k}$
		
• Uniform amplitude • Oscillations move power • Considered to be a propagating wave	• Decaying amplitude • No oscillations, no flow of power • Considered to be evanescent	• Decaying amplitude • Oscillations move power • Considered to be a propagating wave (not evanescent)

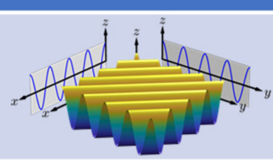
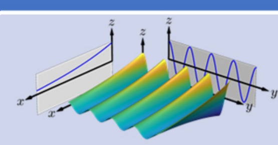
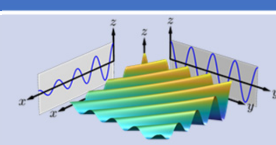
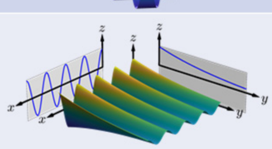
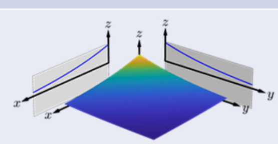
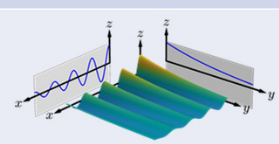
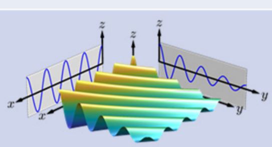
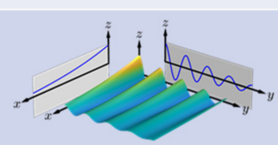
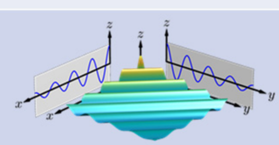
These are the only **2.5 configurations** that electrodynamic fields can take on.

EMPossible

Slide 13

13

2D Waves with Complex Wave Vector $\vec{\tilde{k}}$

	Real $\tilde{k}_x$ $\tilde{k}_x = k'_x$	Imaginary $\tilde{k}_x$ $\tilde{k}_x = -jk''_x$	Complex $\tilde{k}_x$ $\tilde{k}_x = k'_x - jk''_x$
Real $\tilde{k}_y$ $\tilde{k}_y = k'_y$			
Imaginary $\tilde{k}_y$ $\tilde{k}_y = -jk''_y$			
Complex $\tilde{k}_y$ $\tilde{k}_y = k'_y - jk''_y$			

EMPossible

Slide 14

14

Complex Impedance $\tilde{\eta}$

The wave impedance $\tilde{\eta}$ is in general a complex number.

$$\tilde{\eta} = |\eta| \measuredangle \theta_\eta = R_0 + jX_0$$

The amplitude/phase form is the most meaningful when substituted into the expression for the magnetic field component of a wave.

$$\vec{H} = \frac{\hat{k} \times \vec{P}}{\tilde{\eta}} e^{-j\vec{k}\bullet\vec{r}} = \frac{\hat{k} \times \vec{P}}{|\eta|} e^{-j\left(\vec{k}\bullet\vec{r}+\theta_\eta\right)}$$

θ_η affects phase

$|\eta|$ affects magnitude

EMPossible Slide 15

15

Complex Impedance $\tilde{\eta}$ in Terms of $\tilde{\mu}$, $\tilde{\varepsilon}$, and σ

Given complex permeability $\tilde{\mu}$ and complex permittivity $\tilde{\varepsilon}$,

$$\tilde{\eta} = \sqrt{\frac{\tilde{\mu}}{\tilde{\varepsilon}}}$$

Given real permeability μ, real permittivity ε and conductivity σ,

$$\tilde{\eta} = \sqrt{\frac{\mu}{\varepsilon + \sigma/j\omega}} = \sqrt{\frac{\mu/\varepsilon}{1 + \sigma/j\omega\varepsilon}}$$

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\sigma/\omega\varepsilon\right)^2\right]^{1/4}} \qquad \measuredangle\eta = \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)$$

$\tilde{\eta}$ collects all amplitude and phase information between $\vec{E}$ and $\vec{H}$ into a single parameter.

The complex impedance $\tilde{\eta}$ is a crazy mix of the fundamental parameters.

EMPossible Slide 16

16

Complex Refractive Index $\tilde{n}$ (3 of 3)

Recall that $k = k_0 n$. However, $\tilde{k}$ is a complex number, so refractive index must be a complex number as well.

$$\tilde{n} = n_o - j\kappa$$

Extinction coefficient, κ

Ordinary refractive index, n_o

The real and imaginary parts of refractive index $\tilde{n}$ can be related to the real and imaginary parts of $\tilde{k}$ as well as to α and β.

$$\tilde{k} = k_0 \tilde{n}$$

$$k' - jk'' = k_0 \left(n_o - j\kappa \right)$$

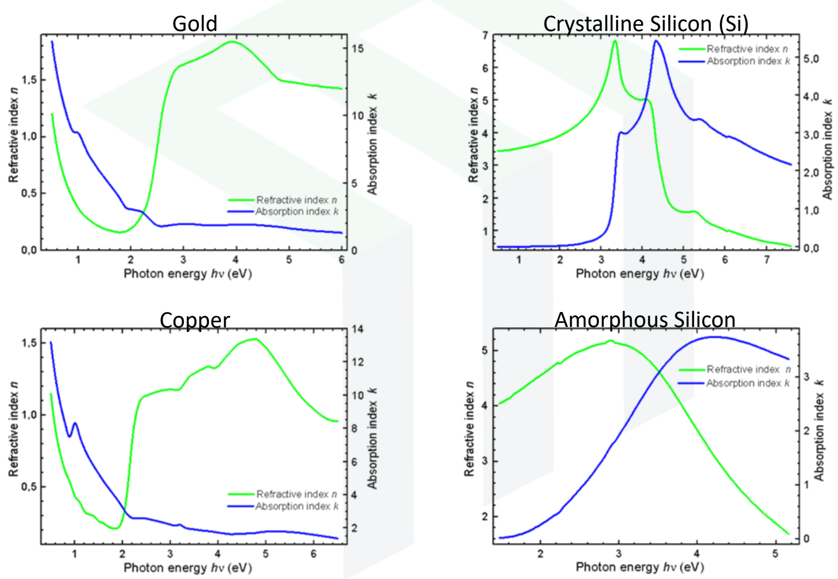
$$\beta - j\alpha = k_0 \left(n_o - j\kappa \right)$$

$$n_o = \frac{\operatorname{Re}\left[\tilde{k}\right]}{k_0} = \frac{\beta}{k_0}$$

$$\kappa = \frac{\operatorname{Im}\left[\tilde{k}\right]}{k_0} = \frac{\alpha}{k_0}$$

EMPossible

Complex Refractive Index for Various Materials



EMPossible

http://www.ioffe.ru/SVA/NSM/nk/

Loss Tangent, $\tan\delta$

Sometimes material loss is given in terms of a "loss tangent" $\tan\delta$.

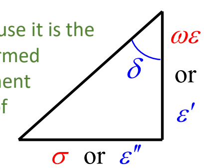
$$\tan\delta = \frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon} \qquad P(z) = P_0 e^{-\delta k_0 n z}$$

Recall that interpreting wave properties (velocity v and loss α) is not intuitive using just the complex dielectric function $\tilde{\varepsilon}$. In this case, the complex refractive index $\tilde{n}$ is preferred.

It turns out that the loss tangent $\tan\delta$ and the extinction coefficient κ are essentially the same quantity.

$$\delta = \frac{2\kappa}{n} = \frac{\alpha_{\text{abs}}}{k_0 n}$$

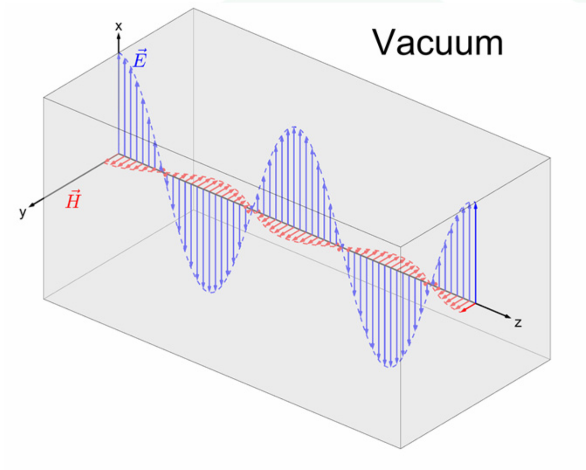
It is called a loss tangent because it is the angle in the complex plane formed between the resistive component and the reactive component of the complex permittivity.



EMPossible

Visualization of EM Waves

Waves in Materials (1 of 3)



Waves in Vacuum

- $\vec{H}$ is 377× smaller than $\vec{E}$.

  $\eta_0 = \frac{E_0}{H_0} \cong 376.73\ \Omega$

- $\vec{E}$ and $\vec{H}$ are in phase

  $\text{Im}[\eta] = 0$

- $\vec{E} \perp \vec{H}$

  $\vec{H} \propto \vec{k} \times \vec{P}$

- Amplitude does not decay

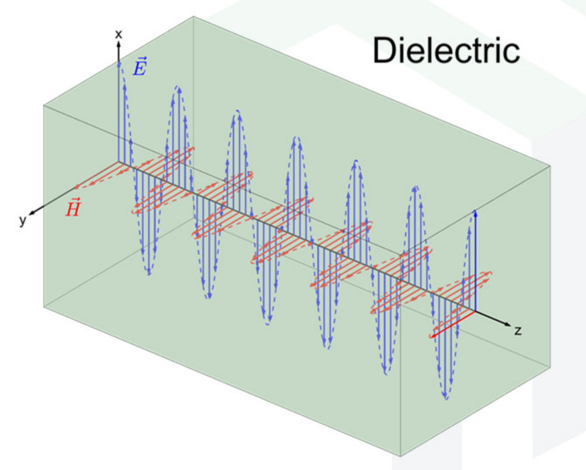
  $\sigma = 0$

EMPossible

Slide 21

21

Waves in Materials (2 of 3)



Waves in Dielectric

- $\vec{H}$ is larger now, but still smaller than $\vec{E}$.

  $\eta \propto \frac{1}{\sqrt{\varepsilon}}$

- $\vec{E}$ and $\vec{H}$ are still in phase

  $\text{Im}[\eta] = 0$

- $\vec{E} \perp \vec{H}$

  $\vec{H} \propto \vec{k} \times \vec{P}$

- Amplitude still does not decay

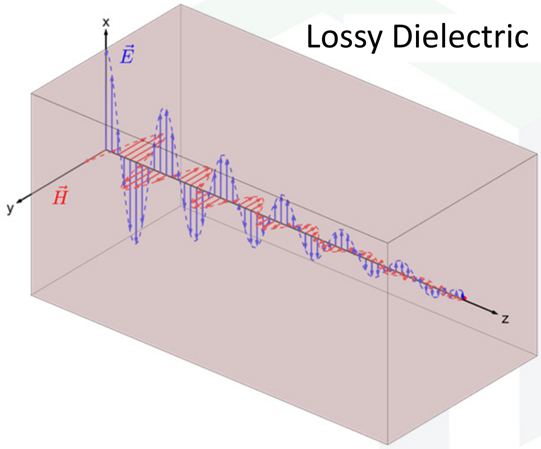
  $\sigma = 0$

EMPossible

Slide 22

22

Waves in Materials (3 of 3)



Waves in Lossy Dielectric

- $\vec{H}$ remains larger, but still smaller than $\vec{E}$.

  $\eta \propto \frac{1}{\sqrt{\varepsilon}}$

- $\vec{E}$ and $\vec{H}$ are out of phase!

  $\mathrm{Im}[\eta] \neq 0$

- $\vec{E} \perp \vec{H}$

  $\vec{H} \propto \vec{k} \times \vec{P}$

- Amplitude decays

  $\sigma \neq 0$

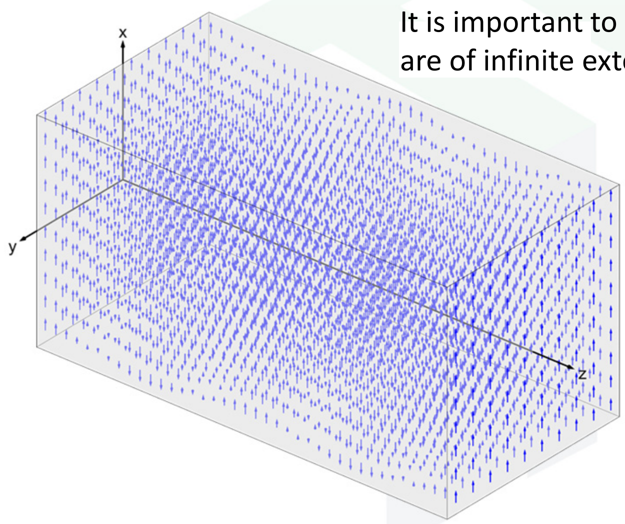
EMPossible

Slide 23

23

More Realistic Wave ($\vec{E}$ Only)

It is important to remember that plane waves are of infinite extent in the x and y directions.



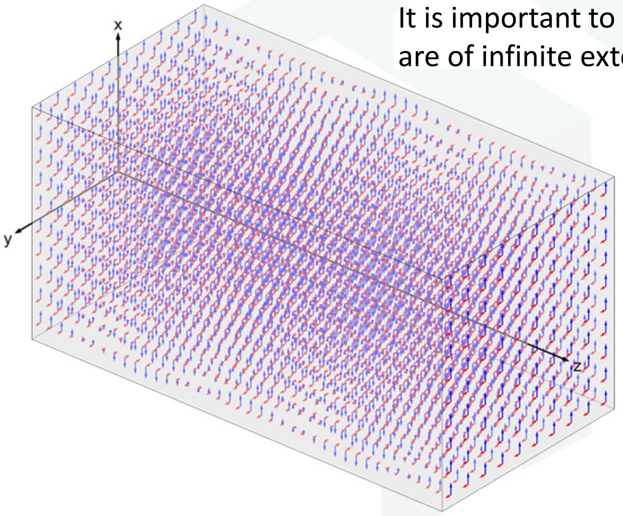
EMPossible

Slide 24

24

More Realistic Wave $\left(\vec{E} \;\&\; \vec{H}\right)$

It is important to remember that plane waves are of infinite extent in the x and y directions.



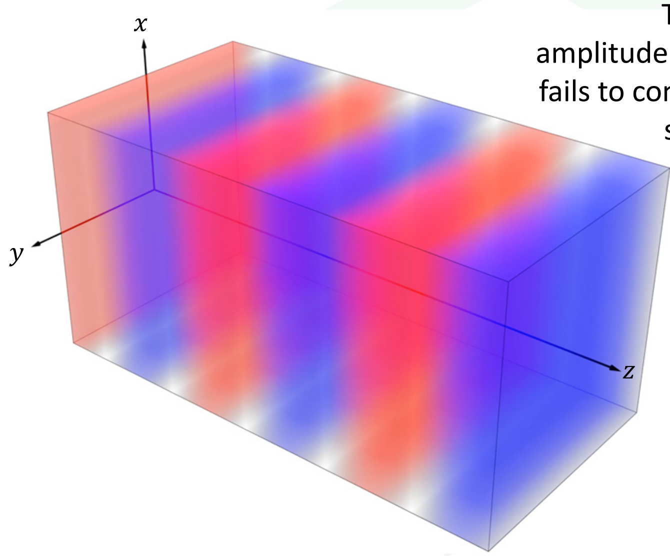
EMPossible

Slide 25

25

More Realistic Wave (Amplitude & Phase)

This representation shows the wave amplitude and phase more realistically, but it fails to convey polarization and that there are separate electric and magnetic field components.



EMPossible

Slide 26

26

Complex Wave Parameters for Special Cases

Summary of Waves in Lossy Dielectrics

Condition: This is the general case. All materials have loss.

Fundamental Parameters: $\sigma,\ \mu = \mu_0\mu_r,\ \tilde{\varepsilon} = \varepsilon_0\tilde{\varepsilon}_r = \varepsilon + \dfrac{\sigma}{j\omega}$

Attenuation Coefficient: $\alpha = -\mathrm{Im}\left[\tilde{k}\right] \qquad \alpha = \omega\sqrt{\dfrac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\dfrac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$

Phase Constant: $\beta = \mathrm{Re}\left[\tilde{k}\right] \qquad \beta = \omega\sqrt{\dfrac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\dfrac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$

Impedance: $\tilde{\eta} = \sqrt{\dfrac{\tilde{\mu}}{\tilde{\varepsilon}}} = \sqrt{\dfrac{\mu/\varepsilon}{1+\sigma/j\omega\varepsilon}}$

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\sigma/\omega\varepsilon\right)^2\right]^{1/4}} \qquad \measuredangle\eta = \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)$$

Summary of Waves in Lossless Dielectrics

Condition: $\sigma \ll \omega\varepsilon$

Fundamental Parameters: $\sigma \approx 0,\ \mu = \mu_0\mu_r,\ \varepsilon = \varepsilon_0\varepsilon_r$

Attenuation Coefficient: $\alpha = 0$

No attenuation

Phase Constant: $\beta = \omega\sqrt{\mu\varepsilon}$

Impedance: $\eta = \sqrt{\frac{\mu}{\varepsilon}} = \eta_0\sqrt{\frac{\mu_r}{\varepsilon_r}}$ $\quad |\eta| = \sqrt{\mu/\varepsilon}$ $\quad \measuredangle\eta = 0$

$\vec{H}$ is ~3× small than $\vec{E}$.

$\vec{E}$ and $\vec{H}$ are in phase

Notes:
- Most commonly analyzed, due to easy math.
- Usually a good approximation for dielectrics.
- Not physically real, except in vacuum. All materials have loss.

EMPossible

Slide 29

29

Summary of Waves in Good Conductors

Condition: $\sigma \gg \omega\varepsilon$

Fundamental Parameters: $\sigma,\ \mu = \mu_0\mu_r,\ \varepsilon = \varepsilon_0\varepsilon_r$

Attenuation Coefficient: $\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$ Strong attenuation

Phase Constant: $\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$

Impedance: $\tilde{\eta} = \sqrt{\frac{j\omega\mu}{\sigma}}$ $\quad |\eta| = \sqrt{\frac{\omega\mu}{\sigma}}$ $\quad \measuredangle\eta = 45°$

$\vec{E}$ and $\vec{H}$ are out of phase.

Notes:
- Very strong attenuation.
- Waves tend to reflect from good conductors so often do not experience the loss.
- $\vec{E}$ leads $\vec{H}$ by 45°.

EMPossible

Slide 30

30